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On the ponderomotive force and the effect of loss reaction on parametric instability*

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In this paper, the growth rate, ponderomotive force and the exciting condition for parametric instability are derived by considering the loss reaction using a new method. On the basis of the hydrodynamic equations, we take the production and loss reactions in plasma into account to derive the coupling equations for the electron plasma oscillation and ion acoustic oscillation, and obtain the growth rate for the parametric instability, the ponderomotive force and the exciting condition. The result shows that (a) the production reaction has no effect on the parametric instability, and the effect of loss reaction on the parametric instability is a damping one, (b) the more intensive the external field or pump is, the larger the growth rate is, (c) there exist two modes of the ponderomotive force, i.e. the high frequency mode and the low frequency mode, and (d) when ponderomotive force counteracts the damping force, the oscillations become non-damping and non-driving. The ratio of the electron plasma oscillation to ion acoustic oscillation is independent of the loss reaction and the external field.

Keywords: parametric instability, loss reaction, ponderomotive force
PACC: 9420R, 9420V, 5235M, 5235P

1. Introduction

In the linear approximation, a plasma supports two distinct, independent modes of oscillation: the high-frequency electron plasma oscillation and low-frequency ion-acoustic oscillation. It is also well known that a strong nonlinear process in which these two basic oscillations are coupled and simultaneously excited by means of a single high-frequency electromagnetic field will take place.

This nonlinear process, namely the parametric instability, has aroused considerable interest since the 1960s. It is now recognized especially that, to a large extent, the wave–wave coupling plays an important role in the ionospheric F-region modification.[1−8] Historically, Alive and Silin[9,10] were the first who made a systematic study of the parametric instability. Based on the hydrodynamic equation, he obtained the interesting results for the cold plasma. An experiment performed by Stern and Tzoar[11] showed the excitation of a couple of electron plasma wave and ion-acoustic wave under a relatively weak external field. Lee and Su[12] investigated the parametric coupling using the hydrodynamic description of the thermal plasma. However, the above authors did not account for such damping actions as the collision and the Landau damping. From the view of Vlasov equation, Jackson[13] took the Landau damping into account and achieved the general description of the problem. Also, Goldman[14,15] estimated the threshold for the excitation by making allowance for the Landau damping, but their work was based on the Green function perturbative method. Nishikawa[16,17] considered the collision frequency as the damping constant and obtained some interesting results for the threshold and the growth rate. In addition, Bezzerides and Weinstock[18] viewed the unstable waves themselves as damping factor which disturbs the moving coherence of particles by perturbing individual particle orbits. In fact, we believe that there are more reactions affecting the growth of the parametric instability in plasma besides the Landau damping and the collision, such as the loss reaction in ionospheric plasma. So, based on the hydrodynamic equations, we derive the coupling equations for the electron plasma oscillation and ion

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acoustic oscillation, and obtain the growth rate for the parametric instability, the ponderomotive force and the exciting condition.

2. Basic equations

We start from the equation of motion

$$m^\alpha n^\alpha \Big(\frac{\partial}{\partial t} + \boldsymbol{v}^\alpha \cdot \frac{\partial}{\partial r}\Big)\boldsymbol{v}^\alpha = e^\alpha n^\alpha \boldsymbol{E}_0 - e^\alpha n^\alpha \nabla \Phi - T^\alpha \nabla n^\alpha \,, \tag{1}$$

the equation of continuity

$$\frac{\partial n^\alpha}{\partial t} + \nabla \cdot (n^\alpha \boldsymbol{v}^\alpha) = q - \beta n^\alpha \,, \tag{2}$$

and the Poisson equation

$$\nabla^2 \Phi = -\frac{1}{\varepsilon_0}\sum (e^\alpha n^\alpha) \,, \tag{3}$$

where the superscript α denotes either electron (no α) or ion ($\alpha = i$), and m the mass, n the density, $\boldsymbol{v}$ the velocity, e the electric charge, T the temperature, Φ the electrostatic potential, q the production coefficient and β the loss coefficient. We consider the plasma to be nonstationary and then take the production and loss coefficient into account in writing the equation of continuity, and write the term of loss in linear type (βn^α).

