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Remarks on interpretations of the Eötvös experiment and misinterpretation of $E = mc^2$

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The Eötvös experiment on the verification of equivalence between inertial mass and gravitational mass of a body is famous for its accuracy. A question is, however, can these experimental results be applied to the case of a physical space in general relativity, where the space coordinates could be arbitrary? It is pointed out that it can be validly applied because it has been proven that Einstein's equivalence principle for a physical space must have a frame of reference with the Euclidean-like structure. Will claimed further that such an overall accuracy can be translated into an accuracy of the equivalence between inertial mass and each type of energy. It is shown that, according to general relativity, such a claim is incorrect. The root of this problem is due to an inadequate understanding of special relativity that produced the famous equation $E = mc^2$, which must be understood in terms of energy conservation. Concurrently, it is pointed out that this error is a problem in Will's book, 'Theory and Experiment in Gravitational Physics'.

Keywords: Einstein's equivalence principle, Einstein–Minkowski condition, Euclidean-like structure, Eötvös experiment, weak equivalence principle, $E = mc^2$

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1. Introduction

General relativity is established on two principles,[1,2] namely: 1) Einstein's equivalence principle, which requires the Einstein–Minkowski condition that a free falling point-like particle in a gravitational field is along a geodesic and results in a co-moving local Minkowski space; and 2) the principle of general relativity, that is 'The law of physics must be of such a nature that they apply to systems of reference in any kind of motion.' However, these two principles are practically replaced[3] with Pauli's version of the equivalence principle,[4] which requires only a mathematical existence of local Minkowski spaces, and unrestricted covariance, which Einstein called the principle of covariance:[1] 'The general laws of nature are to be expressed by equations which hold good for all systems of co-ordinates, that is, are co-variant with respect to any substitutions whatever (generally co-variant).' Consequently, many accepted Lorentz manifolds are not valid in physics.[5]

Einstein's principles actually require a physical meaning of coordinates that Einstein's theory did not provide,[1,2] but the substitutions require no physical meaning of the coordinates. Thus, Einstein's equivalence principle is practically not applicable. Perhaps this is why essentially only the Pauli version, which is a corrupted simplification of theorems in Riemannian geometry, is used beyond Einstein's own work. In spite of Einstein's objection,[3] his equivalence principle was commonly but mistakenly regarded the same as Pauli's[4] version: 'For every infinitely small world region(i.e. a world region which is so small that the space- and time-variation of gravity can be neglected in it) there always exists a coordinate system $K_0(X_1, X_2, X_3, X_4)$ in which gravitation has no influence either in the motion of particles or any physical process.' Apparently, Pauli,[4] Misner *et al*,[6] Will,[7] Straumann,[8] and others overlooked (or disagreed with) Einstein's remark, 'For it is clear that, e.g., the gravitational field generated by a material point in its environment certainly cannot be 'transformed away' by any choice of the system of coordinates ...'. (Ref.[1], p144)

Moreover, once Pauli's version, which considers only the metric signature, is used as a 'physical principle', unrestricted covariance has to be adapted; and this necessarily leads to the notion that space-time coordinates have no physical meaning. Thus, a vicious circle of logic was established within the theory. However, the price for ignoring the Einstein–Minkowski condition is often not free. For instance, Misner *et*

al[6] and Ohanian and Ruffini,[9] different from others, have made the incorrect conclusion on the local time of the earth.[1)]

If the coordinates were arbitrary, there would be no definite logical deduction in such a theory because every physical quantity has to be measurable and therefore must be expressed in terms of coordinates. Thus, in current theory, inevitably there are inconsistent results and statements that cannot be verified. An example, as shown by Logunov and Mestvirishvili,[10] that diffeomorphic coordinate systems could lead to inconsistent results (positive or negative) of gravitational radiation. Another example is that attempts to obtain a metric for uniform gravity in support of Einstein's equivalence principle had failed.[11]

