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Probabilistic teleportation of an unknown N-atom state using a two-atom nonmaximally entangled state in cavity QED*

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We present a scheme for realizing probabilistic teleportation of an unknown N-atom state via cavity QED. This scheme requires only a nonmaximally entangled pair to be used as a quantum channel, so the requirement of entanglement is reduced. In addition, our scheme does not involve the Bell-state measurement and is insensitive to the cavity decay, which is important from the experimental point of view. If the quantum channel is a two-atom maximally entangled state, teleportation of an unknown N-atom state can be realized by a simpler scheme via cavity QED.

Keywords: probabilistic teleportation without Bell-state measurement, atomic entangled state, nonmaximally entangled pair, cavity QED
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1. Introduction

Quantum teleportation, proposed by Bennett *et al*[1] and experimentally realized by Bouwmeester *et al*,[2] is the process transmitting unknown quantum information from a sender to a remote receiver via a quantum channel with the help of some classical information and without transmission of the carrier. In the original teleportation scheme the sender and the receiver share a bipartite maximally entangled state serving as a quantum channel, which is standard teleportation. But in real experiments, the actual available quantum channels are usually nonmaximally entangled states, such as pure nonmaximally entangled states and mixed entangled states, etc. Using nonmaximally entangled quantum state as a quantum channel results in probabilistic teleportation.[3−5] Quantum teleportation has paid more and more attention both theoretically[6] and experimentally[7] in recent years, due to its promising applications in many branches of quantum information processing.

We note that the cavity QED system is a qualified candidate for demonstrating quantum information processing.[8] But the joint-Bell state measurement is very complex for experimental realization,[9] and thus has been a troublemaker. In order to avoid the difficulty, Zheng *et al*[10−14] have proposed a scheme for the teleportation of an unknown atomic state without Bell-state measurement based cavity QED respectively. Ye and Guo[11] and Jin *et al*[15] proposed schemes for teleporting a two-atom entangled state without the Bell-state measurement. Wang *et al*[16] suggested implementing teleportation of a N-atom state via a two-atom entangled state without the Bell-state measurement in cavity QED. These schemes[10−16] all use maximally entangled states as a quantum channel. Recently, Dong *et al*[17] and Xue *et al*[18] have presented a scheme of teleporting an atomic entangled state with a nonmaximally entangled state as a quantum channel, separately. The schemes suffer submit to the affection of cavity decay.

In the present paper, we propose an experimental scheme for teleporting an unknown N-atom state using a two-atom nonmaximally entangled state without the Bell-state measurement in cavity QED. The considerable advantage of the scheme is that teleporting an unknown N-atom state only needs a two-atom nonmaximally entangled state, which greatly reduces the amount of entanglement and thus needs less classical bits. In addition, during the passage of the atoms through the cavity field, the cavity is only virtually excited and no transfer of quantum information will happen between the atoms and the cavity, thus the

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scheme is insensitive to cavity decay, which is important from the experimental point of view. Meanwhile, our scheme does not involve the Bell-state measurement.

The rest of this paper is organized as follows: in Section 2 we describe the evolution of atomic states in two types of cavities involved in our scheme for teleportation; Section 3 is devoted to the part for probabilistic teleportation of an unknown N-atom state; Section 4 gives the conclusion and remarks on the experimental feasibility of our scheme.

2. Evolution of atomic states in two types of cavities

First we give the evolution of the atomic states in a thermal cavity. Considering K identical two-level atoms simultaneously interacting with a single-mode cavity field, driven by a classical field. In the strong driving regime $2\Omega \gg \delta, g$ and in the case of $\delta \gg g/2$ (Ω is the Rabi frequency of the classical field, g is the atom-cavity coupling strength and δ is the detuning between the atomic transition frequency and the cavity frequency), there exists no energy exchange between the atomic system and the cavity. Then in the interaction picture, the effective interaction Hamilton (let $\hbar = 1$) reads[19]

