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Chaos synchronization between two different 4D hyperchaotic Chen systems*

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This paper presents chaos synchronization between two different four-dimensional (4D) hyperchaotic Chen systems by nonlinear feedback control laws. A modified 4D hyperchaotic Chen system is obtained by changing the nonlinear function of the 4D hyperchaotic Chen system, furthermore, an electronic circuit to realize two different 4D hyperchaotic Chen systems is designed. With nonlinear feedback control method, chaos synchronization between two different 4D hyperchaotic Chen systems is achieved. Based on the stability theory, the functions of the nonlinear feedback control for synchronization of two different 4D hyperchaotic Chen systems is derived, the range of feedback gains is determined. Numerical simulations are shown to verify the theoretical results.

Keywords: nonlinear feedback control, chaos synchronization, the 4D hyperchaotic Chen system
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1. Introduction

Chaotic attractor is a well-known phenomenon in physics, engineering, and many other scientific disciplines. In 1963, Lorenz found the first chaotic attractor in the three-dimensional (3D) autonomous chaotic system when he studied a model of the Earth's atmospheric convection flow heated from below and cooled from above;[1] such system is called the Lorenz system. As the first chaotic model, the Lorenz system has become a paradigm of chaos research. On the other hand, the Lorenz system has been used as an important model for studying chaos because of its diversiform dynamic behaviours.[2−8] Based on the Lorenz system, a number of modified chaotic systems were proposed in succession. Chen and Lü presented Chen's system[9] and Lü's system[10] with changing system parameters. They belong to the Lorenz system family, but are not topologically equivalent to each other.[11] Very recently, Liu proposed a new 3D chaotic system which is similar to the Lorenz system.[12] Cuomo obtained 4D and 5D Lorenz systems with the synthesizing method.[13] The investigation results show that the high-dimensional chaotic system (more than 3D) produces the hyperchaos[14−18] which can be used to secure communication.[19] It is a meaningful job to construct a high-dimensional chaos system and to achieve chaos synchronization. In recent years, chaos synchronization between two different chaotic systems has been widely noticed.[20−26] It is believed that secure communications via chaos synchronization in the different chaotic systems are more secure on account of their better flexibility.

This work constructs a modified 4D hyperchaotic Chen system by changing nonlinear property of the 4D hyperchaotic Chen system.[15] At the same time, a circuit is designed to achieve these systems. We can easily transform the 4D hyperchaotic Chen system into the modified 4D hyperchaotic Chen system by operating a switch on the circuit. Therefore the circuit can achieve the function of two different 4D hyperchaotic Chen systems. By analysing the property of these systems, we find a way to realize the chaos synchronization between two different 4D hyperchaotic Chen systems by means of the method of nonlinear feedback control. On the stable theory of systems, a nonlinear feedback controller is constructed for chaos synchronization. Furthermore, a range of feedback gain is obtained. The numerical results have been given to verify the theoretical results of this paper. It proves

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that more flexibility and convenience are the main advantages of this method.

2. System constitution and property

The system in Ref.[15] is a typical 4D autonomous nonlinear system. Because of the restriction in the acceptable voltage of amplifier and multiplier, we must decrease the amplitude of every variable of the hyperchaotic Chen system. According to Refs.[15,16], the 4D hyperchaotic Chen system equation is described as follows:

$$\begin{cases} \dot{x}_1 = -ax_1 + ay_1 + w_1, \\ \dot{y}_1 = dx_1 + cy_1 - 10x_1z_1, \\ \dot{z}_1 = -bz_1 + 10x_1y_1, \\ \dot{w}_1 = rw_1 + 10y_1z_1, \end{cases} \tag{1}$$

where x_1, y_1, z_1 and w_1 are state variables, and a, b, c, d and r are the real constants.

