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Scheme for entanglement concentration of unknown W class states via linear optics*

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This paper proposes a scheme for entanglement concentration of unknown triparticle W class states with a certain probability. This protocol is mainly based on the coincidences of single-photon detectors and requires single-photon detectors and linear optical elements. The scheme is feasible within current technology.

Keywords: entanglement concentration, W state, linear optical elements
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1. Introduction

Quantum entanglement plays an important role in the development of quantum computation and quantum information,[1−6] such as quantum teleportation, quantum dense coding, quantum cryptography, and quantum games. Entanglement involving bipartite systems such as Bell states has been well understood[7] while entanglement of multipartite systems is still undergoing intense research.[8−10] Entanglement of the three qubits is more complicated than that of two qubits. Reference [11] has shown that the entanglement of three qubits could be classified into GHZ class states and W class states. The W state is a more robust entanglement state than the GHZ state when one of the three qubits is traced out. Several schemes for the preparation of W state have been proposed,[10,12,13] and the W state of three photons has also been observed in an experiment.[8] Recently, W state was widely utilized in quantum information processing (QIP), such as quantum teleportation,[14−16] quantum dense coding,[17,18] and quantum cryptography.[19]

To fulfil QIP, distribution of the entangled states between distant locations is essential over large distances. But, during the transmission through a noisy channel or storage in a noisy environment, the quality of entangled states usually decreases, for example, the maximally entangled states could collapse into nonmaximally entangled ones or even mixed states. So, we need extract the larger or maximally entangled state from the nonmaximally entangled ones. There are many theoretical protocols which can achieve this conversion,[20−25] and several schemes for entanglement concentration have also been reported[26−29] experimentally.

In the previous papers,[22] we have discussed distillation of the W class states where the coefficients of the states were known for the involved parties, which will inevitably pose some limitations for the realistic application of the concentration scheme. Here we will discuss the concentration of unknown W class states with linear optical elements.

2. The scheme for concentration of unknown photonic W class states via linear optics

In this section, we will propose a scheme in which three identical partially entangled tripartite W class states can be concentrated into a W state with linear optical elements. We suppose that three W class states to be concentrated can be expressed as

$$|W'\rangle_{123} = a|VHH\rangle_{123} + b|HVH\rangle_{123} + c|HHV\rangle_{123}, \quad (1a)$$

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$$|W'\rangle_{456} = a|VHH\rangle_{456} + b|HVH\rangle_{456} + c|HHV\rangle_{456}, \quad (1b)$$

$$|W'\rangle_{789} = a|VHH\rangle_{789} + b|HVH\rangle_{789} + c|HHV\rangle_{789}, \quad (1c)$$

where H and V denotes horizontal and vertical linear polarizations respectively, subscripts $i(i = 1 \sim 9)$ denote the spatial modes of photons, a, b and c are unknown coefficients and $|a|^2 + |b|^2 + |c|^2 = 1$. Photons 1, 5 and 9 belong to Alice, photons 3, 4 and 8 belong to Bob, and photons 2, 6 and 7 belong to Charlie, respectively (shown in Fig.1). Then the joint state of the nine photons is

$$\begin{aligned}|\Phi\rangle_{123456789} &= (a|VHH\rangle_{123} + b|HVH\rangle_{123} \\ &+ c|HHV\rangle_{123}) \otimes (a|VHH\rangle_{456} + b|HVH\rangle_{456} \\ &+ c|HHV\rangle_{456}) \otimes (a|VHH\rangle_{789} \\ &+ b|HVH\rangle_{789} + c|HHV\rangle_{789}). \end{aligned} \quad (2)$$

Then we let the six photons 1, 9, 3, 4, 6 and 7 pass through three polarizing beam splitters (PBSs): i.e., photons 1 and 9 pass through the PBS1, photons 3 and 4 pass through the PBS2, photons 6 and 7 pass through the PBS3, respectively. Suppose that these six photons arrive at the three PBSs simultaneously so that the photons 1 and 9 interfere at the PBS1, photons 3 and 4 interfere at the PBS2, photons 6 and 7 interfere at the PBS3, respectively. Since the PBSs transmit only the horizontal polarization component and reflect the vertical component, after photons 1, 9, 3, 4, 6, 7 pass through the three PBSs, respectively, if there is one and only one photon in the one of the two output modes of every PBS, the above state will be projected into the following one:

$$\begin{aligned}|\Psi\rangle &= \frac{1}{\sqrt{3|abc|^2 + |b|^6}} \\ &\times [abc(|VV\rangle_{1'9'}|HH\rangle_{4'3'}|HH\rangle_{6'7'}|HVH\rangle_{258} \\ &+ |HH\rangle_{9'1'}|HH\rangle_{3'4'}|VV\rangle_{6'7'}|VHH\rangle_{258} \\ &+ |HH\rangle_{1'9'}|VV\rangle_{3'4'}|HH\rangle_{6'7'}|HHV\rangle_{258}) \\ &+ b^3|HH\rangle_{1'9'}|HH\rangle_{3'4'}|HH\rangle_{6'7'}|VVV\rangle_{258}], \end{aligned} \quad (3)$$

with a probability of $3|abc|^2 + |b|^6$. In the above description, the PBS plays the double role of both overlapping the two photons and erasing the 'which way' information.[21]