We consider the system of a weakly and singly ionized and collisionless plasma located in a high frequency external transverse field

$$\boldsymbol{E}_0 = \frac{1}{2}(\boldsymbol{E}_{\boldsymbol{k}_0} + \boldsymbol{E}_{-\boldsymbol{k}_0}) \,, \tag{4}$$

where

$$\boldsymbol{E}_{\pm\boldsymbol{k}_0} = \boldsymbol{E}_0 \exp[\pm \mathrm{j}(\boldsymbol{k}_0 \cdot \boldsymbol{r} - \omega_0 t)],$$

and suppose that $\boldsymbol{E}_0$ is spatially homogeneous (i.e. $\boldsymbol{k}_0 \approx 0$) and ω_0 is slightly above the electron frequency ω_{pe}. In addition, we assume $(\boldsymbol{k}_0, \omega_0)$ to satisfy the matching relation, namely, $\boldsymbol{k}_0 = \boldsymbol{k} + \Delta\boldsymbol{k}$, $\omega_0 = \omega + \Delta\omega$, where $(\boldsymbol{k}, \omega)$ and $(\Delta\boldsymbol{k}, \Delta\omega)$ represent the electron plasma wave and ion-acoustic wave, respectively.

The external field or pump causes the charged particles of plasma to oscillate back and forth. By linearizing Eq.(1) and then taking the $(\boldsymbol{k}_0, \omega_0)$ Fourier component, we obtain

$$n^\alpha_{\boldsymbol{k}_0} = \Phi_{\boldsymbol{k}_0} = 0 \tag{5}$$

and

$$\boldsymbol{v}^\alpha_{\pm\boldsymbol{k}_0} = \pm \mathrm{j}\frac{e^\alpha \boldsymbol{E}_{\pm\boldsymbol{k}_0}}{2\omega_0 m^\alpha} \,, \tag{6}$$

where $\mathrm{j} = \sqrt{-1}$.

We take the $(\boldsymbol{k}, \omega)$ Fourier component of Eq.(1) for electron and retain the first order of nonlinear term, and then take the scalar product of $\boldsymbol{k}$ with it, thus have

$$n_0[\omega(\boldsymbol{k}\cdot\boldsymbol{v}_{\boldsymbol{k}}) + (\Delta\boldsymbol{k}\cdot\boldsymbol{v}_{\boldsymbol{k}_0})(\boldsymbol{k}\cdot\boldsymbol{v}_{-\Delta\boldsymbol{k}})] + \omega_0(\boldsymbol{k}\cdot\boldsymbol{v}_{\boldsymbol{k}_0})n_{-\Delta k} = -\mathrm{j}\frac{e(\boldsymbol{k}\cdot\boldsymbol{E}_{\boldsymbol{k}_0})}{2m} n_{-\Delta k} - \omega^2_{\mathrm{pe}}(n^{\mathrm{i}}_k - n_k) + k^2 v^2_{T_{\mathrm{e}}} n_k \,. \tag{7}$$

Similarly we have the $(l\Delta\boldsymbol{k}, l\Delta\omega)$ Fourier component for the electron and the ion and take the scalar product of $\Delta\boldsymbol{k}$ with them, then obtain

