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Trigger and control entanglement by atoms with different interaction times*

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The generation of the entanglement between two two-level atoms interacting with the third atom driven by white noise is investigated when the coupling between atoms is modulated by a pulse function. This paper finds that the initial triggering time and the width of the pulse can generate a peak in the entanglement. There is an optimal width of the pulse for which the entanglement can reach a maximum. The asymmetry of the coupling between atoms can generate different entanglement in the system. The multiple triggers can generate multiple peaks in the entanglement. The separation between two peaks is increased as the width of the pulse is increased.

Keywords: two-level atom, trigger entanglement, control entanglement, peak
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Quantum entanglement has played an important role in quantum information processing.[1−4] Much attention has been paid to how to create, control, and exploit the entanglement both theoretically and experimentally.[5−14] However, real quantum systems cannot be separated from the influences of the surrounding environment in a decoherence process. It is necessary to prevent, minimize or engineer the impact of environmental noise in the quantum information processing. Recently, the effects of noise on the quantum information processing have been investigated extensively. For example, the loop control strategies use an ancillary system coupled to the quantum processor to improve the performance,[15−18] and quantum error correction[19−21] uses redundant coding to protect quantum states from noisy environment. Meanwhile, it is very interesting that the noise can also assist the generation of entanglement.[22−28] It is shown that entanglement can be maximized for intermediate values of cavity damping rates in a system consisting of two distinct leaking optical cavities coupled with one atom driven by noise.[26] Through interaction with a cavity driven by noise, two separate atoms can be entangled together.[27] The generation of entangled states of two-level atoms coupled simultaneously with a dissipated atom was discussed. The white noise and the squeezed white noise were used to play a constructive role in the entanglement preparation.[28]

In this paper, the effect of the time variation on the generation of entanglement is investigated when the noise is included in the system. An atom driven by white noise or white squeezed noise is used to trigger the generation of the entanglement between the other two atoms when the coupling between the atoms is modulated by a pulse function. First, the entanglement between the two atoms is calculated when the initial triggering time, the width of the pulse and the asymmetric coupling parameters are varied. Then the effects of two pulses, the triggering time of the second pulse, and the width of the pulses on the entanglement are investigated. The effects of the squeezed white noise on the generation of the entanglement are also discussed. Finally some discussions conclude the paper.

The system contains three two-level atoms A, B and D as shown in Fig.1. The atom D is driven by a white noise. The atoms A and B are coupled to atom D through the coupling parameters $g_{\rm AD}$ and $g_{\rm BD}$. The master equation governing the time evolution of the system is given by ($\hbar = 1$)

$$\frac{{\rm d}\rho}{{\rm d}t} = {\rm i}[H, \rho] + L(\rho). \tag{1}$$

Where the Hamiltonian H describes the internal energies of the three atoms. The Liouvillean $L(\rho)$ describes the decay of the atoms A and B, as well as the interaction of atom D with the noise. The two two-

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level atoms of A and B are localized distantly enough to reasonably consider that there is no interaction between each other.[26−28] The Hamiltonian of the three-atom system can be constructed as

$$H = \sum_{i=\mathrm{A,B,D}} \frac{\omega_i}{2}\sigma_i^{\mathrm{z}} + \sum_{i=\mathrm{A,B}} g_{i\mathrm{D}}(\sigma_{\mathrm{D}}^{+}\sigma_i^{-} + \mathrm{h.c.}), \quad (2)$$

while the Liouvillean with time dependent noise is given by

$$\begin{aligned} L(\rho) = &-\gamma_{\mathrm{D}}(n_{\mathrm{T}}+1)(\sigma_{\mathrm{D}}^{+}\sigma_{\mathrm{D}}^{-}\rho + \rho\sigma_{\mathrm{D}}^{+}\sigma_{\mathrm{D}}^{-} - 2\sigma_{\mathrm{D}}^{-}\rho\sigma_{\mathrm{D}}^{+}) \\ &- \gamma_{\mathrm{D}} n_{\mathrm{T}}(\sigma_{\mathrm{D}}^{-}\sigma_{\mathrm{D}}^{+}\rho + \rho\sigma_{\mathrm{D}}^{-}\sigma_{\mathrm{D}}^{+} - 2\sigma_{\mathrm{D}}^{+}\rho\sigma_{\mathrm{D}}^{-}) \\ &- \sum_{i=\mathrm{A,B}} \gamma_i(\sigma_i^{+}\sigma_i^{-}\rho + \rho\sigma_i^{+}\sigma_i^{-} - 2\sigma_i^{-}\rho\sigma_i^{+}), \quad (3) \end{aligned}$$

