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The solution of the time-dependent Fokker–Planck equation of non-degenerate optical parametric amplification and its application to the optimum realization of EPR paradox*

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In this paper, the solution of the time-dependent Fokker–Planck equation of non-degenerate optical parametric amplification is used to deduce the condition demonstrating the Einstein–Podolsky–Rosen (EPR) paradox. The analytics and numerical calculation show the influence of pump depletion on the error in the measurement of continuous variables. The optimum realization of EPR paradox can be achieved by adjusting the parameter of squeezing. This result is of practical importance when the realistic experimental conditions are taken into consideration.

Keywords: the non-degenerate optical parametric amplification (NOPA), the time-dependent Fokker–Planck equation, EPR paradox

PACC: 0367, 4250, 4265K

1. Introduction

In 1935, Einstein, Podolsky, and Rosen first brought forward that quantum mechanics does not provide a complete description of the physical reality, and they argued that in the measurement of correlations of continuous observables of two separated particles, quantum mechanics results in contradiction with physics reality, it is called the EPR paradox.[1] In 1951, Bohm presented a modified discrete version of the paradox; he considered a pair of spatially separated spin 1/2 particles which show high correlation between their various spin components[2,3] and later stimulated Bell to derive his famous inequalities, now called the Bell inequalities.[4,5] Over the past twenty years, a variety of experiments have investigated the violations by quantum mechanics of the constraints imposed by local realism as codified by the Bell inequalities, these measurements have followed Bohm's suggestion and the Bell inequalities are applicable, but they are not the original EPR paradox of continuous observables. On account of this, in 1988, Reid pointed out the experimental possibility of demonstrating of the original EPR paradox in the non-degenerate optical parametric amplification (NOPA) system.[6] Subsequently, a detail analysis and derivation was given.[7,8] Later, the original EPR paradox was demonstrated experimentally by Ou *et al* as the product of variances $V_1V_2 = \Delta^2_{\rm inf}X_1\Delta^2_{\rm inf}X_2 = 0.70 \pm 0.01$,[9,10] Silberhorn *et al* obtained $V_1V_2 = 0.64 \pm 0.08$,[11] and some other investigators obtained $V_1V_2 = 0.727$ and 0.63.[12–17] Reid's argument is based on the Langevin equation, where the pump is assumed to be un-depleted and of sufficient intensity that it may be modelled classically. In fact, near the threshold region, quantum fluctuation is very large, the nonlinear effect becomes dominating, and the influence of pump depletion cannot be neglected. In this paper, we use the solution of the time-dependent Fokker–Planck equation for NOPA under the consideration of pump depletion to deduce the optimum condition demonstration of the EPR paradox. The numerical calculation shows that the optimum realization of EPR paradox can be achieved by adjusting the degree of squeezing and this is the limit to demonstration of the EPR paradox for a certain finite loss k. The results obtained is compared to that of the ideal case (no loss $k = 0$), the optimum realization appears with squeezed parameter $\bar{r}$ very

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large.[7]

2. The time-dependent Fokker–Planck equation and the solution for NOPA system

We consider the non-degenerate parametric amplifier (the two modes with the same frequencies but different polarizations), and prove that a NOPA system is equivalent to two DOPA systems with different parameters.

The total Hamiltonian describing this system may be written as

$$H = H_0 + V + W,$$

where

$$\begin{aligned} H_0 &= \hbar\omega a_1^\dagger a_1 + \hbar\omega a_2^\dagger a_2 + \sum \hbar\omega_j B_j^\dagger B_j, \\ V &= \hbar \sum k_j B_j \frac{a_1^\dagger + a_2^\dagger}{2} + \hbar \sum k_j^* B_j^\dagger \frac{a_1 + a_2}{2}, \\ W &= \frac{\mathrm{i}\hbar}{2}(\varepsilon a_1^\dagger a_2^\dagger - \varepsilon^* a_1 a_2). \end{aligned} \tag{1}$$