$$\begin{aligned} &n_0[\Delta\omega(\Delta\boldsymbol{k}\cdot\boldsymbol{v}^\alpha_{l\Delta\boldsymbol{k}}) + (\boldsymbol{k}\cdot\boldsymbol{v}^\alpha_{l\boldsymbol{k}_0})(\Delta\boldsymbol{k}\cdot\boldsymbol{v}^\alpha_{-l\boldsymbol{k}})] \\ &+\omega_0(\Delta\boldsymbol{k}\cdot\boldsymbol{v}^\alpha_{l\boldsymbol{k}_0})n^\alpha_{-lk} = l\mathrm{j}\frac{e(\Delta\boldsymbol{k}\cdot\boldsymbol{E}_{l\boldsymbol{k}_0})}{2m^\alpha} n^\alpha_{-lk} \\ &+(-1)^\gamma \omega^2_{\mathrm{p}\alpha}(n^{\mathrm{i}}_{l\Delta k} - n_{l\Delta k}) + \Delta k^2 v^2_{T_\alpha} n^\alpha_{l\Delta k} \,, \end{aligned} \tag{8}$$

where

$$\omega^2_{\mathrm{p}\alpha} = \frac{n_0 e^2}{\varepsilon_0 m^\alpha}$$

and

$$v^2_{T_\alpha} = \frac{T_\alpha}{m^\alpha}$$

denote the plasma frequency and the thermal velocity of the particle α, respectively, and $\gamma = 0$ for ion, $\gamma = 1$ for electron, and $l = \pm 1$.

Now we find the relation between the quantities $(l\boldsymbol{k}\cdot\boldsymbol{v}^\alpha_{l\boldsymbol{k}})$ or $(l\Delta\boldsymbol{k}\cdot\boldsymbol{v}^\alpha_{l\Delta\boldsymbol{k}})$ and n^α_{lk} or $n^\alpha_{l\Delta k}$, respectively. By taking Fourier transform of Eq.(2) for electron and ion and retaining the first order of nonlinear term, and then synthesizing them, we have the $(l\boldsymbol{k}, l\omega)$ Fourier component

$$n_0(l\boldsymbol{k}\cdot\boldsymbol{v}^\alpha_{l\boldsymbol{k}}) = (l\omega + \mathrm{j}\beta)n^\alpha_{lk} - l(\boldsymbol{k}\cdot\boldsymbol{v}^\alpha_{l\boldsymbol{k}_0})n^\alpha_{-l\Delta k} \tag{9}$$

and the $(l\Delta\boldsymbol{k}, l\Delta\omega)$ Fourier component

$$n_0(l\Delta\boldsymbol{k}\cdot\boldsymbol{v}^\alpha_{l\Delta\boldsymbol{k}}) = (l\Delta\omega + \mathrm{j}\beta)n^\alpha_{l\Delta k} + l(\boldsymbol{k}\cdot\boldsymbol{v}^\alpha_{l\boldsymbol{k}_0})n^\alpha_{-lk} \,. \tag{10}$$

Substituting Eqs.(9) and (10) for electron into Eq.(7) and neglecting $\omega^2_{\mathrm{pe}} n^{\mathrm{i}}_k$ due to $m^{\mathrm{i}} \gg m$ and the high frequency $(l\boldsymbol{k}, l\omega)$, then

$$\Big(\omega^2 + \mathrm{j}\beta\omega - \omega^2_{\mathrm{pe}} - k^2 v^2_{T_{\mathrm{e}}} + \frac{e^2 \Sigma_{k_0}\Sigma_{-k_0}}{m^2}\Big)n_k + \mathrm{j}(\omega_0 - 2\Delta\omega + \mathrm{j}\beta)\frac{e\Sigma_{k_0}}{m} n_{-\Delta k} = 0 \,, \tag{11}$$

where

$$\Sigma_{l\boldsymbol{k}_0} = \frac{(\boldsymbol{k}\cdot\boldsymbol{E}_{l\boldsymbol{k}_0})}{2\omega_0}.$$

Similarly, we substitute Eqs.(9) and (10) into Eq.(8), and obtain

$$\Big(\Delta\omega^2+\mathrm{j}l\beta\Delta\omega-\omega_{\mathrm{p}\alpha}^2-\Delta k^2v_{T_\alpha}^2+\frac{e^2\Sigma_{k_0}\Sigma_{-k_0}}{(m^\alpha)^2}\Big)n_{l\Delta k}^\alpha+(-1)^\gamma\mathrm{j}(-l\omega_0+2l\Delta\omega+\mathrm{j}\beta)\frac{e\Sigma_{lk_0}}{m^\alpha}n_{-lk}^\alpha+\omega_{\mathrm{p}\alpha}^2n_{l\Delta k}^{\alpha^*}=0\,,\quad(12)$$

where γ and l are the same as the above.