where n_{T} is an effective photon number which will characterize the white noise, $\gamma_i (i = \mathrm{A,B,D})$ describes the atomic decay rate, $2\gamma_i (i = \mathrm{A,B,D})$ is the spontaneous emission rate from the upper level, and $\gamma_{\mathrm{A}} = \gamma_{\mathrm{B}} = \gamma_{\mathrm{D}}$ is assumed in the following calculations. To simplify the derivation, the interaction picture is chosen and the three atoms are identical with the same free Rabi frequency ω_i. Using the transformation, we find that the Liouvillean part remains unchanged, while the Hamiltonian part is changed to

$$H_{\mathrm{I}} = \sum_{i=\mathrm{A,B}} g_{i\mathrm{D}}(\sigma_{\mathrm{D}}^{+}\sigma_i^{-} + \mathrm{h.c.}). \quad (4)$$

The analytical solution of Eq.(1) is extremely tedious, and some numerical result is required.

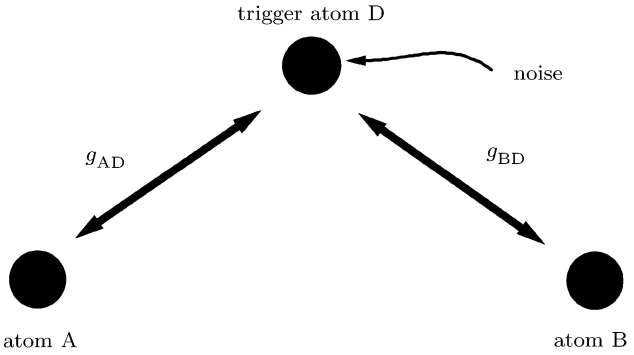


Fig.1. The schematic diagram of the atoms A and B interacted with atom D when atom D is driven by a white noise.

The numerical simulation is presented for the original master equation (1). The atomic ground and excited states are denoted by $|g\rangle$ and $|e\rangle$. For the original master equation of the three-atom system, the initial state of the system is $|g\rangle_{\mathrm{A}}|g\rangle_{\mathrm{B}}|g\rangle_{\mathrm{D}}$. The concurrence C is chosen as the measure of the entanglement. The concurrence C is defined as[29]

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad (5)$$

where λ_i $(i = \mathrm{A,B,D})$ are the square roots of the eigenvalues of the matrix $\rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ in the decreasing order.

The atom D can be used to stimulate the entanglement production between atoms A and B. The coupling parameter g is considered as a pulse function of $g(t) = g_0 f(t)$ with

$$f(t) = \begin{cases} 0 & t < t_1, \\ 1 & t_1 \le t \le t_2, \\ 0 & t > t_2. \end{cases} \quad (6)$$

The entanglement C is plotted in Fig.2 as a function of the time t. Figure 2(a) is a plot of C as functions of the initial injection time t_1 and the time t when $g_{\mathrm{AD}} = g_{\mathrm{BD}} = g_0 f(t)$. From Fig.2(a), it is seen that the peak in C is increased until it reaches a constant value when t_1 is increased. It is clear that the entanglement can be generated no matter what time the atom D is injected. Because the initial state of the atoms A and B is $|g\rangle_{\mathrm{A}}|g\rangle_{\mathrm{B}}$, the states of the atoms A and B will not change until the atom D interacts with them. Figure 2(b) is a plot of C as functions of the initial injection time t_1 and the time t when $g_{\mathrm{AD}} \neq g_{\mathrm{BD}}$ with $g_{\mathrm{AD}} = g_0$ and $g_{\mathrm{BD}} = g_0 f(t)$. From Fig.2(b), it is seen that the entanglement C is increased to a peak and then decreased to zero as the initial injection time t_1 is increased from zero. When t_1 is greater than a certain value, there will be no entanglement. Since the atom D is driven by white noise, the entanglement will be induced by noise when the atom D interacts with atoms A and B. The interaction parameter g between atoms is just the trigger. Figure 2(c) is a plot of C as functions of the width $\Delta t = t_2 - t_1$ of the pulse when $g_{\mathrm{AD}} = g_{\mathrm{BD}} = g_0 f(t)$. For large values of the time t, it is seen that the entanglement C increases and reaches a peak and then decreases to zero as Δt increases from zero. For small values of the time t, the entanglement C increases from zero and then reaches a constant value when Δt increases. The projection of the peak of C on $(\Delta t, t)$ plane likes a mirror picture of the letter "C". Figure 2(d) is a plot of C as functions of the width $\Delta t = t_2 - t_1$ of the pulse when $g_{\mathrm{AD}} = g_0, g_{\mathrm{BD}} = g_0 f(t)$. The curve of C increases from zero and reaches a constant value when Δt increases for small value of t. The projection of the peak of C on $(\Delta t, t)$ plane likes a straight line parallel to the axis of Δt.