Here $a_1(a_1^\dagger)$ and $a_2(a_2^\dagger)$ denote the creation and annihilation operators of the signal and idler field, respectively. Because of frequency degenerate, we have $\hbar\omega_1 = \hbar\omega_2 = \hbar\omega$. But the signal and idler field polarizations are non-degenerate (such as left and right rotation circular polarization or x-, y- directional linear polarization). The former two terms in H_0 correspond to the Hamiltonian of the signal and idler light, the third term describes the Hamiltonian of the heat bath. $B_j(B_j^\dagger)$ describes the creation and annihilation operators. V indicates the interaction Hamiltonian of the signal, idler field and the heat bath. W represents the interaction Hamiltonian between the pump field (included in the gain factor $\varepsilon = \chi\alpha_0$) and the signal, idler photons $a_1^\dagger$ and $a_2^\dagger$. $a_1(a_1^\dagger)$ and $a_2(a_2^\dagger)$ belong to different polarization modes, which are commutative. Setting

$$\alpha_1 = \frac{b_1 + b_2}{\sqrt{2}}, \quad \alpha_2 = \frac{b_1 - b_2}{\sqrt{2}}$$

$$\begin{aligned} b_1 b_1^\dagger - b_1^\dagger b_1 = &\left(\frac{a_1 + a_2}{\sqrt{2}}\right)\left(\frac{a_1^\dagger + a_2^\dagger}{\sqrt{2}}\right) \\ &- \left(\frac{a_1^\dagger + a_2^\dagger}{\sqrt{2}}\right)\left(\frac{a_1 + a_2}{\sqrt{2}}\right) \\ = &\ 1, \\ b_2 b_2^\dagger - b_2^\dagger b_2 = &\ 1, \end{aligned}$$

but

$$b_1 b_2^\dagger - b_2^\dagger b_1 = b_2 b_1^\dagger - b_1^\dagger b_2 = 0, \tag{2}$$

on account of $b_1 = (a_1 + a_2)/\sqrt{2}$, $\quad b_2 = (a_1 - a_2)/\sqrt{2}$, thus we may call b_1 and b_2 the in-phase and out-phase state operators, respectively. It is easy to prove

$$\begin{aligned} V &= \hbar \sum k_j B_j \left(\frac{a_1^\dagger + a_2^\dagger}{2}\right) + \hbar \sum k_j^* B_j^\dagger \left(\frac{a_1 + a_2}{2}\right) \\ &= \hbar\left(\sum \frac{k_j}{\sqrt{2}} B_j b_1^\dagger + \sum \frac{k_j^*}{\sqrt{2}} B_j^\dagger b_1\right) = V_1, \\ W &= \frac{\mathrm{i}\hbar}{4}(\varepsilon b_1^{\dagger 2} - \varepsilon^* b_1^2) - \frac{\mathrm{i}\hbar}{4}(\varepsilon b_2^{\dagger 2} - \varepsilon^* b_2^2) \\ &= W_1 + W_2 \end{aligned} \tag{3}$$

Comparing Eq.(1) with Eq.(3), we can immediately arrive at that a non-degenerate parametric down-conversion system (ε, k) may be looked as a combination of two coherent degenerate parametric down-conversion system 1 $(\varepsilon/2, k/\sqrt{2})$ and system 2 $(-\varepsilon/2, 0)$.

The NOPA system density matrix ρ can be expressed by the product of the DOPA system 1 density matrix ρ_1 and the DOPA system 2 density matrix ρ_2, namely $\rho = \rho_1\rho_2$. In the positive-P representation, the quasi-probability p for NOPA system can be written as the product of the quasi-probability p_1 and the quasi-probability p_2 for the coherent DOPA1 and DOPA 2 system, respectively, namely $p = p_1 p_2$. Furthermore, the Fokker–Planck equation for NOPA system can be written as combined system of DOPA 1 and DOPA 2:[18]