Due to $m^{\mathrm{i}}\gg m$ and the high frequency$(l\boldsymbol{k},l\omega)$, it is reasonable to assume $n_k^{\mathrm{i}}\approx 0$. Then from Eq.(12) for the ion, we have

$$n_{l\Delta k}^{\mathrm{i}}\approx\frac{-\omega_{\mathrm{pi}}^2}{\Delta\omega^2+\mathrm{j}l\beta\Delta\omega-\omega_{\mathrm{pi}}^2-\Delta k^2v_{T_{\mathrm{i}}}^2+e^2\Sigma_{k_0}\Sigma_{-k_0}/(m^{\mathrm{i}})^2}n_{l\Delta k}\,.\quad(13)$$

By substituting Eq.(13) into Eq.(12) for the electron, and supposing $\Delta\omega(\Delta\omega+\mathrm{j}\beta)\approx\Delta k^2v_{T_{\mathrm{i}}}^2$ and $m^{\mathrm{i}}/m\gg 1$, we have

$$\Big[-\Delta\omega^2+(-1)\mathrm{j}l\beta\Delta\omega+k^2c_s^2-\frac{e^2\Sigma_{k_0}\Sigma_{-k_0}}{mm^{\mathrm{i}}}\Big]n_{l\Delta k}+\mathrm{j}(-l\omega_0+2l\Delta\omega+\mathrm{j}\beta)\frac{e\Sigma_{lk_0}}{m^{\mathrm{i}}}n_{-lk}=0\,,\quad(14)$$

where $c_{\mathrm{s}}^2=(T_{\mathrm{e}}+T_{\mathrm{i}})/m^{\mathrm{i}}$.

Now we rewrite Eqs.(11) and (14) for $l=1$, and for simplification, neglect $2\Delta\omega$ and β compared with ω_0, then obtain

$$\frac{\partial^2}{\partial t^2}n_k+\beta\frac{\partial}{\partial t}n_k+\omega_{\mathrm{L}}'^2n_k=\mathrm{j}\omega_0\frac{e\Sigma_{k_0}}{m}n_{-\Delta k}\quad(15)$$

and

$$\frac{\partial^2}{\partial t^2}n_{\Delta k}+\beta\frac{\partial}{\partial t}n_{\Delta k}+\omega_{\mathrm{S}}'^2n_{\Delta k}=\mathrm{j}\omega_0\frac{e\Sigma_{k_0}}{m^{\mathrm{i}}}n_{-k}\,,\quad(16)$$

where

$$\omega_{\mathrm{L}}'^2=\omega_{\mathrm{L}}^2-\frac{e^2\Sigma_{k_0}\Sigma_{-k_0}}{m^2}\,,$$

$$\omega_{\mathrm{S}}'^2=\omega_{\mathrm{S}}^2-\frac{e^2\Sigma_{k_0}\Sigma_{-k_0}}{mm^{\mathrm{i}}}$$

and $\omega_{\mathrm{L}}^2=\omega_{\mathrm{pe}}^2+k^2v_{T_{\mathrm{e}}}^2$, $\omega_{\mathrm{S}}^2=k^2c_{\mathrm{S}}^2$, which are the dispersion relation of high-frequency electron plasma oscillation and low-frequency ion-acoustic oscillation in the absence of the external field or pump, respectively.

