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# Persistent spin current in a quantum wire with weak Dresselhaus spin–orbit coupling*

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The spin current in a parabolically confined semiconductor heterojunction quantum wire with Dresselhaus spin–orbit coupling is theoretically studied by using the perturbation method. The formulae of the elements for linear and angular spin current densities are derived by using the recent definition for spin current based on spin continuity equation. It is found that the spin current in this Dresselhaus spin–orbit coupling quantum wire is antisymmetrical, which is different from that in Rashba model due to the difference in symmetry between these two models. Some numerical examples for the result are also demonstrated and discussed.

**Keywords:** semiconductor heterojunction quantum wire, Dresslhuaus spin–orbit coupling, spin current

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Recently, spintronics has become one of the hotspots of the condensed matter physics, and attracted a great deal of attention of all.[1] Generating and measuring spin current is the primary research task in this field. The spin Hall effect (SHE)[2−4] is one of the most important topics of spintronics that is considered to be a hopeful way to produce the spin current. Recent intensive interest comes from the SHE and the spin precession in low-dimensional semiconductor heterostructures in the presence of spin–orbit coupling (SOC).[3] Some important results and interesting phenomena have been found and are under further study now.

In recent studies,[5−7] the so-called quantum spin continuity equation has been defined as

$$\frac{\partial \boldsymbol{s}(r,t)}{\partial t} + \nabla \cdot \boldsymbol{J}^{\mathrm{s}} = \boldsymbol{J}^{\omega}, \tag{1}$$

where $\boldsymbol{s}(r,t) = \Psi^{\dagger}\boldsymbol{s}\Psi$ is the local spin density with the spin operator $\boldsymbol{s} = \frac{\hbar}{2}\boldsymbol{\sigma}$ ($\boldsymbol{\sigma}$ is the pauli matrix) and electron wavefunction $\Psi$; $\boldsymbol{J}^{\mathrm{s}}$ is the conventional linear spin current density tensor defined[8,9] as

$$\boldsymbol{J}^{\mathrm{s}} = \mathrm{Re}\{\Psi^{\dagger}\boldsymbol{v}\boldsymbol{s}\Psi\}, \tag{2}$$

where $\boldsymbol{v} = \partial \boldsymbol{r}/\partial t = \frac{1}{\mathrm{i}\hbar}[\boldsymbol{r}, H]$ is the velocity operator, $\boldsymbol{J}^{\omega}$ is the torque density or the so-called angular spin current density vector which adds a torque dipole term to the conventional spin current and is defined[8,9] as

$$\boldsymbol{J}^{\omega} = \mathrm{Re}\{\Psi^{\dagger}\frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}t}\Psi\}, \tag{3}$$

where $\mathrm{d}\boldsymbol{s}/\mathrm{d}t = \frac{1}{\mathrm{i}\hbar}[\boldsymbol{s}, H]$. Equation (1) is based on the quantum mechanical principle and is very similar to the classic vector continuity equation. However, the above defined spin current is independent of the choice of Hamiltonian,[9] and is a straightforward generalization of the usual particle current that is generally accepted in the literature.[10] Both linear and angular spin currents play a parallel role in contributing to a physical quantity. They describe the translational and rotational (precession) motions of an electron spin, respectively.

The recent work[10] has studied a system of a hard-wall confining potential determined quantum wire in the presence of Rashaba SOC, and the persistent spin current has been discussed. Parallel to this work, in the present paper we study a parabolically confined quantum wire with Dresselhaus SOC[11,12] and derive the formulas of spin current densities for the system. All the nonzero elements of spin current density are calculated analytically and several numerical examples are demonstrated. We have found some

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results to be different from those of Rashba effect. These results have not yet been reported previously for a quantum wire system. Moreover, Dresselhaus model has aroused great interest of all[13] since the SHE was demonstrated experimentally in some semiconductor materials.