$$H_{\rm eff} = 2(\Omega S_{\rm x} + \lambda S_{\rm x}^2)\,. \tag{1}$$

Where $\lambda = g^2/2\delta$, $S_{\rm x} = \frac{1}{2}\sum_{j=1}^{K}(S_j^+ + S_j^-)$, $S_j^+ = |e_j\rangle\langle g_j|$, and $S_j^- = |g_j\rangle\langle e_j|$, with $|e_j\rangle$ and $|g_j\rangle (j = 1, 2, \ldots, K)$ being the excited and ground states of the jth atom respectively. Here we have assumed that the transition frequency of atoms is equal to the frequency of classical field. We note that the effective Hamilton is independent of the cavity field state, allowing it to be in a thermal state. Then the evolution operator of the system is given by

$$U(t) = {\rm e}^{-{\rm i}H_{\rm eff}t} = {\rm e}^{-{\rm i}2(\Omega S_{\rm x} + \lambda S_{\rm x}^2)t}. \tag{2}$$

Using the representation of the operator S_z, the atomic states $|g_1 g_2 \ldots g_K\rangle$ and $|e_1 e_2 \ldots e_K\rangle$ can be expressed as $|K/2, -K/2\rangle$ and $|K/2, K/2\rangle$, respectively. On the other hand, such states can be expanded in terms of the eigenstates of $S_{\rm x}$[19] as follows:

$$\begin{aligned} |K/2, -K/2\rangle &= \sum_{M=-K/2}^{M=K/2} C_M |K/2, M\rangle_{\rm x}, \\ |K/2, K/2\rangle &= \sum_{M=-K/2}^{M=K/2} C_M (-1)^{K/2-M} |K/2, M\rangle_{\rm x}\,. \end{aligned} \tag{3}$$

Thus, if the K atoms are initially in the state $|g_1 g_2 \ldots g_K\rangle$, the evolution of the system will be

$$\sum_{M=-K/2}^{M=K/2} C_M {\rm e}^{-{\rm i}2(\Omega M + \lambda M^2)t} |K/2, M\rangle_{\rm x}\,.$$

When K is even, then M is an integer. With the choice of $\lambda t = \pi/4$ and $\Omega t = n\pi$, the system will evolve to[19]

$$\begin{aligned} &\frac{1}{\sqrt{2}} \sum_{M=-K/2}^{M=K/2} C_M [{\rm e}^{-{\rm i}\pi/4} + {\rm e}^{{\rm i}\pi/4}(-1)^M] |K/2, M\rangle_{\rm x} \\ &\quad = \frac{1}{\sqrt{2}} [{\rm e}^{-{\rm i}\pi/4} |g_1 g_2 \ldots g_K\rangle \\ &\qquad + {\rm e}^{{\rm i}\pi/4}(-1)^{K/2} |e_1 e_2 \ldots e_K\rangle]\,. \end{aligned} \tag{4}$$

On the other hand, for the case where K is odd, we choose $\lambda t = \pi/4$ and $\Omega t = (2n + 3/4)\pi$. Then the system will evolve to[19]

$$\frac{1}{\sqrt{2}} {\rm e}^{{\rm i}(7/8)\pi} [{\rm e}^{-{\rm i}\pi/4} |g_1 g_2 \ldots g_K\rangle + {\rm e}^{{\rm i}\pi/4}(-1)^{(1+K)/2} |e_1 e_2 \ldots e_K\rangle]\,. \tag{5}$$