It is obvious that Eqs.(15) and (16) are two oscillating formulas interacting with each other, whose frequencies are ω_{L}' and ω_{S}' and the driving terms $\mathrm{j}\omega_0(e\Sigma_{k_0}/m)n_{-\Delta k}$ and $\mathrm{j}\omega_0(e\Sigma_{k_0}/m^{\mathrm{i}})n_{-k}$, respectively. The damping terms of Eqs.(15) and (16) are $\beta(\partial/\partial t)n_k$ and $\beta(\partial/\partial t)n_{\Delta k}$, respectively, so the loss reaction in plasma will result in the damping of high-frequency electron plasma oscillation and low-frequency ion-acoustic oscillation. Also, Fig.1 illustrates the coupling shown by Eqs.(15) and (16).

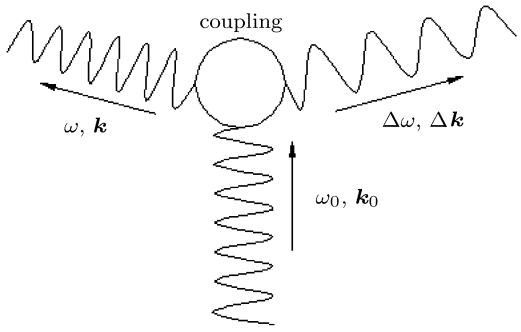


Fig.1. The diagram for nonlinear coupling between plasma mode, ion mode, and external field.

3. Dispersion relation and growth rate

Equation (11) together with Eq.(14) for $l=-1$ determines the dispersion relation

$$\begin{vmatrix}\omega^2+\mathrm{j}\beta\omega-\omega_{\mathrm{L}}'^2 & j\omega_0\dfrac{e\Sigma_{k_0}}{m}\\ \mathrm{j}\omega_0\dfrac{e\Sigma_{-k_0}}{m^{\mathrm{i}}} & -(\omega_0-\omega)^2+\mathrm{j}\beta(\omega_0-\omega)+\omega_{\mathrm{S}}'^2\end{vmatrix}=0,\quad(17)$$

where we have neglected $\omega_{\mathrm{pe}}^2n_k^{\mathrm{i}}$.

We substitute $\omega=x+\mathrm{j}y$ into Eq.(17) and separate the real and imaginary parts, then obtain the real part

$$(x^2-y^2-\beta y-\omega_{\mathrm{L}}'^2)[-(\omega_0-x)^2+y^2+\beta y+\omega_{\mathrm{S}}'^2]=x(\omega_0-x)(2y+\beta)^2-\Lambda\,,\quad(18)$$

where $\Lambda = \omega_0^2 e^2 \Sigma_{k_0} \Sigma_{-k_0}/mm^{\rm i}$, and the imaginary part

$$(x^2 - y^2 - \beta y - \omega_{\rm L}'^2)(\omega_0 - x)(2y + \beta) = x(2y + \beta)[(\omega_0 - x)^2 - y^2 - \beta y - \omega_{\rm S}'^2]\,. \tag{19}$$

From Eqs.(15) and (16) and the frequency matching relation $\omega_0 = \omega + \Delta\omega$, we believe that n_k and $n_{\Delta k}$ satisfy the dispersion relation Eq.(17), and $\omega_{\rm L}'$ will be very close to the solution of Eqs.(18) and (19), namely, $x \approx \omega_{\rm L}'$ and $\omega_0 - x \approx \omega_{\rm S}'$.

Substituting $x \approx \omega_{\rm L}'$ and $\omega_0 - x \approx \omega_{\rm S}'$ into Eq.(18), we arrive at

$$y^4 + 2\beta y^3 + (4\omega_{\rm L}'\omega_{\rm S}' + \beta^2)y^2 + 4\beta\omega_{\rm L}'\omega_{\rm S}' y + (\beta^2\omega_{\rm L}'\omega_{\rm S}' - \Lambda) = 0\,. \tag{20}$$