The single-electron Hamiltonian of our system is

$$H = \frac{\boldsymbol{p}^2}{2m^*} + V(x) + \frac{\beta}{\hbar}(\sigma_y p_y - \sigma_x p_x), \tag{4}$$

where $\boldsymbol{p} = (p_x, p_y)$ is the two-dimensional momentum vector, $m^*$ is the effective mass of the electron, and the lateral confining potential[12] $V(x) = \frac{1}{2}m^*\omega^2 x^2$ with an oscillator frequency $\omega$ determining an effective width of the wire. The last Hamiltonian term originates from the Dresselhaus SOC due to the bulk-inversion symmetry of zinc-blende type crystal.[11] In some semiconductor heterostructures, such as GaAs quantum well,[14] the Dresselhaus SOC is visible[15] and the strength parameter $\beta$ is $\sim$0.09 eVÅ in value.

From Eq.(4), it is obvious that the total Hamiltonian is translationally invariant in the wire direction, so its corresponding wavefunction can be a product of a plane wave in the longitudinal $y$-direction and a transverse eigenfunction in the $x$-coordinate as follows:

$$\Psi_{\uparrow\downarrow}(r) = e^{\mathrm{i}k_y y}\,\Phi_{\uparrow\downarrow}(u), \tag{5}$$

where $k_y$ is the longitudinal wave number, the subscripts $\uparrow$ and $\downarrow$ denote respectively the spin-up and spin-down, and $u = x/l_\omega$ is the dimensionless transverse coordinate with $l_\omega = (\hbar/m^*\omega)^{1/2}$ (typical spatial scale in relation to the confining potential). The substitution of Eqs.(4) and (5) into the definitions (2) and (3) leads to the general expressions for the elements of linear spin current density

$$J^{\mathrm{s}}_{ij} = \mathrm{Re}\{\,\Psi^\dagger \left(\frac{\boldsymbol{p}_i}{m^*} + \frac{\beta}{\mathrm{i}\hbar^2}\sigma_{x,y}[\boldsymbol{r}, p_{x,y}]\right)\boldsymbol{s}_j\,\Psi\} \tag{6}$$

and angular spin current density

$$J^{\omega}_i = \frac{\beta}{2\mathrm{i}\hbar}\mathrm{Re}\{\,\Psi^\dagger([\sigma_i, \sigma_y p_y] - [\sigma_i, \sigma_x p_x])\,\Psi\}, \tag{7}$$

where $i, j = x, y, z$. The element of linear spin current density, e.g. $J^{\mathrm{s}}_{xy}$, represents the spin current of an electron moving along the $x$-direction with its spin in the $y$-direction. While each component of angular spin current density $J^{\omega}_i$ represents the electron spin precession in the $i$-direction. In the equilibrium case, the persistent spin current can be obtained straightforwardly by summing over all the occupied states and integrating over the energy from 0 to Fermi energy $E_{\mathrm{F}}$.

For a weak SOC case, the transverse eigenfunction and the total eigenenergy can be solved by using the perturbation method[12]

$$\begin{aligned}\Phi^n_{\uparrow\downarrow}(u) = \;& \psi^n_{\uparrow\downarrow}(u) \pm \frac{\mathrm{i}}{2\sqrt{2}}\frac{l_\omega}{l_\beta}\{\sqrt{n}\psi^{n-1}_{\uparrow\downarrow}(u) \\ & + \sqrt{n+1}\psi^{n+1}_{\uparrow\downarrow}(u)\},\end{aligned} \tag{8}$$

$$E_{n\uparrow\downarrow} = \frac{\hbar\omega}{2}[(2n+1) + (k_y l_\omega)^2 \pm \frac{l_\omega}{l_\beta}(k_y l_\omega)], \tag{9}$$

where

$$\psi^n_{\uparrow\downarrow}(u) = \frac{\mathrm{e}^{-u^2/2}}{\sqrt{2^n n!\sqrt{\pi}}}\mathrm{H}_n(u) \tag{10}$$

is the eigenfunction of an oscillator with Hermite Polynomial $\mathrm{H}_n(u)$, and $l_\beta = \hbar^2/(2m^*\beta)$ is the characteristic spatial scale associated with the Dresselhaus SOC. Note that the perturbation method is workable only on condition that $(l_\omega/l_\beta)(k_y l_\omega) \ll 1$.[12] Here we choose $l_\omega/l_\beta \le 0.1$ to ensure that our system can fulfill this inequality.