When $a = 35, b = 3, c = 12, d = 7$ and $r = 0.7$, the system (1) is hyperchaotic.[15] If the parameters of system (1) are kept to be constant and a variable function of the system (1) is changed,[12] a modified 4D hyperchaotic Chen system is obtained

$$\begin{cases} \dot{x}_2 = -ax_2 + ay_2 + w_2, \\ \dot{y}_2 = dx_2 + cy_2 - 10x_2z_2, \\ \dot{z}_2 = -bz_2 + 10x_2^2, \\ \dot{w}_2 = rw_2 + 10y_2z_2. \end{cases} \tag{2}$$

Let $a = 35, b = 3, c = 12, d = 7$ and $r = 0.7$, the modified 4D Chen system (2) is hyperchaotic too. Its Lyapunov exponent can be obtained

$$\lambda_{\mathrm{L}1} = 1.5916, \quad \lambda_{\mathrm{L}2} = 0.2852,$$

$$\lambda_{\mathrm{L}3} = 0, \quad \lambda_{\mathrm{L}4} = -11.6174.$$

Two 4D hyperchaotic Chen systems with cross correlation have same parameters but different forms of nonlinear functions. Because every system has different nonlinear property, their manifold evolutions in phase space are determined to be totally different. Figures 1 and 2 show the parts of phase pictures of the projection x–y, x–z, x–w, z–w of the chaotic attractor when the parameters of two 4D hyperchaotic Chen systems are $a = 35, b = 3, c = 12, d = 7$ and $r = 0.7$.

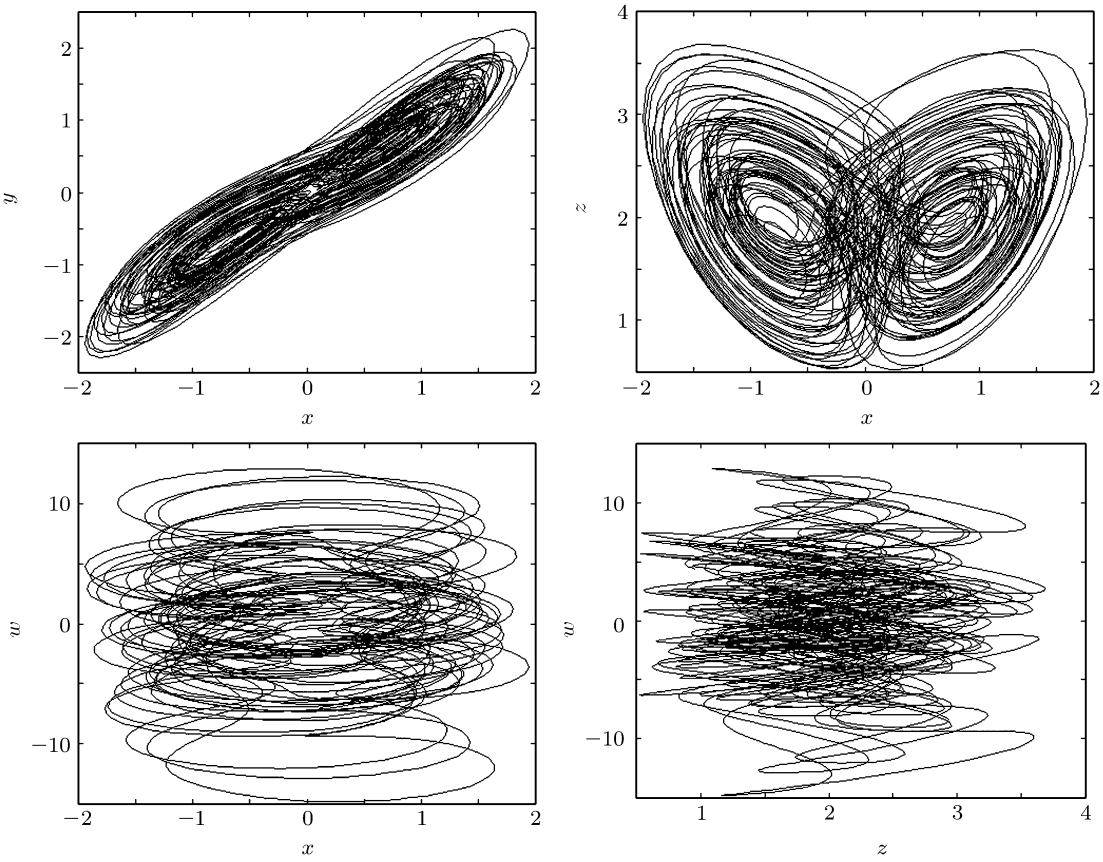


Fig.1. Phase portrait of hyperchaotic system (1).