Moreover, the equivalence of accelerated mass and gravitational mass (weak equivalence principle or WEP for short in Ref.[7]), the foundation of Einstein's equivalence principle, would have to be justified anew in terms of general relativity since the supporting experiments were not based on arbitrary coordinate systems. As Morrison[12,13] pointed out, the 'covariance principle' disrupts the necessary physical continuity from special relativity to general relativity. Moreover, Zhou[14] pointed out, 'The concept that coordinates does not matter in the interpretation of Einstein's theory $\cdots$ necessarily leads to mathematical results which can hardly have a physical interpretation and are therefore a mystification of the theory.' Since this problem would be applicable to almost any issue, it was not clear that Einstein's theory is compatible with experiments.

Fortunately, it has been proven that Einstein's equivalence principle necessarily implies an existence of the Euclidean-like structure for a physical space.[15,16] Thus, the physical meaning of space coordinates has been clarified, and the local time can be determined. In addition, it has been shown that Einstein's 'principle of covariance' has no theoretical basis in physics or observational support beyond what is allowed by the principle of general relativity.[12] Moreover, a clear counter example against general covariance is given in Ref.[17]. In addition, based on the Einstein–Minkowski condition, Einstein's theory of measurement is proven invalid.[13] However, this only means that some of the fundamental problems have been addressed.

Unfortunately, some misunderstanding of general relativity actually started from special relativity. Now let us examine the classic torsion-balance measurements of Eötvös, Dicke, and Braginsky and their collaborators.[18,19]

2. The Eötvös-type experiment

For the convenience of discussion, we use the description of Will.[7] Two objects of different compositions are connected by a rod of length r, and suspended in a horizon orientation by a fine wire ('torsion balance'). If the gravitational acceleration of the bodies differs, there will be a torque N induced on the suspension wire, given by

$$N = \eta r(\boldsymbol{g} \times \hat{\boldsymbol{e}}_w) \cdot \hat{\boldsymbol{e}}_r, \tag{1}$$

where $\eta \equiv 2\dfrac{|a_1 - a_2|}{|a_1 + a_2|}$, a_1 and a_2 are the accelerations of the two bodies, $\boldsymbol{g}$ is the gravitational acceleration, and $\hat{\boldsymbol{e}}_w$ and $\hat{\boldsymbol{e}}_r$ are unit vectors along the wire and the rod respectively. Here η is the dimensionless parameter that measures the strength of the violation of WEP. If the entire apparatus is rotated about a direction ω with angular velocity $|\omega|$, the torque will be modulated with a period $2\pi/\omega$. In the experiment of Barron Roland von Eötvös, g was the acceleration of the Earth. In the Princeton[20] and Moscow[19] experiments, g was that of the Sun and the rotation of the earth provided the modulation N at the period of 24 h. The modulated torque was determined either by measuring the torsional motion of the rod (Moscow) or by measuring the force required to counteract the torque and keep the rod in place (Princeton). The resulting upper limits on measurable torques $|N|$ yielded limits on η given by

$$|\eta| < \begin{cases} 1 \times 10^{-11}[\text{Princeton}], \\ 1 \times 10^{-12}[\text{Moscow}], \end{cases} \tag{2}$$

where the limits are formal standard deviations.

It should be noted that since a physical space has a frame of reference with the Euclidean-like structure, these experimental results can be directly applied to a physical space in general relativity. If the space coordinates were ambiguous as in Einstein's theory of measurement, first one has to clarify at least what acceleration means in an arbitrary coordinate system.

1) See end notes on page 638.

3. Will's invalid interpretation and misunderstanding in relativity

Will further assumed that the passive gravitational mass $m_{\rm p}$ of a body is

$$m_{\rm p} = m_{\rm I} + \sum_A \eta^A E^A/c^2, \tag{3}$$

where $m_{\rm I}$ is the inertial mass of the body, E^A is the internal energy of the energy of the body generated by interaction A, and η^A is a dimensionless parameter that measures the strength of the violation of WEP induced by that interaction, and c is the speed of light in vacuum.