$$\begin{aligned} \frac{\partial p_1}{\partial t} &= \left[\frac{k}{\sqrt{2}}\left(\frac{\partial}{\partial b_1} b_1 + \frac{\partial}{\partial b_1^*} b_1^*\right) - \left(\frac{\varepsilon}{2} b_1^* \frac{\partial}{\partial b_1} + \frac{\varepsilon^*}{2} b_1 \frac{\partial}{\partial b_1^*}\right) + \frac{1}{2}\left(\frac{\varepsilon}{2}\frac{\partial^2}{\partial b_1^2} + \frac{\varepsilon^*}{2}\frac{\partial^2}{\partial b_1^{*2}}\right)\right] p_1, \\ \frac{\partial p_2}{\partial t} &= \left[\left(\frac{\varepsilon}{2} b_2^* \frac{\partial}{\partial b_2} + \frac{\varepsilon^*}{2} b_2 \frac{\partial}{\partial b_2^*}\right) - \frac{1}{2}\left(\frac{\varepsilon}{2}\frac{\partial^2}{\partial b_2^2} + \frac{\varepsilon^*}{2}\frac{\partial^2}{\partial b_2^{*2}}\right)\right] p_2. \end{aligned} \tag{4}$$

Usually, the Fokker–Planck equation for DOPA has the following form

$$\frac{\partial p}{\partial t}=\left[k\left(\frac{\partial}{\partial\alpha}\alpha+\frac{\partial}{\partial\alpha^*}\alpha^*\right)-\varepsilon\left(\alpha^*\frac{\partial}{\partial\alpha}+\alpha\frac{\partial}{\partial\alpha^*}\right)+\frac{\varepsilon}{2}\left(\frac{\partial^2}{\partial\alpha^2}+\frac{\partial^2}{\partial\alpha^{*2}}\right)\right]p. \tag{5}$$

Thus the solution of DOPA system 1 and 2 can be written immediately as

$$p_1=p\left(\frac{k}{\sqrt{2}},\ \frac{\varepsilon}{2},\ \alpha_1\right),\ \text{and}\ p_2=p\left(0,\ -\frac{\varepsilon}{2},\ \alpha_2\right). \tag{6}$$

When the pump depletion is neglected, the pump amplitude α_0 and therefore the gain coefficient $\varepsilon=\chi\alpha_0$ is constant, and the NOPA (ε,k) system is time-independent. However, considering pump depletion $\alpha_0=\alpha_0(\tau)$. NOPA (ε,k) and DOPA1 $(\varepsilon/2,k/\sqrt{2})$, DOPA2 $(-\varepsilon/2,0)$ are all time-dependent, where $\varepsilon=\chi\alpha_0(\tau)$.

In order to determine $\alpha_0(\tau)$, we should study the interaction between pump wave and parametric waves. In the positive-P representation, substitute the operators b_1,b_2 by the c numbers α_1,α_2, we derive the semi-classical equations describing the interaction between the pump wave α_0 and two degenerate parametric modes α_1,α_2,[19] with the initial value $\alpha_{10}=\alpha_{20}=1,\alpha_{00}=1$, as follows

$$\begin{aligned}\frac{\mathrm{d}\alpha_1}{\mathrm{d}\tau}&=-\alpha_1+\frac{\chi}{\gamma_1}\alpha_0\alpha_1^*,\\ \frac{\mathrm{d}\alpha_2}{\mathrm{d}\tau}&=-\frac{\chi}{\gamma_1}\alpha_0\alpha_2^*,\\ \frac{\mathrm{d}\alpha_0}{\mathrm{d}\tau}&=-\gamma_r\alpha_0+\frac{\bar{\varepsilon}}{\gamma_1}-\frac{\chi}{2\gamma_1}\left(\frac{\alpha_1^2-\alpha_2^2}{2}\right).\end{aligned} \tag{7}$$

Here we use the dimensionless time $\tau=kt/\sqrt{2}=\gamma_1 t$, and $\gamma_{\mathrm{r}}=\gamma_0/\gamma_1,\gamma_0$ is loss of pump α_0. Integrating Eq.(5) numerically, the function of $\alpha_0=\alpha_0(\tau)$ is determined.