By this way we obtain a multi-atom GHZ state. Similarly, we obtain the following evolutions:

$$\begin{aligned} |e_1 e_2 \ldots e_{K-1} e_K\rangle &\xrightarrow{U(t)} \frac{1}{\sqrt{2}} [{\rm e}^{-{\rm i}\pi/4} |e_1 e_2 \ldots e_{K-1} e_K\rangle + {\rm e}^{{\rm i}\pi/4}(-1)^{K/2} |g_1 g_2 \ldots g_{K-1} g_K\rangle]\,, \\ |e_1 e_2 \ldots e_{K-1} g_K\rangle &\xrightarrow{U(t)} \frac{1}{\sqrt{2}} [{\rm e}^{-{\rm i}\pi/4} |e_1 e_2 \ldots e_{K-1} g_K\rangle + {\rm e}^{{\rm i}\pi/4}(-1)^{K/2} |g_1 g_2 \ldots g_{K-1} e_K\rangle]\,, \\ |g_1 g_2 \ldots g_{K-1} e_K\rangle &\xrightarrow{U(t)} \frac{1}{\sqrt{2}} [{\rm e}^{-{\rm i}\pi/4} |g_1 g_2 \ldots g_{K-1} e_K\rangle + {\rm e}^{{\rm i}\pi/4}(-1)^{K/2} |e_1 e_2 \ldots e_{K-1} g_K\rangle]\,, \end{aligned} \tag{6}$$

with K being even ($\lambda t=\pi/4$, $\Omega t=n\pi$), and

$$\begin{aligned}
&|e_1e_2\ldots e_{K-1}e_K\rangle \xrightarrow{U(t)} \frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}(7/8)\pi}[\mathrm{e}^{-\mathrm{i}\pi/4}|e_1e_2\ldots e_{K-1}e_K\rangle+\mathrm{e}^{\mathrm{i}\pi/4}(-1)^{(K+1)/2}|g_1g_2\ldots g_{K-1}g_K\rangle],\\
&|e_1e_2\ldots e_{K-1}g_K\rangle \xrightarrow{U(t)} \frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}(7/8)\pi}[\mathrm{e}^{-\mathrm{i}\pi/4}|e_1e_2\ldots e_{K-1}g_K\rangle+\mathrm{e}^{\mathrm{i}\pi/4}(-1)^{(K+1)/2}|g_1g_2\ldots g_{K-1}e_K\rangle],\\
&|g_1g_2\ldots g_{K-1}e_K\rangle \xrightarrow{U(t)} \frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}(7/8)\pi}[\mathrm{e}^{-\mathrm{i}\pi/4}|g_1g_2\ldots g_{K-1}e_K\rangle+\mathrm{e}^{\mathrm{i}\pi/4}(-1)^{(K+1)/2}|e_1e_2\ldots e_{K-1}g_K\rangle], \qquad (7)
\end{aligned}$$

with K being odd ($\lambda t=\pi/4$, $\Omega t=2n\pi+3\pi/4$).

Now we describe the evolutions of two identical two-level atoms in a vacuum cavity. Considering the two identical atoms 1, 2 simultaneously interacting with a single-mode vacuum cavity field. In the case of $\delta\gg g$ (δ is the detuning between the atomic transition frequency and the cavity frequency, g is the atom-cavity coupling strength), there exists no energy exchange between the atomic system and the cavity. In the interaction picture, the effective interaction Hamilton for the system is (let $\hbar=1$)[20]

$$H'_{\mathrm{eff}}=\lambda'\left(\sum_{j=1}^{2}|e_j\rangle\langle e_j|+\sum_{i,j=1,i\neq j}^{2}S_i^+S_j^-\right), \qquad (8)$$

where $\lambda'=g^2/\delta$. So the corresponding evolution operator of the system is

$$\begin{aligned}
U(t')&=\mathrm{e}^{-\mathrm{i}H'_{\mathrm{eff}}t'}\\
&=\exp\left[-\mathrm{i}\lambda't'\left(\sum_{j=1}^{2}|e_j\rangle\langle e_j|+\sum_{i,j=1,i\neq j}^{2}S_i^+S_j^-\right)\right]. \qquad (9)
\end{aligned}$$