As the growth rate and the loss coefficient y and β are so small that

$$y^4 + 2\beta y^3 \ll (4\omega_{\rm L}\omega_{\rm S} + \beta^2)y^2 + 4\beta\omega_{\rm L}\omega_{\rm S} y + (\beta^2\omega_{\rm L}\omega_{\rm S} - \Lambda)\,, \tag{21}$$

the first two terms on the left side of Eq.(20) can be neglected for simplicity. In addition, we assume

$$\omega_{\rm pe}^2 + k^2 v_{T_{\rm e}}^2 \gg \frac{e^2\Sigma_{k_0}\Sigma_{-k_0}}{m^2} = \frac{m^{\rm i}}{m\omega_0^2}\Lambda$$

and

$$k^2 c_{\rm s}^2 \gg \frac{e^2\Sigma_{k_0}\Sigma_{-k_0}}{mm^{\rm i}} = \frac{1}{\omega_0^2}\Lambda,$$

namely, $\omega_{\rm S} \approx \omega_{\rm S}'$ and $\omega_{\rm L} \approx \omega_{\rm L}'$, thus Eq.(20) is rewritten as

$$(4\omega_{\rm L}\omega_{\rm S} + \beta^2)y^2 + 4\beta\omega_{\rm L}\omega_{\rm S} y + (\beta^2\omega_{\rm L}\omega_{\rm S} - \Lambda) = 0. \tag{22}$$

By assuming $4\omega_{\rm L}\omega_{\rm S} \gg \beta^2$, one can obtain the growth rate

$$y = \frac{1}{2}\left(\sqrt{\frac{\Lambda}{\omega_{\rm L}\omega_{\rm S}}} - \beta\right). \tag{23}$$

The parametric instability taking place in F region in ionosphere is taken for example. Figure 2 depicts the growth rate as a function of the normalized intensity of pump. In the numerical simulation, we set $m^{\rm i} = 2.68 \times 10^{-26}$ kg due to o^+ dominating in F region, $k = 11.5$ m^{-1} by $|k| \approx |\Delta k| = \Delta\omega/v_{T_{\rm i}}$, where $\Delta\omega = 10$ kHz and $v_{T_{\rm i}} = 8.66 \times 10^2$ m/s. In addition, $\omega_{\rm pe}$ is on the order of 10^7 Hz, and the threshold value $|\boldsymbol{E}_0|$ 0.1 v/m. The above parameters are typical in F region in ionosphere.[4,6] Then the normalized threshold Λ can be set to be of the order of 10^8 if ignoring the collision (collision frequency, of the order of 10^4,[4] and β is on the order of 10^{-4} Hz.[19]

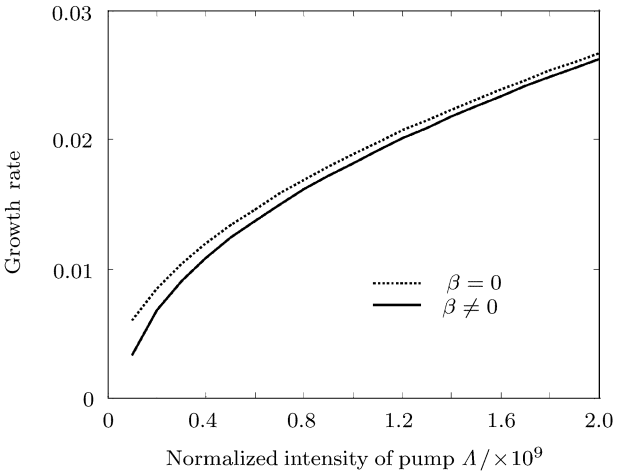


Fig.2. The growth rate versus the normalized intensity of pump.