Following the method of solution of time-dependent Fokker–Planck given in previous paper,[20] we write out the variances of the quantum fluctuation of the real part Δx_1 and imaginary part Δy_1 as follows:

$$\begin{aligned}\langle(\Delta x_1)^2\rangle&=\frac{1}{4}+\frac{\langle(\Delta\beta_1)^2\rangle}{2}=\frac{1}{4}+(A_1-A_{10}\alpha_1^2),\\ \langle(\Delta y_1)^2\rangle&=\frac{1}{4}+\frac{\langle(\Delta i\tilde{\beta}_1)^2\rangle}{2}=\frac{1}{4}+(\tilde{A}_1-\tilde{A}_{10}\tilde{\alpha}_1^2),\end{aligned} \tag{8}$$

where

$$A_1=\alpha_1^2(A_{10}+\frac{1}{4})-\frac{1}{4}+\frac{\alpha_1^2k}{2\sqrt{2}}\int_0^t\alpha_1^{-2}\mathrm{d}t',$$

$$\tilde{A}_1=\tilde{\alpha}_1^2\left(\tilde{A}_{10}+\frac{1}{4}\right)-\frac{1}{4}+\frac{\tilde{\alpha}_1^2k}{2\sqrt{2}}\int_0^t\tilde{\alpha}_1^{-2}\mathrm{d}t',$$

$$\alpha_1(t)=\exp\left[-\int_0^t\left(\frac{k}{\sqrt{2}}-\frac{\varepsilon}{2}\right)\mathrm{d}t'\right],$$

and

$$\begin{aligned}\tilde{\alpha}_1(t)=&\ \exp\left[-\int_0^t\left(\frac{k}{\sqrt{2}}+\frac{\varepsilon}{2}\right)\mathrm{d}t'\right]\\ &=\exp\left[-2\int_0^t\frac{k}{\sqrt{2}}\mathrm{d}t'\right]\alpha_1^{-1}(t).\end{aligned}$$

Similarly, the variances of the quantum fluctuation Δx_2 and Δy_2 read

$$\begin{aligned}\langle(\Delta x_2)^2\rangle&=\frac{1}{4}+\frac{\langle(\Delta\beta_2)^2\rangle}{2}=\frac{1}{4}+(A_2-A_{20}\alpha_2^2),\\ \langle(\Delta y_2)^2\rangle&=\frac{1}{4}+\frac{\langle(\Delta i\tilde{\beta}_2)^2\rangle}{2}=\frac{1}{4}+(\tilde{A}_2-\tilde{A}_{20}\tilde{\alpha}_2^2),\end{aligned} \tag{9}$$

where

$$\alpha_2(t)=\exp\left[-\int_0^t\frac{\varepsilon}{2}\mathrm{d}t'\right],$$

$$\tilde{\alpha}_2(t)=\exp\left[\int_0^t\frac{\varepsilon}{2}\mathrm{d}t'\right]=\alpha_2^{-1}(t),$$

$A_2=\alpha_2^2\left(A_{20}+\frac{1}{4}\right)-\frac{1}{4}$, and $\tilde{A}_2=\tilde{\alpha}_2^2\left(\tilde{A}_{20}+\frac{1}{4}\right)-\frac{1}{4}$. Furthermore, the quantum fluctuation of the time-dependent linearly driven NOPA system are

$$\begin{aligned}\langle(\Delta X_1)^2\rangle&=\frac{1}{2}\left(4\langle(\Delta x_1)^2\rangle+4\langle(\Delta x_2)^2\rangle\right)\\ &=1+2(A_1-A_{10}\alpha_1^2)+2(A_2-A_{20}\alpha_2^2),\\ \langle(\Delta Y_2)^2\rangle&=\frac{1}{2}\left(4\langle(\Delta y_1)^2\rangle+4\langle(\Delta y_2)^2\rangle\right)\\ &=1+2(\tilde{A}_1-\tilde{A}_{10}\tilde{\alpha}_1^2)+2(\tilde{A}_2-\tilde{A}_{20}\tilde{\alpha}_2^2).\end{aligned} \tag{10}$$