In the interaction time t', we can obtain the evolution with different initial states

$$|g_1g_2\rangle \xrightarrow{U(t')} |g_1g_2\rangle, \qquad (10a)$$

$$\begin{aligned}
|g_1e_2\rangle \xrightarrow{U(t')} &\mathrm{e}^{-\mathrm{i}\lambda't'}(\cos(\lambda't')|g_1e_2\rangle\\
&-\mathrm{i}\sin(\lambda't')|e_1g_2\rangle), \qquad (10b)
\end{aligned}$$

$$\begin{aligned}
|e_1g_2\rangle \xrightarrow{U(t')} &\mathrm{e}^{-\mathrm{i}\lambda't'}(\cos(\lambda't')|e_1g_2\rangle\\
&-\mathrm{i}\sin(\lambda't')|g_1e_2\rangle), \qquad (10c)
\end{aligned}$$

$$|e_1e_2\rangle \xrightarrow{U(t')} \mathrm{e}^{-\mathrm{i}2\lambda't'}|e_1e_2\rangle. \qquad (10d)$$

3. Process for realizing probabilistic teleportation

In this section, we describe how to implement the teleportation of an unknown N-atom state via only one two-atom nonmaximally entangled state without the Bell-state measurement. Alice has N ($N\geq 1$) atoms numbered $1,2,\ldots,N$ in an unknown entangled state, which is to be teleported to Bob. We suppose the state is

$$|\phi\rangle=\alpha|g_1g_2\ldots g_N\rangle+\beta|e_1e_2\ldots e_N\rangle, \qquad (11)$$

where α and β are unknown coefficients, and $|\alpha|^2+|\beta|^2=1$. Alice and Bob share a nonmaximally entangled pair (atom A on Alice's side, atom B on Bob's side) used as a quantum channel

$$|\Phi\rangle=a|g_{\mathrm{A}}e_{\mathrm{B}}\rangle+b|e_{\mathrm{A}}g_{\mathrm{B}}\rangle, \qquad (12)$$

where $|a|^2+|b|^2=1$. So the whole system is in the state of

$$\begin{aligned}
|\Psi\rangle=&(\alpha|g_1g_2\ldots g_N\rangle+\beta|e_1e_2\ldots e_N\rangle)\\
&\otimes(a|g_{\mathrm{A}}e_{\mathrm{B}}\rangle+b|e_{\mathrm{A}}g_{\mathrm{B}}\rangle). \qquad (13)
\end{aligned}$$

Suppose that atoms $1,2,\ldots,N$ and the two entangled atoms A, B are identical. Alice sends atoms $1,2,\ldots,(N-1)$ simultaneously across a single-mode cavity in a thermal state, and at the same time these atoms are driven by a classical field. The system evolution is described by Eq.(2), with the choice of $\lambda t=\pi/4$ and $\Omega t=n\pi$ ($\Omega t=2n\pi+3\pi/4$) when N is odd (even), the whole state evolves to

$$\begin{aligned}
|\Psi\rangle=&\frac{1}{\sqrt{2}}[|g_1g_2\ldots g_{N-1}\rangle\mathrm{e}^{\mathrm{i}\varphi}(\alpha|g_N\rangle+\mathrm{i}d_1(N)\beta|e_N\rangle)\\
&+|e_1e_2\ldots e_{N-1}\rangle\mathrm{e}^{\mathrm{i}\psi_1(N)}(\alpha|g_N\rangle\\
&-\mathrm{i}d_1(N)\beta|e_N\rangle)](a|g_{\mathrm{A}}e_{\mathrm{B}}\rangle+b|e_{\mathrm{A}}g_{\mathrm{B}}\rangle), \qquad (14)
\end{aligned}$$

where

$$\varphi=-\pi/4(\varphi=5\pi/8)$$

when N is odd (even), $\psi_1(N)$ and $d_1(N)$, which are functions of N, are defined as

$$\psi_1(N)=\begin{cases}\pi/4, & \text{if} \quad N=4k-3,\\ 5\pi/4, & \text{if} \quad N=4k-1,\\ \pi/8, & \text{if} \quad N=4k-2,\\ 9\pi/8, & \text{if} \quad N=4k,\end{cases} \qquad (15)$$