To extract the W state from the joint state, $|\Phi\rangle_{123456789}$, Alice, Bob and Charlie rotate the polarization of the photons $1'$, $9'$, $3'$, $4'$, $6'$ and $7'$ by 45° with six half-wave plates (HWPi, $i = 1 \sim 6$, shown in Fig.1), respectively. The transformation of the HWPi is given by

$$\begin{aligned}|H\rangle_i &\to \frac{1}{\sqrt{2}}(|H\rangle_i + |V\rangle_i), \\ |V\rangle_i &\to \frac{1}{\sqrt{2}}(|H\rangle_i - |V\rangle_i). \end{aligned} \quad (4)$$

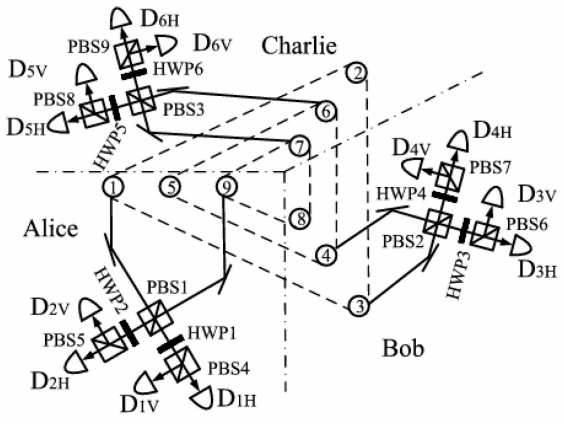


Fig.1. Schematic set-up for concentration of the W states. Photons 1, 5, 9 (3, 4, 8 / 2, 6, 7) are all belong to Alice (Bob / Charlie). Nine polarizing beam splitters (PBSi, $i = 1 \sim 9$), six half-wave plates (HWPi, $i = 1 \sim 6$), and twelve single-photon detectors are used.

After these operations, the joint state of the nine photons will become

$$\begin{aligned}|\Psi'\rangle &= \frac{1}{8\sqrt{3|abc|^2 + |b|^6}} \\ &\otimes\{abc[(|H\rangle_{1'} - |V\rangle_{1'})(|H\rangle_{9'} - |V\rangle_{9'})(|H\rangle_{3'} + |V\rangle_{3'})(|H\rangle_{4'} + |V\rangle_{4'})(|H\rangle_{6'} + |V\rangle_{6'})(|H\rangle_{7'} + |V\rangle_{7'})|HVH\rangle_{258} \\ &+(|H\rangle_{1'} + |V\rangle_{1'})(|H\rangle_{9'} + |V\rangle_{9'})(|H\rangle_{3'} + |V\rangle_{3'})(|H\rangle_{4'} + |V\rangle_{4'})(|H\rangle_{6'} - |V\rangle_{6'})(|H\rangle_{7'} - |V\rangle_{7'})|VHH\rangle_{258} \\ &+(|H\rangle_{1'} + |V\rangle_{1'})(|H\rangle_{9'} + |V\rangle_{9'})(|H\rangle_{3'} - |V\rangle_{3'})(|H\rangle_{4'} - |V\rangle_{4'})(|H\rangle_{6'} + |V\rangle_{6'})(|H\rangle_{7'} + |V\rangle_{7'})|HHV\rangle_{258}] \\ &+b^3(|H\rangle_{1'} + |V\rangle_{1'})(|H\rangle_{9'} + |V\rangle_{9'})(|H\rangle_{3'} + |V\rangle_{3'})(|H\rangle_{4'} + |V\rangle_{4'})(|H\rangle_{6'} + |V\rangle_{6'})(|H\rangle_{7'} + |V\rangle_{7'})|VVV\rangle_{258}\}. \end{aligned} \quad (5)$$

Now, Alice, Bob and Charlie let the photons $1'$, $9'$, $3'$, $4'$, $6'$, and $7'$ pass through the other six PBSs (PBSi, $i = 4 \sim 9$, in Fig.1) respectively. At this time, they observe the coincidence of the single-photon detectors

among D_{1H} (or D_{1V}) and D_{2H} (or D_{2V}) and D_{3H} (or D_{3V}) and D_{4H} (or D_{4V}) and D_{5H} (or D_{5V}) and D_{6H} (or D_{6V}), apparently, there are sixty-four possible coincidence detections. Basing on Eq.(5), if both photons of the every photon pairs $1'$ and $9'$, $3'$ and $4'$, $6'$ and $7'$ are observed to be in same polarization state or in the different polarization one, we can obtain the states of the photons 2, 5, 8 which are shown in Table 1.