The factor 1/2 introduced manifests the probabilities of the system, half in the 'in phase state' and half in the 'out of phase state'. Substituting Eqs.(8) and (9) into Eq.(10), the quantum fluctuation of the time-dependent linearly driven NOPA system are

$$\begin{aligned}\langle(\Delta X_1)^2\rangle&=\frac{\alpha_1^2(t)}{2}+\frac{\alpha_2^2(t)}{2}+kf(t),\\ \langle(\Delta Y_2)^2\rangle&=\frac{\tilde{\alpha}_1^2(t)}{2}+\frac{\tilde{\alpha}_2^2(t)}{2}+k\tilde{f}(t),\end{aligned} \tag{11}$$

where

$$f(t)=\alpha_1^2(t)\int_0^t\alpha_1^{-2}(t')\mathrm{d}t',$$

and

$$\tilde{f}(t)=\tilde{\alpha}_1^2(t)\int_0^t\tilde{\alpha}_1^{-2}(t')\mathrm{d}t'.$$

The solutions X_1, Y_2, satisfying Eq.(11) are

$$\begin{aligned} X_1 = & X_{10}\frac{\alpha_1(t)+\alpha_2(t)}{2} - Y_{20}\frac{\alpha_1(t)-\alpha_2(t)}{2} \\ & + \int_0^t a(t')\xi(t')\mathrm{d}t', \\ Y_2 = & Y_{20}\frac{\tilde{\alpha}_1(t)+\tilde{\alpha}_2(t)}{2} + X_{10}\frac{\tilde{\alpha}_1(t)-\tilde{\alpha}_2(t)}{2} \\ & + \int_0^t \tilde{a}(t')\xi(t')\mathrm{d}t', \end{aligned} \tag{12}$$

where

$$a(t') = \exp\left[-\int_{t'}^t (k/\sqrt{2}-\varepsilon/2)\mathrm{d}t\right],$$

$$\tilde{a}(t') = \exp\left[-\int_{t'}^t (k/\sqrt{2}+\varepsilon/2)\mathrm{d}t\right].$$

$\xi(t)$ is the random force. $\langle\xi(t')\xi(t'')\rangle = k\delta(t'-t'')$, X_{10}, Y_{20} are the vacuum fluctuations, with $\langle X_{10}\rangle = \langle Y_{20}\rangle = 0$, $\langle X_{10}^2\rangle = \langle Y_{20}^2\rangle = 1$.

Using Eq.(12), we can calculate the variance

$$\begin{aligned} \langle(\Delta X_1)^2\rangle = & \langle X_1^2\rangle - \langle X_{10}^2\rangle = \langle X_1^2\rangle = \frac{\alpha_1^2(t)}{2} + \frac{\alpha_2^2(t)}{2} \\ & + \int_0^t\int_0^t a(t')a(t'')\xi(t')\xi(t'')\mathrm{d}t'\mathrm{d}t'' \\ = & \frac{\alpha_1^2(t)}{2} + \frac{\alpha_2^2(t)}{2} + kf(t). \end{aligned} \tag{13}$$

Thus Eq.(12) is the solution of Eq.(11). Using Eq.(12) we also obtain the correlation between X_1 and Y_2 as follows:

$$\begin{aligned} \langle X_1Y_2\rangle = & \left(\frac{\alpha_1(t)+\alpha_2(t)}{2}\right)\times\left(\frac{\tilde{\alpha}_1(t)-\tilde{\alpha}_2(t)}{2}\right) \\ & - \left(\frac{\alpha_1(t)-\alpha_2(t)}{2}\right)\times\left(\frac{\tilde{\alpha}_1(t)+\tilde{\alpha}_2(t)}{2}\right) \\ & + \frac{1}{2}(1-\exp[-\sqrt{2}kt]). \end{aligned} \tag{14}$$

For simplification, we set $\mathrm{d}\tau = k\mathrm{d}t/\sqrt{2}$, $1/\eta = \varepsilon/2/k/\sqrt{2}$, so

$$\begin{aligned} \alpha_1(\tau) = & \exp\left(-\int_0^\tau\left(\frac{k}{\sqrt{2}}-\frac{\varepsilon}{2}\right)\mathrm{d}t\right) \\ = & \exp\left(-\int_0^\tau\left(1-\frac{1}{\eta}\right)\mathrm{d}\tau'\right). \end{aligned} \tag{15}$$