4. Ponderomotive force and the exciting condition

On multiplying Eq.(11) by $-(m/k^2)\nabla$, it can be rewritten as

$$-\frac{m\omega^2}{k^2}\nabla n_k = -m\frac{\omega_{\rm pe}^2}{k^2}\nabla n_k - T_{\rm e}\nabla n_k + \frac{e^2\Sigma_{k_0}\Sigma_{-k_0}}{mk^2}\nabla n_k + {\rm j}\beta\frac{m\omega}{k^2}\nabla n_k + {\rm j}\frac{e(\omega_0 - 2\Delta\omega + {\rm j}\beta)}{k^2}\nabla[\Sigma_{k_0} n_{-\Delta k}], \tag{24}$$

where each term of Eq.(24) has the dimensions of force, that is, their units are the Newton. The term on the left side of equation is the resultant force $\boldsymbol{F}$, and the first term on the right side of equation is the Coulomb force introduced by electrostatic field $\boldsymbol{F}_{\rm int}$, the second term the thermal pressure $\boldsymbol{F}_{\rm pre}$, the third term the Coulomb force by external field $\boldsymbol{F}_{\rm ext}$, the fourth the damping force $\boldsymbol{F}_{\rm d}$ due to the loss reaction and the fifth the ponderomotive force $\boldsymbol{F}_{\rm p}$ driving the parametric instability.

Now we derive the relationship between $n_{-\Delta k}$ and $\boldsymbol{E}_{-\Delta k}$ for a clearer expression of ponderomotive force. Substituting Eq.(13) for $l=-1$ into the Poisson equation

$$\nabla\cdot\boldsymbol{E}_{-\Delta k}=-\frac{e}{\varepsilon_0}(n^{\mathrm{i}}_{-\Delta k}-n_{-\Delta k})\,, \tag{25}$$

we obtain

$$n_{-\Delta k}=-\frac{\varepsilon_0}{e}\left(\frac{-\omega_{\mathrm{pi}}^2}{\Delta\omega^2-\mathrm{j}\beta\Delta\omega-\omega_{\mathrm{pi}}^2-\Delta k^2v_{T_{\mathrm{i}}}^2+e^2\Sigma_{k_0}\Sigma_{-k_0}/(m^{\mathrm{i}})^2}-1\right)\nabla\cdot\boldsymbol{E}_{-\Delta k}\,. \tag{26}$$

It is obvious that $\nabla\cdot\boldsymbol{E}_{-\Delta k}$ will grow in the same way as $n_{-\Delta k}$ and with the growth rate of instability. Similarly, we can obtain $n_k=(\varepsilon_0/e)\nabla\cdot\boldsymbol{E}_k$.

Substituting Eq.(26) into the ponderomotive force term of Eq.(24), one arrives at the ponderomotive force in $\boldsymbol{k}$,

$$\boldsymbol{F}_{\mathrm{p}k}=\frac{\varepsilon_0}{2k^2}\nabla[(\boldsymbol{k}\cdot\boldsymbol{E}_{k_0})(\nabla\cdot\boldsymbol{E}_{-\Delta k})]\,, \tag{27}$$

where $2\Delta\omega+\mathrm{j}\beta$, $\beta\Delta\omega$ and ω_{pi} have been neglected. Meanwhile, we can obtain the damping force in $\boldsymbol{k}$,

$$\boldsymbol{F}_{\mathrm{d}k}=\frac{\beta\varepsilon_0 m\omega}{ek^2}\nabla(\nabla\cdot\boldsymbol{E}_k). \tag{28}$$

It is easy to show the equivalence relation between Eq.(27) and the expression of ponderomotive force in Ref.[6]. So we will not deal with it here.

In the same way as above, one can have the ponderomotive force in $\Delta\boldsymbol{k}$,

$$\boldsymbol{F}_{\mathrm{p}\Delta k}=\frac{\varepsilon_0 m}{2\Delta k^2 m^{\mathrm{i}}}\nabla[(\boldsymbol{k}\cdot\boldsymbol{E}_{k_0})(\nabla\cdot\boldsymbol{E}_{-k})], \tag{29}$$

and the damping force in $\Delta\boldsymbol{k}$

$$\boldsymbol{F}_{\mathrm{d}\,\Delta k}=\frac{\beta\varepsilon_0 m\Delta\omega}{e\Delta k^2}\nabla(\nabla\cdot\boldsymbol{E}_{\Delta k}). \tag{30}$$