Table 1. The relation between the states of the photons 2, 5, 8 and the sameness and differences of the polarization states for both photons of the every photon pairs $1'$ and $9'$, $3'$ and $4'$, $6'$ and $7'$.

Sameness and differences of the polarization states of			The states of the photons 2, 5, 8
photons $1'$ and $9'$	photons $3'$ and $4'$	photons $6'$ and $7'$	
same	same	same	$[abc(\|HVH\rangle_{258}+\|VHH\rangle_{258}+\|HHV\rangle_{258})+b^3\|VVV\rangle_{258}]/\sqrt{3\|abc\|^2+\|b\|^6}$
difference	same	same	$[abc(-\|HVH\rangle_{258}+\|VHH\rangle_{258}+\|HHV\rangle_{258})+b^3\|VVV\rangle_{258}]/\sqrt{3\|abc\|^2+\|b\|^6}$
same	difference	same	$[abc(\|HVH\rangle_{258}+\|VHH\rangle_{258}-\|HHV\rangle_{258})+b^3\|VVV\rangle_{258}]/\sqrt{3\|abc\|^2+\|b\|^6}$
same	same	difference	$[abc(\|HVH\rangle_{258}-\|VHH\rangle_{258}+\|HHV\rangle_{258})+b^3\|VVV\rangle_{258}]/\sqrt{3\|abc\|^2+\|b\|^6}$
difference	difference	same	$[abc(-\|HVH\rangle_{258}+\|VHH\rangle_{258}-\|HHV\rangle_{258})+b^3\|VVV\rangle_{258}]/\sqrt{3\|abc\|^2+\|b\|^6}$
difference	same	difference	$[abc(-\|HVH\rangle_{258}-\|VHH\rangle_{258}+\|HHV\rangle_{258})+b^3\|VVV\rangle_{258}]/\sqrt{3\|abc\|^2+\|b\|^6}$
same	difference	difference	$[abc(\|HVH\rangle_{258}-\|VHH\rangle_{258}-\|HHV\rangle_{258})+b^3\|VVV\rangle_{258}]/\sqrt{3\|abc\|^2+\|b\|^6}$
difference	difference	difference	$[abc(-\|HVH\rangle_{258}-\|VHH\rangle_{258}-\|HHV\rangle_{258})+b^3\|VVV\rangle_{258}]/\sqrt{3\|abc\|^2+\|b\|^6}$

Following Table 1, conditions for the different polarization state of the photons $1'$ and $9'$ (or $3'$ and $4'$, or $6'$ and $7'$) are detected, Alice or Bob or Charlie could perform an additional π phase shift via local operation and classical communication, the states of the photons 2, 5, 8 (shown in Table 1) can be transformed into the following states:

$$|\psi'\rangle = [abc(|VHH\rangle_{258}+|HVH\rangle_{258}+|HHV\rangle_{258}) \pm b^3|VVV\rangle_{258}]/\sqrt{3|abc|^2+|b|^6}. \quad (6)$$

Suppose $|b|^2 \ll |ac|$, then the state $|\Psi'\rangle_{258}$ can be approximately expressed as

$$|\psi''\rangle = \frac{1}{\sqrt{3}}(|VHH\rangle_{258}+|HVH\rangle_{258}+|HHV\rangle_{258}) = |W\rangle, \quad (7)$$

the above state is a W state, which is extracted from the three identical W class states. And the successful probability is $3|abc|^2+|b|^6 \approx 3|abc|^2$ when $|b|^2 \ll |ac|$.

3. Discussions and conclusion

We will discuss the experiment feasibility of the physical scheme briefly. The experimental observation of three-photon polarization-entangled W state by means of spontaneous down-conversation has been reported.[8] In Ref.[21], Zhao presented a practical scheme for entanglement concentration of two-photon entangled states, and in Ref.[29], entanglement purification of arbitrary unknown states were realized experimentally. Furthermore, the main elements used in our scheme are waveplates, PBSs and single-photon detectors, which has been used widely in these and other QIP. So we can consider that our proposal is feasible to be realized within current technology.

In conclusion, we have proposed an entanglement concentration protocol for unknown W class states with the help of coincidence detections of single-photon detectors, and a W state could be obtained with a certain probability from the three identical W class states, which are all initially unknown partially entangled states.

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