According to theorem of the mean value, the integrate $\int_0^\tau \mathrm{d}\tau'/\eta$ can be expressed as: $\tau/\bar{\eta} = \int_0^\tau \mathrm{d}\tau'/\eta$, and Eq.(15) as

$$\alpha_1(\tau) = \exp\left(-\tau+\frac{1}{\bar{\eta}}\tau\right),$$

where

$$\frac{1}{\bar{\eta}}\tau = \ln(\exp(\tau)\alpha_1(\tau)). \tag{16}$$

Considering $\tau = kt/\sqrt{2} = \eta\times\varepsilon t/2 = \eta r/2$, namely $r = \varepsilon t = 2\tau/\eta$, we have

$$\bar{r} = \frac{2\tau}{\bar{\eta}} = 2\ln(\exp(\tau)\alpha_1(\tau)). \tag{17}$$

The product of gain ε and interaction time t, r is squeezed parameter, which determines the squeezing degree of the field. In short, for a time-dependent system, the corresponding squeezed parameter $\bar{r}$ can be expressed as a function of dimensionless time τ and the numerical solution of $\alpha_1(\tau)$. Similarity $\tilde{\alpha}_1(\tau), \alpha_2(\tau)$, and $\tilde{\alpha}_2(\tau)$ can be expressed in terms of τ and $\bar{r}$ as shown below

$$\begin{aligned} & \alpha_1(\tau) = \exp\left(-\tau+\frac{\bar{r}}{2}\right), \quad \tilde{\alpha}_1(\tau) = \exp\left(-\tau-\frac{\bar{r}}{2}\right), \\ & \alpha_2(\tau) = \exp\left(-\frac{\bar{r}}{2}\right), \quad \tilde{\alpha}_2(\tau) = \exp\left(\frac{\bar{r}}{2}\right). \end{aligned} \tag{18}$$

Substituting Eq.(18) into Eqs.(11) and (13), we derive

$$\begin{aligned} \langle X_1^2\rangle = & \frac{1}{2}(\exp(-2\tau+\bar{r})+\exp(-\bar{r})) \\ & + \exp(-2\tau+\bar{r})\int_0^\tau \exp(2\tau'-\bar{r})\mathrm{d}\tau', \\ \langle Y_2^2\rangle = & \frac{1}{2}(\exp(-2\tau-\bar{r})+\exp(\bar{r})) \\ & + \exp(-2\tau-\bar{r})\int_0^\tau \exp(2\tau'+\bar{r})\mathrm{d}\tau', \\ \langle X_1Y_2\rangle = & \frac{1}{2}[\exp(-\tau-\bar{r})-\exp(-\tau+\bar{r}) \\ & +(1-\exp(-2\tau))]. \end{aligned} \tag{19}$$

Substituting the above results into the formula evaluating the variance V_1 and V_2[7] yield

$$\begin{aligned} V_1 = \Delta_{\mathrm{inf}}^2 X_1 = \langle X_1^2\rangle - \frac{\langle X_1Y_2\rangle^2}{\langle Y_2^2\rangle}, \\ V_2 = \Delta_{\mathrm{inf}}^2 X_2 = \langle X_2^2\rangle - \frac{\langle X_2Y_1\rangle^2}{\langle Y_1^2\rangle}. \end{aligned} \tag{20}$$

It is easy to find that V_1 is the same as V_2.