$$d_1(N)=\begin{cases}1, & \text{if} \quad N=4k-3,\\ -1, & \text{if} \quad N=4k-1,\\ -1, & \text{if} \quad N=4k-2,\\ 1, & \text{if} \quad N=4k,\end{cases} \tag{16}$$

where k is a natural number. Then a cavity initially prepared in vacuum state $|0\rangle$ is introduced in Alice's location. Alice will let atoms N and A pass through the cavity simultaneously. The system evolution is described by Eq.(9), the whole state evolves to

$$\begin{aligned}|\Psi\rangle=\frac{1}{\sqrt{2}}\{&\alpha a\mathrm{e}^{\mathrm{i}\varphi}|g_1g_2\ldots g_{N-1}g_Ng_\mathrm{A}\rangle|e_\mathrm{B}\rangle+\mathrm{i}\beta bd_1(N)\mathrm{e}^{\mathrm{i}\varphi}\mathrm{e}^{-\mathrm{i}2\lambda' t'}|g_1g_2\ldots g_{N-1}e_Ne_\mathrm{A}\rangle|g_\mathrm{B}\rangle\\ &+\alpha a\mathrm{e}^{\mathrm{i}\psi_1(N)}|e_1e_2\ldots e_{N-1}g_Ng_\mathrm{A}\rangle|e_\mathrm{B}\rangle-\mathrm{i}\beta bd_1(N)\mathrm{e}^{\mathrm{i}\psi_1(N)}\mathrm{e}^{-\mathrm{i}2\lambda' t'}|e_1e_2\ldots e_{N-1}e_Ne_\mathrm{A}\rangle|g_\mathrm{B}\rangle\\ &+\mathrm{e}^{-\mathrm{i}\lambda' t'}\mathrm{e}^{\mathrm{i}\varphi}|g_1g_2\ldots g_{N-1}g_Ne_\mathrm{A}\rangle[\alpha b\cos(\lambda' t')|g_\mathrm{B}\rangle+\beta ad_1(N)\sin(\lambda' t')|e_\mathrm{B}\rangle]\\ &+\mathrm{e}^{\mathrm{i}\varphi}\mathrm{e}^{-\mathrm{i}\lambda' t'}\mathrm{e}^{-\mathrm{i}\pi/2}|g_1g_2\ldots g_{N-1}e_Ng_\mathrm{A}\rangle[\alpha b\sin(\lambda' t')|g_\mathrm{B}\rangle-\beta ad_1(N)\cos(\lambda' t')|e_\mathrm{B}\rangle]\\ &+\mathrm{e}^{\mathrm{i}\psi_1(N)}\mathrm{e}^{-\mathrm{i}\lambda' t'}|e_1e_2\ldots e_{N-1}g_Ne_\mathrm{A}\rangle[\alpha b\cos(\lambda' t')|g_\mathrm{B}\rangle-\beta ad_1(N)\sin(\lambda' t')|e_\mathrm{B}\rangle]\\ &+\mathrm{e}^{\mathrm{i}\psi_1(N)}\mathrm{e}^{-\mathrm{i}\lambda' t'}\mathrm{e}^{-\mathrm{i}\pi/2}|e_1e_2\ldots e_{N-1}e_Ng_\mathrm{A}\rangle[\alpha b\sin(\lambda' t')|g_\mathrm{B}\rangle+\beta ad_1(N)\cos(\lambda' t')|e_\mathrm{B}\rangle]\}.\end{aligned} \tag{17}$$

After interaction time $\lambda' t'=\arctan|b|/|a|$, atoms N and A fly out of the cavity. Then Alice carries out local measurements on the singled atoms $1,2,\ldots,N$, and A (in fact, Alice only needs to measure the atoms $N-1,N$,A) with the base $\{|g\rangle,|e\rangle\}$. If atoms $1,2,\ldots,N$, and A are in the state $|g_1g_2\ldots g_{N-1}g_Ne_\mathrm{A}\rangle$ or $|e_1e_2\ldots e_{N-1}g_Ne_\mathrm{A}\rangle$, the state of atom B will collapse into the following state with the aid of a rotational operation: $\alpha|g_\mathrm{B}\rangle+\beta|e_\mathrm{B}\rangle$.