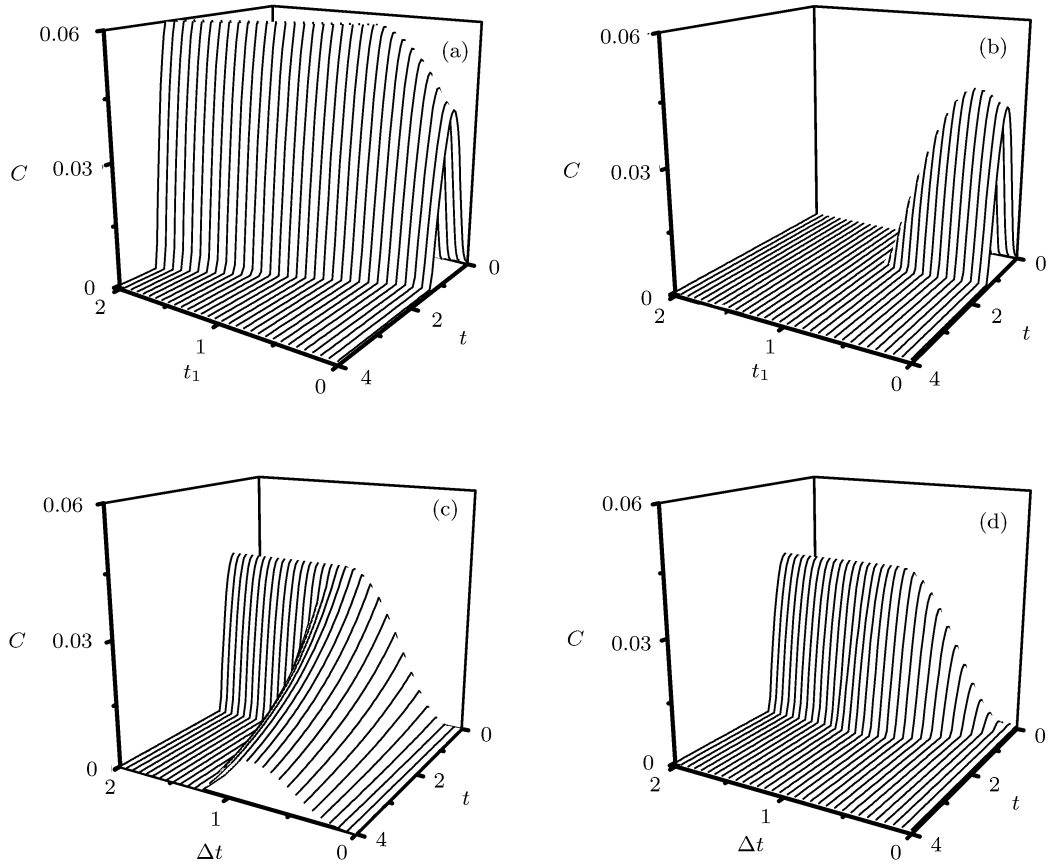


Fig.2. The entanglement C between atoms A and B is plotted as a function of the time t. The parameters are chosen to be $n_{\rm T} = 5.0$, $\gamma = 0.2$, $g_0 = 1.0$. (a) The entanglement C is plotted when the initial time t_1 is varied. The coupling parameter is chosen to be $g_{\rm AD} = g_{\rm BD} = g_0 f(t)$. (b) The entanglement C is plotted when the initial time t_1 is varied. The coupling parameter is chosen to be $g_{\rm AD} = g_0$, $g_{\rm BD} = g_0 f(t)$. (c) The entanglement C is plotted when the width of the pulse Δt is varied. The parameters are chosen to be $t_1 = 0.5$, $g_{\rm AD} = g_{\rm BD} = g_0 f(t)$. (d) The entanglement C is plotted when the width of the pulse Δt is varied. The parameters are chosen to be $t_1 = 0.5$, $g_{\rm AD} = g_0$, $g_{\rm BD} = g_0 f(t)$.