3. Numerical calculation

Now we discuss how to calculate Eq.(5) numerically. The parameters $\bar{\varepsilon}/\gamma_1, \chi/\gamma_1$, in Eq.(5) can be expressed by γ_{r}, $g = \chi^2/2\gamma_0\gamma_1$ and $\mu = \bar{\varepsilon}/\varepsilon_{\mathrm{c}}$, ($\varepsilon_{\mathrm{c}} = \alpha_0\gamma_0 = \gamma_0\gamma_1/\chi$), so $\chi/\gamma_1 = \sqrt{2\gamma_{\mathrm{r}}g}$, $\bar{\varepsilon}/\gamma_1 = \varepsilon_{\mathrm{c}}\bar{\varepsilon}/\gamma_1\varepsilon_{\mathrm{c}} = \mu\sqrt{\gamma_{\mathrm{r}}/2g}$. The numerical solutions of $\alpha_0(\tau)$ and $\alpha_1(\tau)$ ($\alpha_2(\tau)$) versus τ are depicted in Fig.1.(a) and (b) with $\gamma_{\mathrm{r}} = 0.5$, $g = 1/1000$ and $\mu = 2$. Using $\alpha_1(\tau)$ and Eqs.(17)–(20), we obtain the relations $\bar{r} = \bar{r}(\tau), V_1 = V_1(\tau)$. Eliminating the variable τ, we have $V_1 = V_1(\bar{r})$, which is shown in curve

3 in Fig.2 for $\mu = 2$. Similarly, changing the value of μ to 0.5, 0.999 and 2.0113 depicting another curves 1 and 2 and solid curve, respectively as shown. Now returning to Fig.1(b), we obtain the ratio of the first peak $\alpha_{0\mathrm{m}}$ to the steady state $\alpha_{0\mathrm{s}}(\tau \to \infty, \alpha_0 \to \alpha_{0\mathrm{s}})$, namely $\alpha_{0\mathrm{m}}/\alpha_{0\mathrm{s}} = 63.246/31.6 \approx 2$. This is in agreement with the theory, just representing the μ value that is used in Eq.(5).

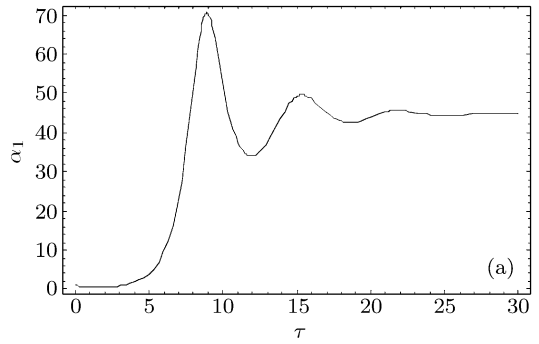


Fig.1. (a) The curve of $\alpha_1(\alpha_2)$ vs τ.

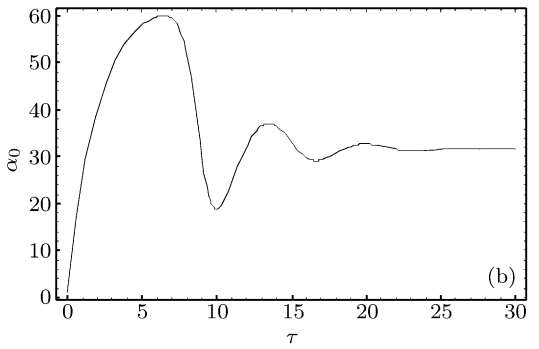


Fig.1. (b) The curve of α_0 vs τ.

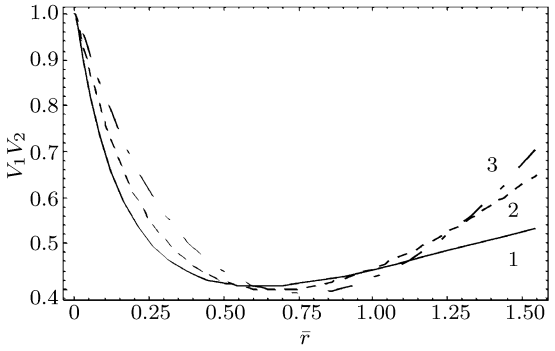


Fig.2. The minimum error V_1V_2 as functions of the squeezing τ. (time-dependent solution)

Inspecting Fig.1(b) shows that α_0 and α_1, α_2 relatively small at the first peak of $\alpha_{0\mathrm{m}} = \bar{\varepsilon}/\gamma_1\gamma_\mathrm{r} = \sqrt{\gamma_\mathrm{r}/2\eta}\mu/\gamma_\mathrm{r} = \mu\sqrt{1/2\eta\gamma_\mathrm{r}} = \alpha_{0\mathrm{s}} = \mu$, thus $\alpha_{0\mathrm{m}}/\lambda_{\mathrm{os}} = \mu$.