Now Bob introduces N additional atoms initially prepared in the state $|g_{N+1}g_{N+2}\ldots g_{2N}\rangle$. Then Bob has atoms B, $N+1,N+2,\ldots$, and $2N$ simultaneously across a single-mode cavity in a thermal state, and at the same time these atoms are driven by a classical field. The system evolution is described by Eq.(2), with the choice of $\lambda t=\pi/4$ and $\Omega t=n\pi$ ($\Omega t=2n\pi+3\pi/4$) when N is odd (even), the whole state evolves to

$$\begin{aligned}|\Psi\rangle=\frac{1}{\sqrt{2}}[&|g_\mathrm{B}\rangle\mathrm{e}^{\mathrm{i}\varphi}(\alpha|g_{N+1}g_{N+2}\ldots g_{2N}\rangle\\ &+\mathrm{i}\beta d_2(N)|e_{N+1}e_{N+2}\ldots e_{2N}\rangle)\\ &+|e_\mathrm{B}\rangle\mathrm{e}^{\mathrm{i}\psi_2(N)}(\alpha|e_{N+1}e_{N+2}\ldots e_{2N}\rangle\\ &-\mathrm{i}\beta d_2(N)|g_{N+1}g_{N+2}\ldots g_{2N}\rangle)],\end{aligned} \tag{18}$$

where $\psi_2(N)$ and $d_2(N)$ are defined as

$$\psi_2(N)=\begin{cases}5\pi/4, & \text{if} \quad N=4k-3,\\ \pi/4, & \text{if} \quad N=4k-1,\\ 9\pi/8, & \text{if} \quad N=4k-2,\\ \pi/8, & \text{if} \quad N=4k,\end{cases} \tag{19}$$

$$d_2(N)=\begin{cases}-1, & \text{if} \quad N=4k-3,\\ 1, & \text{if} \quad N=4k-1,\\ 1, & \text{if} \quad N=4k-2,\\ -1, & \text{if} \quad N=4k.\end{cases} \tag{20}$$

We perform the following unitary transformations respectively, on atoms $N+1$ and $N+2$:

$$R=\begin{pmatrix}\mathrm{e}^{-\mathrm{i}\pi/4} & 0\\ 0 & 1\end{pmatrix}.$$

The transformation can be achieved by using the Stark effects in a static electric field.[21] Then we have

$$\begin{aligned}|\Psi\rangle=\frac{1}{\sqrt{2}}[&|g_\mathrm{B}\rangle\mathrm{e}^{\mathrm{i}\varphi}\mathrm{e}^{-\mathrm{i}\pi/2}(\alpha|g_{N+1}g_{N+2}\ldots g_{2N-1}\rangle-\beta d_2(N)|e_{N+1}e_{N+2}\ldots e_{2N-1}\rangle)\\ &+|e_\mathrm{B}\rangle\mathrm{e}^{\mathrm{i}\psi_2(N)}(\alpha|e_{N+1}e_{N+2}\ldots e_{2N-1}\rangle-\beta d_2(N)|g_{N+1}g_{N+2}\ldots g_{2N-1}\rangle)\,.\end{aligned}$$

Bob carries out local measurements on the singled atom B with the base $\{|g\rangle, |e\rangle\}$ and performs corresponding unitary transformations on the atoms $N+1$, $N+2, \ldots$, $2N$ (see Table 1), which can be easily implemented,[22]

Table 1. Corresponding unitary transformations on the atoms $N+1$, $N+2, \ldots$, $2N$ according to the results of measurement on atom B.