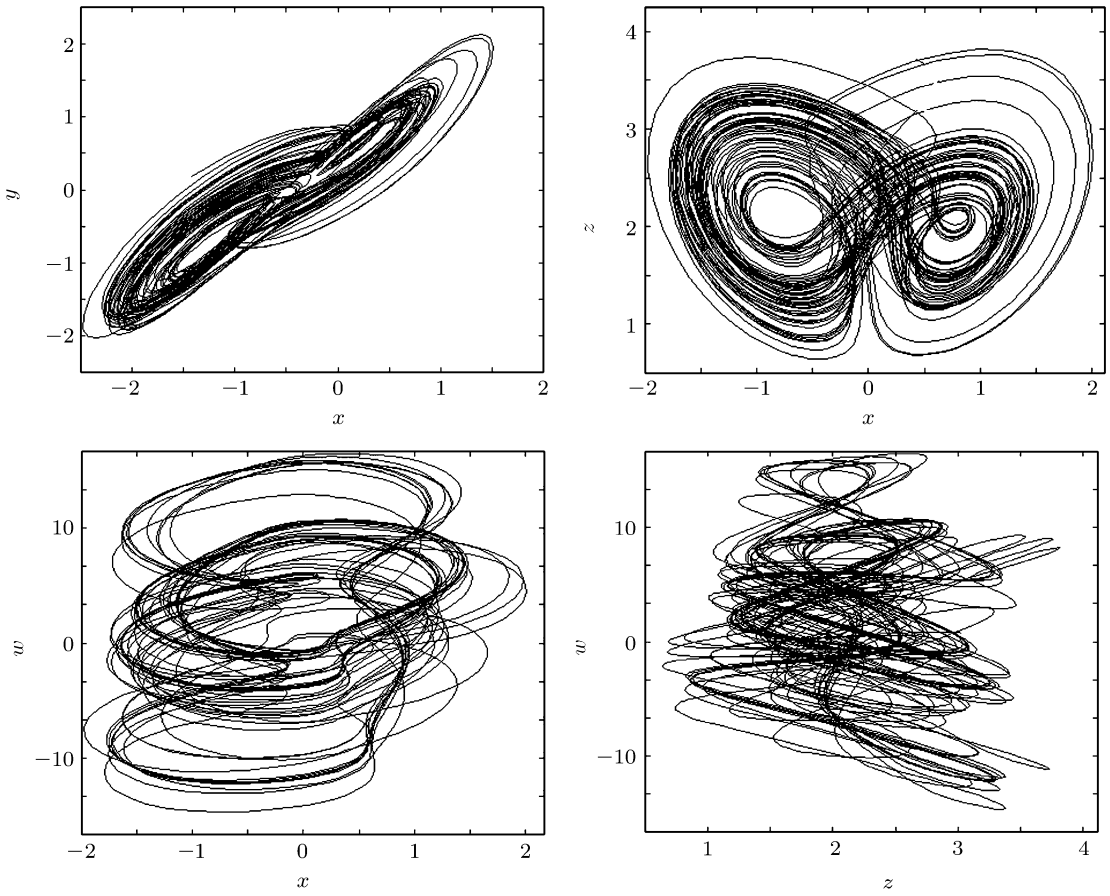


Fig.2. Phase portrait of hyperchaotic system (2).