Thus, the accelerations of bodies 1 and 2 are respectively,

$$\begin{aligned} a_1 &= \left(1 + \sum \eta^A E_1^A/m_1c^2\right) g, \text{ and} \\ a_2 &= \left(1 + \sum \eta^A E_2^A/m_2c^2\right) g, \end{aligned} \tag{4}$$

where the subscript I on m_1 and m_2 have been dropped. It follows from Eqs.(1) and (4)

$$\eta \equiv 2\frac{|a_1 - a_2|}{|a_1 + a_2|} = \sum_A \eta^A \left(\frac{E_1^A}{m_1c^2} - \frac{E_2^A}{m_2c^2}\right). \tag{5}$$

Thus, based on the experimental limits on η, Will could placed limits on the WEP-violation parameters η^A. The basis of assumption (3) seems to be the famous equation $E = mc^2$. However, to be more precise, special relativity implies only that mass and energy are equivalent in connection with energy–momentum conservation.[21,22]

Thus, assumption (3) actually has an implicit assumption, i.e., any type of energy is equivalent to mass $m = E/c^2$. Unfortunately, according to general relativity, this is simply not true. This is due to that the source of an Einstein equation is an energy–stress tensor,[23] and thus the equivalence of energy and mass is restricted. For example, the electromagnetic energy and mass are not equivalent, since an electromagnetic stress tensor is traceless. This has been explicitly manifested by the Reissner–Nordstrom metric,[23,24]

$$\begin{aligned} {\rm d}s^2 = &\left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right) {\rm d}t^2 \\ &- \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} {\rm d}r^2 - r^2 {\rm d}\Omega^2, \end{aligned} \tag{6}$$

where q and M are the charge and mass of a particle and r is the radial distance from the particle. In this metric (6), the gravitational components generated by mass and electricity have different signs and, furthermore, very different radial coordinate dependence.

Some might argue that the total electric energy outside a sphere of radius r is $q^2/2r$, and thus the effective mass is

$$M - \frac{q^2}{2r}. \tag{7}$$

Thus Eq.(7) could be interpreted as supporting $m = E/c^2$ at least for electromagnetic energy. Such a view aims at a narrow point but misses the whole picture. There are several difficulties arising from such a view:

A) If any energy has a mass equivalence, an increase of energy should lead to an increment of gravitational strength. However, although energy increases by the presence of a charge, the strength of a gravitational force, as shown by metric (6), decreases everywhere.

B) If the electric energy is assigned a mass, should it be considered as part of the gravitational mass of the particle or not? If it is then gravitational mass and inertial mass are different. If it is not, it means that any electromagnetic energy should be assigned a mass.

C) If any electromagnetic energy should assign a mass equivalence, then this means a rejection of the notion that a photon is massless and that special relativity is valid.

As in the above, problems are created because one ignores the fact that the electromagnetic energy–stress tensor is traceless, but the massive energy–stress tensor is not. However, to interpret that electromagnetic energy has a mass equivalence, the coincidence of Eq.(7) is far from adequate. In fact, it is invalid in physics that a particle with additional energy would result in less mass since of Eq.(7) was interpreted as the mass inside a sphere of radius r. Thus, such a frivolous conclusion of equivalence is a manifestation of inadequate understanding of general relativity. In physics, inadequate deliberations are often the source of errors even for seasoned theorists.

Since not every type of energy is equivalent to mass, Will's assumption (3) is actually invalid. It is impossible to justify a violation of general relativity as a theoretical basis of general relativity. Nevertheless, this nonequivalence remains compatible with Einstein's famous equation, relating the total energy $E_{\rm T}$ and mass $M_{\rm T}$,

$$E_{\rm T} = M_{\rm T}c^2. \tag{8}$$

It is crucial to note that $E_{\rm T}$ is the total energy of the particle. (However, $E_{\rm T}$ does not include the energy of the field out side the particle.) The massive energy–stress tensor has a very specific form that not each type of internal energy is compatible with. Obviously, there are physical differences for massive and massless energy forms. Note that electromagnetic energy such as photons has no rest mass equivalence. This means that there should be cancellations among gravitational effects due to different types of energy.