Result of measurement on atom B	Corresponding unitary transformations on the atoms $N+1$, $N+2, \ldots$, $2N$
$\|g_{\rm B}\rangle$	$(I_{N+1})^{[1-d_2(N)]/2}[\sigma_{z(N+1)}]^{[1+d_2(N)]/2} \otimes I_{N+2} \otimes \cdots \otimes I_{2N}$
$\|e_{\rm B}\rangle$	$[\sigma_{x(N+1)}]^{[1-d_2(N)]/2}[{\rm i}\sigma_{y(N+1)}]^{[1+d_2(N)]/2} \otimes \sigma_{x(N+2)} \otimes \cdots \otimes \sigma_{x(2N)}$

Then we obtain

$$\alpha|g_{N+1}g_{N+2}\ldots g_{2N}\rangle + \beta|e_{N+1}e_{N+2}\ldots e_{2N}\rangle,$$

which is exactly the initial state and indicates the success in the teleportation. If atoms $1, 2, \ldots, N$, and A are in the states

$$|g_1g_2\ldots g_{N-1}g_Ng_{\rm A}\rangle, \quad |g_1g_2\ldots g_{N-1}e_Ne_{\rm A}\rangle,$$
$$|e_1e_2\ldots e_{N-1}g_Ng_{\rm A}\rangle, \quad |e_1e_2\ldots e_{N-1}e_Ne_{\rm A}\rangle,$$
$$|g_1g_2\ldots g_{N-1}e_Ng_{\rm A}\rangle, \quad |e_1e_2\ldots e_{N-1}e_Ng_{\rm A}\rangle,$$

the teleportation will fail. The total probability of the successful teleportation is $|a|^2|b|^2$. If $|a| = |b| = 1/\sqrt{2}$, i.e. for a quantum channel composed of a two-atom maximally entangled state, the total probability equals 1/4. In fact, if the quantum channel is a two-atom maximally entangled state, we can reconstruct the original entangled state in another way without introducing a vacuum cavity. The detailed procedure is given as follows. Firstly, Alice sends atoms $1, 2, \ldots, N$, and A simultaneously across a single-mode cavity in a thermal state, and at the same time these atoms are driven by a classical field. Secondly, Alice conducts local measurements on the singled atoms $1, 2, \ldots$, N, and A and informs Bob of her measurement results via a classical communication. Thirdly, according to the measurement results, Bob performs a corresponding unitary transformation on the atom B and the state of atom B will collapse into the state $\alpha|g_{\rm B}\rangle + {\rm i}\beta|e_{\rm B}\rangle$; Fourthly, Bob introduces N additional atoms initially prepared in the state $|g_{N+1}g_{N+2}\ldots g_{2N}\rangle$ and has atoms B, $N+1$, $N+2, \ldots$, and $2N$ simultaneously across a single-mode cavity in a thermal state, and at the same time these atoms are driven by a classical field. Finally, Bob carries out local measurements on the singled atom B and can reconstruct the original atomic entangled state by performing the corresponding unitary transformations on the atoms $N+1$, $N+2, \ldots$, and $2N$. In this case, the total probability of the successful teleportation is 1. Obviously, when the quantum channel is a two-atom maximally entangled state, this scheme is much simpler than the former.

4. Discussion and summary

Now we give a brief discussion on the experiment. For the Rydberg atoms with principal quantum numbers being 49, 50 and 51, the radiative time is about $T_{\rm r} = 3 \times 10^{-2}$ s, and the coupling constant is $g = 2\pi \times 24$ kHz.[23] With the choice of $\delta = 10$ g,[23] the required atom-thermal-cavity-field interaction time is on the order of $T \approx 10^{-4}$ s and atom-vacuum-cavity-field interaction time is about $T' \approx 2 \times 10^{-4}$ s. Then the time required to complete the whole procedure is much shorter than $T_{\rm r}$. The scheme requires $N-1$ or $N+1$ atoms to be sent simultaneously through a thermal cavity and two atoms be sent simultaneously through a vacuum cavity. But in real case, we cannot achieve simultaneousness to perfection. Calculation on the error suggests that it only slightly affects the fidelity of the reconstruct state.[20,24]