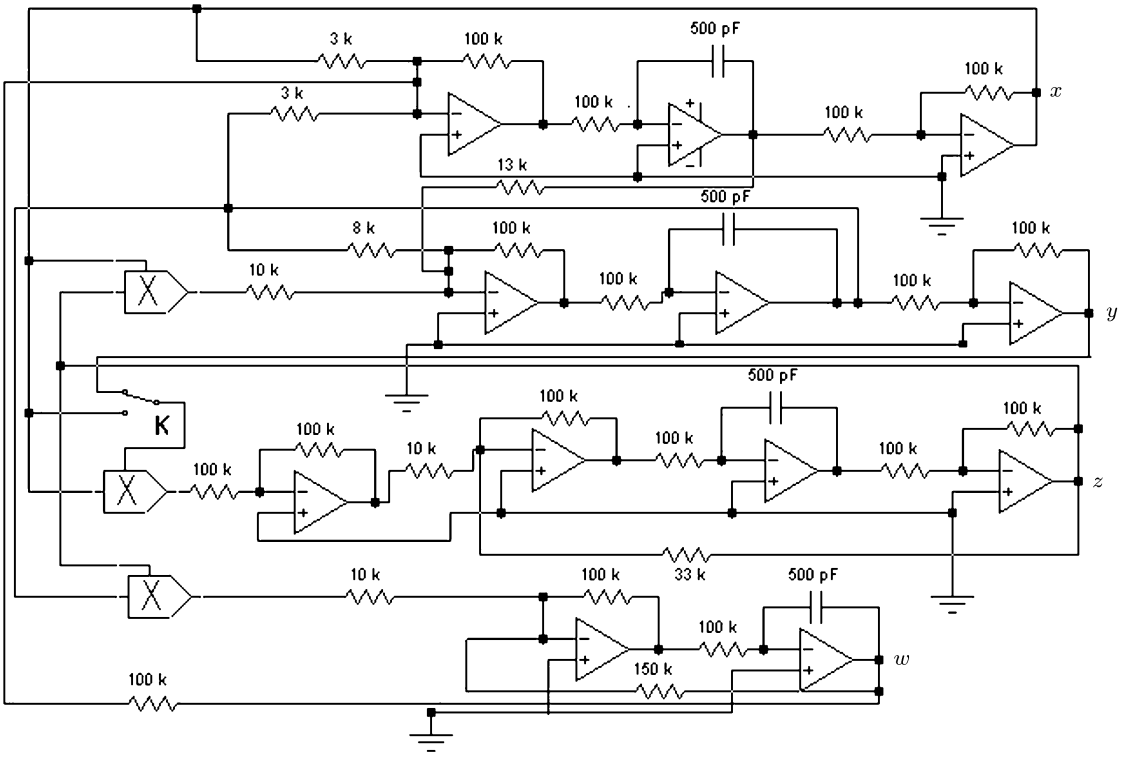


Fig.3. The actual circuit of two four-dimensional hyperchaotic Chen systems.

It is notable that the above-mentioned two 4D hyperchaotic Chen systems are different but tightly correlative. The nonlinear functions of system (1) and system (2) are the functions of system variables as multipliers or quadratic, so they can be realized by analogous multiplier easily. Since two 4D hyperchaotic Chen systems with cross correlation have same parameters but different forms of nonlinear functions. We can design an electronic circuit for realizing two 4D hyperchaotic Chen systems. Two hyperchaotic systems can be altered only through operating the switch. The actual circuit is shown as Fig.3. Switching the on-off K can change the function of system variables xy or x^2. It can be seen that the operation is very convenient.