Moreover, if mass–energy equivalence was assumed, Newtonian theory would imply that an electromagnetic plane wave could generate an infinitely divergent gravitational potential because the electromagnetic energy, on the average, is constant. However, the approximate validity of classical electrodynamics and special relativity requires that such a potential be not only finite but also bounded, as general relativity implies.[25] In addition, in an atom, the total energy includes at least the nuclear energy; the related electromagnetic energy is comparatively small. Thus, general relativity also requires the verified fact of a small isospin dependence of the nuclear force. A complete isospin independence of nuclear energy is not supported by nuclear physics.[26]

4. Discussions and conclusions

The paper points out that a direct application of well-known accurate experiments to general relativity should be addressed if the coordinates in general relativity were arbitrary as in Einstein's theory. This important issue is overlooked in many textbooks of general relativity. Moreover, it is probably useless to address this problem without solving the physical meaning of space coordinates. However, this issue must be solved because it is related to the validity of general relativity since the 'covariance principle' is not generally valid. Thus, the discovery of that a physical space necessarily has a frame of reference with the Euclidean-like structure is important.

This paper points out also another fundamental error in the current theory of general relativity due to inadequate understanding of special relativity. Einstein's famous equation, $E = mc^2$, was often misunderstood as any form of energy has a related mass although Einstein[27] has made clear that such a relation must be understood in term of energy conservation. In this paper, it is shown that some type of energy such as electromagnetic energy does not have a mass. Moreover, it is illustrated that $m = E/c^2$ is in fact inconsistent with Einstein's field equation in general relativity. A profound consequence is that the coupling constant of gravity needs not always the same.[21] Thus, it is possible that an anti-gravity coupling may exist as mentioned by Pauli.[4]

The misinterpretation of $E = mc^2$ as any energy has a mass correspondence, i.e., $m = E/c^2$, is a rather common one. For instance, Tolman (Ref.[28] p49), Fock (Ref.[29] p111), and Hawking (Ref.[30] p107) have also made this mistake explicitly. Will has made this mistake along with other invalid assumptions[22]2) in his 1976 interpretation of the Kreuzer experiment.[31] In view of that Will's book 'Theory and experiment in gravitational physics' gets only positive comments from two top journals,3)4) this error seems to have been rather universally committed. However, there are few exceptions, such as Bohm.[32] It is hoped that this paper would call the attention to this important issue of over simplifying the relationship between mass and energy.

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End notes

1)Misner, Thorne, and Wheeler[6] claimed that Einstein's equivalence principle is as follows: 'In any and every local Lorentz frame, anywhere and anytime in the universe, all the (Nongravitational) laws of physics must take on their familiar special-relativistic form. Equivalently, there is no way, by experiments confined to infinitestimally small regions of spacetime, to distinguish one local Lorentz frame in one region of spacetime frame any other local Lorentz frame in the same or any other region.' However, this is only an alternative version of Pauli's because the Einstein–Minkowski condition is ignored. In their Eq.(40.14),[6] they obtained a physically incorrect conclusion on the local time of the earth in the solar system because they did not understand Einstein's equivalence principle and related theorems in Riemannian space. Unfortunately, Ohanian & Ruffini[9] also ignored the Einstein–Minkowski condition and had the same problems as shown in their Eq.(50). However, Liu,[33] Straumann,[8] Wald,[24] and Weinberg[23] did not make the same mistake. Note that Ohanian, Ruffini, and Wheeler became non-believer of Einstein's principles.[9]

2)Will was an open referee of Ref.[22].

3)'Consolidates much of the literatures on experimental gravity and should be invaluable to researchers in gravitation.'—*Science*, on Will's book.[7]

4)'A concise meaty book ··· and a most useful reference work ... researchers and serious students of gravitation should be pleased with it.'—*Nature*, on Will's book.[7]

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