In summary, we have proposed a simple scheme to realize the teleportation of an unknown multi-atom state via only one two-atom nonmaximally entangled state in cavity QED. The successful probability of the teleportation is dependent on the superposition coefficient of the state used as a quantum channel. The scheme does not involve the Bell-state measurement, which can largely reduce the experimental steps and time and thus will increase the fidelity of teleportation in experiment. Another important feature is that teleporting a multi-atom state requires only a nonmaximally entangled pair to be used as a quantum channel, so we reduce the requirement for entanglement. In contrast to the schemes of probabilistic teleportation in Ref.[17,18], in our protocol, we introduce a cavity initially prepared in vacuum state. The cavity is only virtually excited and no transfer of quantum information will happen between the cavity and the atoms during the passage of the atoms through the

vacuum cavity. Thus our scheme is insensitive to cavity decay and the requirement for the quality factor of the cavity is greatly loosened, whereas in the previous scheme,[17,18] receiver needs to introduce a vacuum cavity with a high-quality factor and the resonant interaction to happen between the atom and the cavity, thus the schemes submit to the affection of cavity decay. If the quantum channel is a two-atom maximally entangled state, we propose another simpler scheme for realizing teleportation of an unknown N-atom state via cavity QED. In contrast to the scheme in Ref.[16], our protocol can simultaneously send the additional atoms and atom B to the cavity, whereas the previous scheme[16] cannot do so simultaneously, so our scheme is simpler and the overall time for the reconstruction of the original entangled state will be much shorter.

References

[1] Bennett C H, Brassard G, Grépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895

[2] Bouwmeester D, Pan J W, Mattle K, Eibl M, Weinfurter H and Zeilinger A 1997 *Nature* **390** 575

[3] Ye L and Guo G C 2002 *Chin. Phys.* **11** 996

[4] Dai H Y, Li C Z and Chen P X 2003 *Chin. Phys.* **12** 1354

[5] Lin X and Li H C 2005 *Chin. Phys.* **14** 1724

[6] Yang M, Song W and Cao Z L 2005 *Phys. Rev.* A **71** 012308

[7] Pan J W, Gasparoni S, Aspelmeyer M, Jennewein T and Zeilinger A 2003 *Nature* **421** 721

[8] Raimond J M, Brune M and Haroche S 2001 *Rev. Mod. Phys.* **73** 565

[9] Barrett M D, Chiaverini J, Schaetz T, Britton J, Itano W M, Jost J D, Knill E, Langer C, Leibfried D, Ozeri R and Wineland D J 2004 *Nature* **429** 737

[10] Zheng S B 2004 *Phys. Rev.* A **69** 064302

[11] Ye L and Guo G C 2004 *Phys. Rev.* A **70** 054303

[12] Yang M and Cao Z L 2004 quant-ph/0411195

[13] Yang Z B 2006 *J. Phys.* B: *At. Mol. Opt. Phys.* **39** 603

[14] Chen C Y 2006 *Commun. Theor. Phys.* **45** 79

[15] Jin L H, Jin X R and Zhang S 2005 *Phys. Rev.* A **72** 024305

[16] Wang X W, Liu X and Fang M F 2006 *Chin. Phys.* **15** 676

[17] Dong P, Yang M and Cao Z L 2006 *Commun. Theor. Phys.* **45** 621

[18] Xue Z Y, Yi Y M and Cao Z L 2006 *Commun. Theor. Phys.* **45** 83

[19] Zheng S B 2003 *Phys. Rev.* A **68** 035801

[20] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392

[21] Domokos P, Paimond J M, Brune M and Haroche S 1995 *Phys. Rev.* A **52** 3554

[22] Scully M O and Zubairy M S 1997 *Quantum Optics* (New York: Cambridge University Press) p145

[23] Osnaghi S, Bertet P, Auffeves A, Maioli P, Brune M, Raimond J M and Haroche S 2001 *Phys. Rev. Lett.* **87** 037902

[24] Zheng S B 2002 *Phys. Rev.* A **66** 060303