3. Chaos synchronization

Let the 4D hyperchaotic Chen system (1) be the drive system; the response system is the modified 4D hyperchaotic Chen system (2). Using nonlinear feedback control method to achieve the chaos synchronization between two different 4D hyperchaotic Chen systems, and adding nonlinear feedback control functions on the response system, we obtain that the equation of the response system is

$$\begin{cases} \dot{x}_2 = -ax_2 + ay_2 + w_2 + u_1, \\ \dot{y}_2 = dx_2 + cy_2 - 10x_2z_2 + u_2, \\ \dot{z}_2 = -bz_2 + 10x_2^2 + u_3, \\ \dot{w}_2 = rw_2 + 10y_2z_2 + u_4, \end{cases} \tag{3}$$

where $u_i(t)(i = 1, 2, 3, 4)$ is the nonlinear feedback control functions. Let us define the state errors $e_1 = x_1 - x_2,\ e_2 = y_1 - y_2,\ e_3 = z_1 - z_2,\ e_4 = w_1 - w_2$, the equation of error system can be obtained from Eqs.(1) and (3)

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + e_4 - u_1, \\ \dot{e}_2 = (d - 10z_2)e_1 + ce_2 - 10x_1e_3 - u_2, \\ \dot{e}_3 = 10(x_1y_1 - x_1^2) + 20x_1e_1 - 10e_1^2 - be_3 - u_3, \\ \dot{e}_4 = 10z_2e_2 + 10y_1e_3 + re_4 - u_4. \end{cases} \tag{4}$$

Set the nonlinear feedback control functions $u_i(t)(i = 1, 2, 3, 4)$ as

$$\begin{cases} u_1 = 0, \\ u_2 = (d - 10z_2)e_1 + k_2e_2, \\ u_3 = 10(x_1y_1 - x_1^2) + 20x_1e_1, \\ u_4 = 10z_2e_2 + 10y_1e_3 + k_4e_4, \end{cases} \tag{5}$$

where k_2, k_4 are feedback gains of the nonlinear feedback control.

Substituting Eq.(5) into Eq.(4) and transferring to the form as $\dot{x} = A(t)x + O(x, t)$,[23] we get the equation of error system

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{pmatrix} = \begin{bmatrix} -a & a & 0 & 0 \\ 0 & c - k_2 & -10x_1 & 0 \\ 0 & 0 & -b & 0 \\ 0 & 0 & 0 & r - k_4 \end{bmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -10e_1^2 \\ 0 \end{pmatrix}. \tag{6}$$

Theorem The different 4D hyperchaotic system (1) and (3) are synchronized completely on the nonlinear feedback control functions (5), when feedback gains are $k_2 > c$ and $k_4 > r$.

Proof It is known from Eq.(6), the zero of error variable is the equilibrium of the error system. According to the linear stability theorem,[23] the linear matrix of the error system denoted by Eq.(6) is

$$A = \begin{bmatrix} -a & a & 0 & 0 \\ 0 & c - k_2 & -10x_1 & 0 \\ 0 & 0 & -b & 0 \\ 0 & 0 & 0 & r - k_4 \end{bmatrix}. \tag{7}$$

It is a time-varying matrix. When the system is in a chaotic state, system variables are varying in the chaos region, thus $A(t)$ is bound for all of t. Because of system parameters a, b, c, d and r are positive, when feedback gains $k_2 > c$ and $k_4 > r$, matrix $A(t)$ has all eigenvalues with negative real parts, so the zero answer of linear system $\dot{e} = A(t)e$ is uniformly asymptotically stable.

The nonlinear term of error system (6) $O(e, t) = [0\ \ 0\ \ -10e_1^2\ \ 0]^{\mathrm{T}}$ and $\lim\limits_{\|e\| \to 0} \dfrac{\|O(e, t)\|}{\|e\|} = 0$ are satisfied for any value of t. This guarantees the asymptotic stability of the error system (6), which implies that system (1) and system (3) is chaos synchronization at the control of the nonlinear feedback. In such a way the theorem is proved.

In numerical experiment, we take the parameters of two 4D hyperchaotic Chen systems to be $a = 35, b = 3, c = 12, d = 7$ and $r = 0.7$ while the initial values of the drive and response systems are

$x_1(0) = y_1(0) = z_1(0) = w_1(0) = 0.6$, $x_2(0) = y_2(0) = z_2(0) = w_2(0) = 0.1$ respectively. The fourth order Runge–Kutta integration method is used to solve two systems of differential equations (1) and (3). In addition, a time step size 0.01 is employed. In the effect of nonlinear feedback control functions, two different 4D hyperchaotic Chen systems achieve chaos synchronization immediately. Figure 4 is the dynamics of synchronization error e_1, e_2, e_3, e_4 for different drive system and response system when the feedback gain $k_2 = 12.5$ and $k_4 = 0.8$.