In the following we calculate the spin current density for the states of $n = 0$ and $n = 1$. By substituting wavefunction (5) together with Eq.(8) into Eqs.(6) and (7), all the nonzero elements of spin current density can be calculated as follows:

$$\begin{aligned}
J^{\mathrm{s}}_{xx} &= -\frac{\beta}{\pi^{1/2}}\mathrm{e}^{-u^2}\left(1 + \frac{l_\omega^2}{4l_\beta^2}u^2\right), \\
J^{\mathrm{s}}_{xz} &= \frac{\hbar^2 l_\omega}{2l_\beta m^*\pi^{1/2}}\mathrm{e}^{-u^2}, \\
J^{\mathrm{s}}_{yx} &= \frac{\hbar^2 k_y}{m^*\pi^{1/2}}\mathrm{e}^{-u^2}\left(1 - \frac{l_\omega^2}{4l_\beta^2}u^2\right), \\
J^{\mathrm{s}}_{yy} &= \frac{\beta}{\pi^{1/2}}\mathrm{e}^{-u^2}\left(1 + \frac{l_\omega^2}{4l_\beta^2}u^2\right) - \frac{\hbar^2 k_y l_\omega}{m^* l_\beta}\frac{\mathrm{e}^{-u^2}}{\pi^{1/2}}u, \\
J^{\omega}_z &= -\frac{2\beta}{\hbar\pi^{1/2}}\mathrm{e}^{-u^2}\left(1 + \frac{l_\omega^2}{4l_\beta^2}u^2\right)
\end{aligned} \tag{11}$$

for $n = 0$, and

$$\begin{aligned}
J^{\mathrm{s}}_{xx} &= -\frac{2\beta}{\pi^{1/2}}\mathrm{e}^{-u^2}u^2\left(1 + \frac{l_\omega^2}{2l_\beta^2}u^2\right), \\
J^{\mathrm{s}}_{xz} &= -\frac{\hbar^2 l_\omega}{m^* l_\beta \pi^{1/2}}\mathrm{e}^{-u^2}u^2, \\
J^{\mathrm{s}}_{yx} &= \frac{2\hbar^2 k_y}{m^*\pi^{1/2}}\mathrm{e}^{-u^2}u^2\left(1 - \frac{l_\omega^2}{4l_\beta^2}u^2\right), \\
J^{\mathrm{s}}_{yy} &= \frac{2\beta}{\pi^{1/2}}\mathrm{e}^{-u^2}u^2\left(1 + \frac{l_\omega^2}{4l_\beta^2}u^2\right) - \frac{2\hbar^2 k_y l_\omega}{m^* l_\beta}\frac{\mathrm{e}^{-u^2}}{\pi^{1/2}}u^3, \\
J^{\omega}_z &= -\frac{4\beta}{\hbar\pi^{1/2}}\mathrm{e}^{-u^2}u^2\left(1 + \frac{l_\omega^2}{4l_\beta^2}u^2\right)
\end{aligned} \tag{12}$$

for $n = 1$. We can also calculate the spin current density for the states of higher sub-bands, but the results

are similar to those for the states of $n = 0$ and $n = 1$ except for some higher terms in $u$.

In the equilibrium case, for each sub-band the probabilities of the four occupied degenerate states are the same.[10] Integrating over energy from 0 to Fermi energy $E_{\rm F}$ and summing over the propagating modes, the zero temperature total spin current density for $n = 0, 1, 2$ can be obtained as follows:

$$J_{xx}^{\rm sT} = -\frac{\beta}{2\pi^{1/2}}{\rm e}^{-u^2}\left(1+\frac{l_\omega^2}{4l_\beta^2}u^2\right)(4u^4+3),$$
$$J_{yy}^{\rm sT} = -J_{xx}^{\rm sT},\quad J_z^{\omega{\rm T}} = (2/\hbar)J_{xx}^{\rm sT}. \tag{13}$$