From Fig.2, it is seen that the entanglement C can be triggered and even controlled by varying the initial injection time t_1, the interaction parameters $g_{\rm AD}$ or $g_{\rm BD}$, and the width Δt of the pulse. When the coupling parameter is asymmetric with $g_{\rm AD} \neq g_{\rm BD}$, the generation of the entanglement is quite different in comparison with that of $g_{\rm AD} = g_{\rm BD}$ [see Figs.2(a) and 2(b), 2(c) and 2(d)].

If there are two triggers in the coupling parameters between atoms A, B, and D, Eq.(6) can be extended to

$$f(t) = \begin{cases} 0 & t < t_1, \\ 1 & t_1 \leq t \leq t_2, \\ 0 & t_2 < t < t_3, \\ 1 & t_3 \leq t \leq t_4, \\ 0 & t > t_4. \end{cases} \tag{7}$$

It is assumed that the second pulse is triggered at the time t_3 and end up at the time t_4. The entanglement C is plotted in Fig.3 as a function of the time t when the coupling parameters are the same with $g_{\rm AD} = g_{\rm BD} = g_0 f(t)$. In Figs.3(a) and 3(b), the entanglement C is plotted as functions of the initial triggering time t_3 of the second pulse and the time t when the width Δt of the pulse is varied. The width Δt of the pulse is defined as $\Delta t = t_2 - t_1 = t_4 - t_3$. That is, the width and height of the second pulse is the same as that of the first one. It is found that there are two peaks in C. The first peak is located at small values of t. The height is almost a constant and not varied as t_3 is increased. The second peak is located as large values of both t_3 and t. The second peak increases from zero and then reaches a constant as t_3

increases. If the width (Δt) of the pulse is small as shown in Fig.3(a) ($\Delta t = 0.5$), the height of the second peak is higher than that of the first one. The two peaks are close to each other. When the width Δt of the pulse increases as shown in Fig.3(b) ($\Delta t = 2.0$), the separation between the two peaks and the heights of the two peaks in C increase. The second peak in C also increases from zero and then reaches a constant. The height of the second peak is almost the same as that of the first one. From Figs.3(a) and 3(b), it is seen that the separation between the two peaks increases as t_3 and t increase. In Figs.3(c) and 3(d), the entanglement C is plotted as functions of the width Δt_2 of the second pulse and the time t when the width Δt_1 of the first pulse is fixed and t_3 is varied. The widths of the two pulses are defined as $\Delta t_1 = t_2 - t_1$ and $\Delta t_2 = t_4 - t_3$. From Fig.3(c), it is found that there are two peaks in C. The first peak with constant height is parallel to the axis of Δt_2. The second peak increases from zero and then reaches a constant and also parallel to the first one. The height of the second peak is lower than that of the first one. When t_3 increases as shown in Fig.3(d), i.e., the separation between the two pulses increases, the constant part of the height in the second peak is higher than that of the first one. While the height of the first peak is almost the same as that in Fig.3(c). This result can be extended to N pulses in $f(t)$. It seems that there will be N peaks in C if there are N pulses in $f(t)$ when the time interval between two nearest neighbouring pulses of the same amplitude is large enough. If the time interval between two nearest pulses is small, the peaks in the entanglement might overlap.