It is well known that when the driving force counteracts the damping force, the oscillation will become non-damping and non-driving. So by setting

$$\boldsymbol{F}_{\mathrm{d}k}+\boldsymbol{F}_{\mathrm{p}k}=0\,, \tag{31}$$

we can arrive at the exciting condition for the parametric instability,

$$\frac{n_k}{n_{-\Delta k}}=-\mu\frac{\Sigma_{k_0}|_{\mathrm{th}}}{\beta}, \tag{32}$$

where μ is the charge-to-mass ratio of electron and the minus sigh denotes that the direction of damping force is opposite to that of driving force.

Similarly, one can obtain

$$\frac{n_{-k}}{n_{\Delta k}}=-\mu\frac{\Sigma_{-k_0}|_{\mathrm{th}}}{\beta}\,. \tag{33}$$

Multiplying Eq.(32) by Eq.(33) one can have the ratio of n_k^2 to $n_{\Delta k}^2$ in the exciting condition,

$$\eta|_{\mathrm{th}}=\frac{|n_k|^2}{|n_{\Delta k}|^2}\Big|_{\mathrm{th}}=\frac{\mu^2}{\beta^2}\Sigma_{k_0}\Big|_{\mathrm{th}}\Sigma_{-k_0}|_{\mathrm{th}}\,. \tag{34}$$

By setting Eq.(23) to zero, we can obtain the normalized threshold of instability,

$$\Lambda_{\mathrm{th}}=\omega_{\mathrm{L}}\omega_{\mathrm{S}}\beta^2\,. \tag{35}$$

Finally, one combines Eq.(34) with Eq.(35) and obtains

$$\eta|_{\mathrm{th}}=\frac{m^{\mathrm{i}}\omega_{\mathrm{L}}\omega_{\mathrm{S}}}{m\omega_0^2}\,. \tag{36}$$

5. Discussion and conclusion

It has been shown from the above derivation that the production reaction has no effect on the parametric instability. This is because the production reaction is only dependent on the density of the neutral components and the intensity of radiation from ionization source. In addition, we can obtain the conclusion from Eqs.(15) and (16) that the effect of loss reaction on the parametric instability is damping and the damping coefficient is the loss coefficient β. Physically, the larger the density of charged particles in the area, the larger the term of loss βn^{α} will be, and it will in turn lead to the decrease in the density of charged particles, namely, dumping of wave. Also, the loss reaction results in the decrease in growth rate.

It follows from Eq.(27) that the ponderomotive force arises from the coupling of the external field or pump and one of daughter waves and grows with the growth rate of instability. There are two modes for the ponderomotive force, whose frequencies and wavenumbers are the same as those of the originally assumed wave. It is also shown by combining Eqs.(15), (16), (17) and (29) that the external field or pump and two daughter waves will form a positive feedback circuit, that is, the interactions between the pump and the daughter waves will give birth to the ponderomotive force, which makes the original particle density

perturbation or the daughter waves increase, which in turn leads to a further increase in the ponderomotive force.

In addition, it is shown by Eq.(36) that when the pump field approaches the threshold, η becomes independent of the loss coefficient β and the intensity of the pump field. Physically, when the ponderomotive force counteracts the damping force, the oscillation becomes non-damping and non-driving.

Although the above analysis disregarded the collision and Landau damping, one can take them into account by replacing β with $(\beta + \nu_{\rm e} + \Gamma_{\rm L})$, where $\nu_{\rm e}$ is the collision frequency, and

$$\Gamma_{\rm L} = \sqrt{\frac{\pi}{8}} \frac{\omega_{\rm L}^4}{k^3 v_{T_{\rm e}}^3} \exp\Big(\frac{\omega_{\rm L}^2}{2k^2 v_{T_{\rm e}}^2}\Big)$$

the Landau damping coefficient.

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