In Fig.2, the curves 1, 2 and 3 correspond to the ideal case, $\mu = 0.5, 0.999$ and 2.011, respectively. In Fig.1 after the first peak, with the increase of α_1 and α_2, the pump amplitude α_0 attenuates gradually to the steady value $\alpha_{0\mathrm{s}} = \sqrt{1/2g\gamma_\mathrm{r}}$. If we assume that α_0 is constant, $\alpha_0 = \alpha_{0\mathrm{m}}$ at the beginning, neglect of pump depletion, so time-dependent NOPA transits to the time-independent NOPA. In this case, $1/\bar{\eta}$ does not change with τ, $1/\bar{\eta} = (\varepsilon/2)/(k/\sqrt{2}) = \chi\alpha_{0\mathrm{m}}/\gamma_1 = \chi\mu\alpha_{0\mathrm{s}}/\gamma_1 = \mu$. Subsequently, in order to calculate the variance V_1, for the time independent case, we only need to insert $\bar{r} = 2\tau/\bar{\eta} = 2\mu\tau$ into Eqs.(18)–(20), the corresponding curves are depicted in Fig.3.

In Fig.3, we take the same parameters as Fig.2, then comparing Fig.2 with Fig.3, we find that either time-dependent solution or time-independent solution, the trend of the curves are the same, the minimum error $(V_1)_{\min}$ decreases with the increase of $1/\bar{\eta} = \mu$, while at the same time, the optimum squeezing $\bar{r}$ for realization of $(V_1)_{\min}$ increases. For the same $\mu, (V_1)_{\min}$, in time-dependent case is larger than the $(V_1)_{\min}$ in time-independent case, while the corresponding optimum squeezing $\bar{r}$ decreases. As a whole, the time-dependent case curves move up and left relative to the time-independent curves.

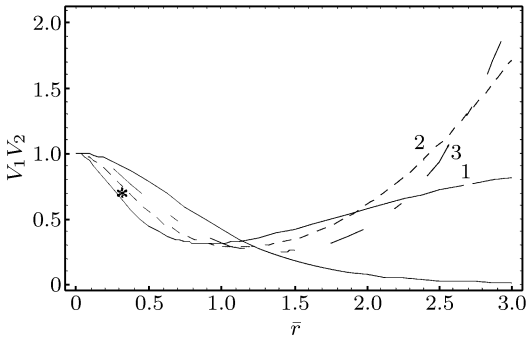


Fig.3. The minimum error V_1V_2 as functions of the squeezing $\bar{r}$. (time in dependent solution)

4. Conclusions

To summarize, in this paper, we use the solution of the time-dependent Fokker–Planck equation for NOPA under consideration of pump depletion to evaluate the optimum realization of the EPR paradox. We find that either time-dependent solution or time-independent solution, the trend of the curves are the same, namely $(V_1)_{\min}$ value decreases with the decrease of relative loss $\bar{\eta} = 1/\mu$, while at the same time,

the optimum squeezing $\bar{r}$ increases with the decrease of relative loss $\bar{\eta}$. As a result, the curves belong to the time-dependent move up and left relative to the curves belong to the time-independent case. The result shows that the optimum realization of EPR paradox can be achieved by adjusting the degree of squeezing and this restricts the demonstration of the EPR paradox for a certain finite loss k. Obviously, comparing the calculated variance with the experimental one, the loss k must be included. For instance, referring to literature [10], where $V_1V_2 = 0.7, g^2 = (\tanh 2r)^2 = 0.58$ easily to evaluate $V = \sqrt{V_1V_2} = 0.836,\ r = 0.499$, this corresponds the point denoted by red star (*) in Fig.3, this point deviates much from the ideal case $\eta = 0$ (solid curve), but very near the threshold case $\eta = 1.001$ (curve 2). If we increase the squeezing parameter r from 0.499 to 1.1, the optimum EPR demonstration $(V = 0.58, V_1V_2 = 0.36)$ will be achieved.

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