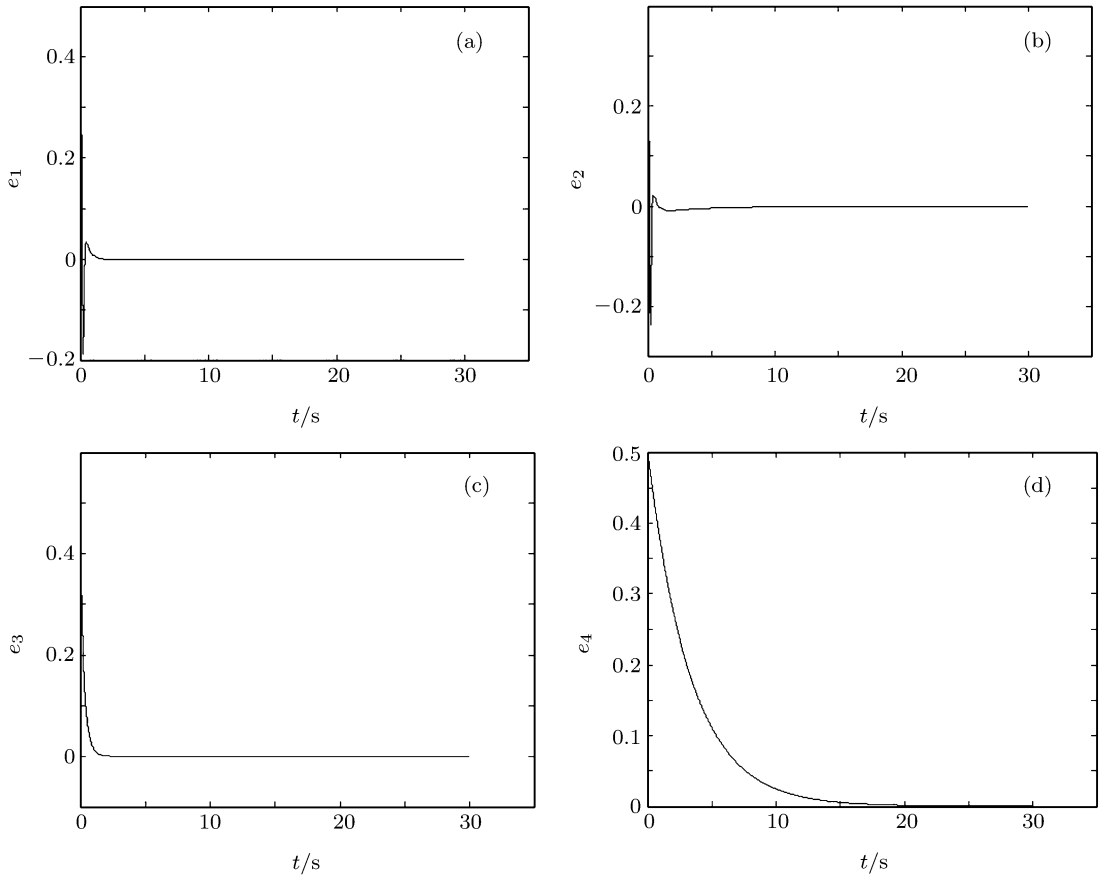


Fig.4. Controlling different systems synchronization.

4. Conclusions

A modified 4D hyperchaotic Chen system is constructed and realization is flexible and convenient. Using nonlinear feedback control method, we can realize the chaos synchronization between two different 4D hyperchaotic Chen systems. The nonlinear feedback control method for chaos synchronization is simple, and realization is easy. This secure communication system with chaotic synchronization based on multi-system switching is better in security than that based on single chaotic system.

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