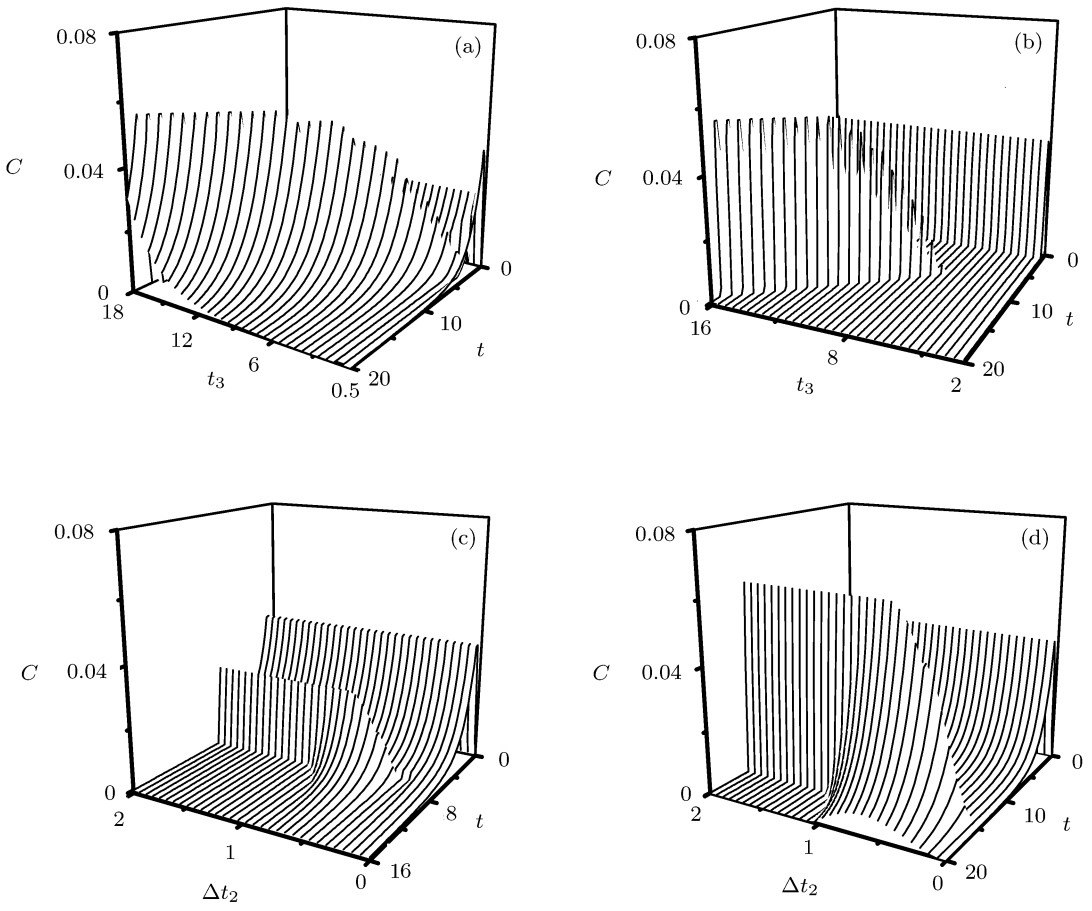


Fig.3. The entanglement C is plotted as a function of the time t. The parameters are chosen to be $n_{\rm T} = 5.0$, $\gamma = 0.2$, $g_0 = 1.0$, $g_{\rm AD} = g_{\rm BD} = g_0 f(t)$, $t_1 = 0.0$. (a) $t_2 - t_1 = t_4 - t_3 = \Delta t = 0.5$; (b) $t_2 - t_1 = t_4 - t_3 = \Delta t = 2.0$; (c) $t_2 - t_1 = \Delta t_1 = 1.0$, $t_3 = 6.0$; (d) $t_2 - t_1 = \Delta t_1 = 1.0$, $t_3 = 14.0$.

The entanglement C is also calculated when the coupling parameters are asymmetric with $g_{\rm AD} = g_0$, $g_{\rm BD} = g_0 f(t)$. The numerical result shows that there is only one peak located at small values of t no matter the initial triggering time t_3 and the width Δt of the pulse are varied. The peak of the entanglement is also parallel to the axis.

When the noise is a squeezed white noise, similar results as that shown in Figs.2 and 3 are obtained.

It is very interesting to note that the generation of the entanglement is mainly due to the noise, i.e. the effective photon number $n_{\rm T}$ in the system. If $n_{\rm T} = 0$, the entanglement in the system will disappear. The noise induced entanglement generation is similar to the phenomenon of stochastic resonance.[30−32] In stochastic resonance, the response of the system to a weak periodic signal can be enhanced by the presence of noise.

In this paper, the entanglement between two two-level atoms triggered by another atom driven by white noise is analysed. The third injected atom is used as a trigger of the entanglement. When the coupling between atoms is modulated by a pulse function, the initial triggering time and the width of the pulse can generate peaks in the entanglement. There is an optimal width of the pulse for which the entanglement is greater than zero and reaches a peak. If the coupling is modulated by two pulses, there are two peaks of different height in the entanglement. The heights of the two peaks depend on the width and the separation of the pulses. The separation between two peaks in the entanglement increases as the width of the pulses increases. It is clear that the multiple triggers can generate multiple peaks in the entanglement. The asymmetry of the coupling parameters can generate quite different entanglement in the system. This effect can be used to control the entanglement between atoms.

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