We note that only the two diagonal elements of linear spin current density tensor are nonzero after summing over all the degenerate states. $J_{yx}^{\rm sT}$ and the second term of $J_{yy}^{\rm sT}$ vanish because they are proportional to the longitudinal wave number $k_y$ [see Eqs.(11) and (12)], and $k_{y\uparrow}^{+}(k_{y\downarrow}^{+})$ is opposite in sign to $k_{y\downarrow}^{-}(k_{y\uparrow}^{-})$. This result is similar to that obtained in Refs.[16,17], where Rashba has found a nonzero spin current in an infinite two-dimensional electron gas (2DEG) system with SOC in the thermodynamic equilibrium. The spin current tensor $J_{ij}$ of the system fulfills a different symmetry with a different choice of SOC type (Rashba or Dresselhaus). For Dresselhaus case, the nonzero elements are $J_{xx}$ and $J_{yy}$ that are related also by $J_{xx} = -J_{yy}$. In our work here we find that this symmetry still holds when the infinite 2DEG is confined into a quantum wire by a parabolic potential. Moreover, the spin current density for a hard-wall potential confined quantum wire with Rashba SOC has also been studied,[10] and the result shows that this symmetry is broken. Furthermore, $J_{xz}^{\rm sT}$ here is very small because $J_{xz}^{\rm s}$ in Eqs.(11) and (12) has a factor of $\hbar^2$, and one can neglect its influence.

Next, we show some numerical examples of the analytically calculated results. Figure 1 demonstrates numerically the dependences of the two linear spin current density elements $J_{xx}^{\rm s}$ and $J_{yy}^{\rm s}$ (in units of $\beta/\sqrt{\pi}$) on the dimensionless transverse coordinate $u$ with $l_\omega/l_\beta = 0.1$ and $E_{\rm F} = 3\hbar\omega$. From Fig.1 we can see that $J_{xx}^{\rm s}$ is the inversion of $J_{yy}^{\rm s}$. For $n = 0, 1, 2$, both $J_{xx}^{\rm s}$ and $J_{yy}^{\rm s}$ oscillate with one, two and three maximum values, behaving like an oscillator eigenfunction. However, the total linear spin current densities of these two elements always have a peak at the centre of the wire and take the main values from $u = -2$ to 2. The dependence of the nonzero $z$-component of angular spin current density $J_z^{\omega{\rm T}}$ on the dimensionless transverse coordinate $u$ is similar to that of $J_{xx}^{\rm s}$, only the units of their longitudinal axes are different.

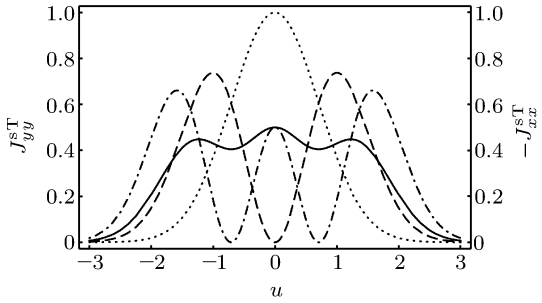



**Fig.1.** Calculated linear spin current density elements $J_{xx}^{\rm s}$ and $J_{yy}^{\rm s}$ (in units of $\beta/\sqrt{\pi}$) as a function of dimensionless transverse coordinate $u$ with $l_\omega/l_\beta = 0.1$ for $n = 0$ (dashed line), $n = 1$ (dotted line), $n = 2$ (dash-dotted line) and total $J_{xx}^{\rm sT}$ and $J_{yy}^{\rm sT}$ (solid line), respectively.

It has been theoretically proved that the persistent spin current can generate an electric field that is proportional to the magnitude of the spin current.[18,19] In our results for a Dresselhaus quantum wire, the spin current induced electric field will peak at the centre of the wire and decrease rapidly to a minimum on the edge of the wire. This character of the induced field may give a way of spin current detection.[18,19]

In conclusion, we have studied theoretically the spin current in a quantum wire modulated by a parabolic potential in the presence of Dresselhaus SOC. Using the generally accepted and widely used definition that fulfills the spin continuity equation, it is found that the persistent spin current exists along both the transverse and longitudinal directions of the wire. The nature of the pure spin current calls for a better understanding of their mechanism, including the properties of the background. From the standpoint of spintronic applications, it is important to understand whether it is a transport current,[20] i.e. whether it can be employed for transporting spins. Further investigation is worthy conducting if these factors are taken into consideration.

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