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Anomalous Hall effect of heavy holes in III−V semiconductor quantum wells*

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The anomalous Hall effect of heavy holes in semiconductor quantum wells is studied in the intrinsic transport regime, where the Berry curvature governs the Hall current properties. Based on the first–order perturbation of wave function the expression of the Hall conductivity the same as that from the semiclassical equation of motion of the Bloch particles is derived. The dependence of Hall conductivity on the system parameters is shown. The amplitude of Hall conductivity is found to be balanced by a competition between the Zeeman splitting and the spin–orbit splitting.

Keywords: anomalous Hall effect, spin-orbit coupling, Rashba model
PACC: 7210B, 7335A, 7360J

Spintronics (spin transport electronics) has become a very active and promising field, which combines the basic quantum mechanics of coherent spin dynamics and technological applications in information processing and storage devices.[1−7] One key issue is how to make use of spin-orbit (SO) coupling in realizing a variety of extraordinary electric and spin transports. Two remarkable examples under active study are the anomalous Hall effect (AHE) with respect to the charge[8−10] and spin[11−17] degrees of freedom. In fact it has been verified that the SO coupling mechanisms in semiconductors have provided a basis for extensive device applications, and a source of interesting physics in a wide range.

For charge degrees of freedom, the AHE is characterized by an additional term in the expression of the transverse Hall resistivity. This term is proportional to the magnetization M of the sample. The total Hall resistivity becomes $\rho_{xy} = R_0 B + R_{\rm H} M$, where B is the magnetic field, $R_0 = 1/ne$ is the ordinary Hall coefficient and $R_{\rm H}$ the anomalous Hall coefficient. The AHE can be observed in magnetic metals or semiconductors in the absence of external magnetic field.[18] The key point of the explanation of AHE depends on the presence of SO coupling. Although the AHE has been known for half a century since the pioneering work of Karplus and Luttinger,[19] its origin has been keenly debated for decades.[20,21] They have emerged anew because of fresh theoretical insights and strong interest in spin currents for spin-based electronics. Recently, the systematic work on metallic ferromagnets[10] and semiconductor ferromagnets[8,9] has attributed the AHE to the real-space or momentum-space Berry-phases. This intrinsic version of the Luttinger theory has received strong experimental support[22] and therefore will be a starting point in this paper.

In this paper we study the AHE of heavy holes in semiconductor quantum wells. In this two-dimensional hole gas (2DHG) system, the Rashba SO interaction arises from the quantum well asymmetry in the growth (z-) direction, and the Dresselhaus term results from the bulk inversion asymmetry. Thus the total spin-orbit interaction is described as[23−26]

$$V_{\rm so} = {\rm i}\frac{\alpha}{2}(k_-^3\sigma_+ - k_+^3\sigma_-). \tag{1}$$

Here $k_\pm = k_x \pm {\rm i}k_y$, and $\sigma_\pm = \sigma_x \pm {\rm i}\sigma_y$, where $\hbar k_i$ and σ_i $(i = x, y, z)$ denote the hole momentum operators and Pauli matrices, respectively. These Pauli matrices operate on the total angular momentum states with spin projection $\pm 3/2$ along the growth direction; in this sense they represent a pseudospin degree of freedom rather than a genuine spin 1/2.[26] α is Rashba spin-orbit coupling coefficient due to structure inversion asymmetry across the quantum well grown along the [001]-direction chosen as the z-axis. For a symmetrically grown quantum well, the coefficient α is essentially proportional to an electric field applied across the well and therefore experimentally tunable.[23−26] Taking into account the kinetic energy, a Zeeman mag-

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netic field normal to the heterostructure, and a uniform electric field, $\boldsymbol{E}$, we can write the $\boldsymbol{k}\cdot\boldsymbol{p}$ Hamiltonian as

$$H = \gamma k^2 + V_{\rm so} + h_0\sigma_z - e\boldsymbol{E}\cdot\boldsymbol{r} \equiv H_0 - e\boldsymbol{E}\cdot\boldsymbol{r}, \quad (2)$$

where $\gamma = \hbar^2/2m^*$ with m^* being band effective mass, $h_0 = g^*B\mu_{\rm B}/2$ denotes Zeeman splitting due to the external weak magnetic field or the internal exchange field, which will occur in the presence of magnetic doping.

The eigenvalues of the unperturbed Hamiltonian H_0 are $\varepsilon_{1,2}(\boldsymbol{k}) = \gamma k^2 \mp \Delta_{\boldsymbol{k}}$ with $\Delta_{\boldsymbol{k}} = \sqrt{h_0^2 + \alpha^2 k^6}$. The corresponding Bloch eigenstates are given by

$$\Phi_1(\boldsymbol{r}) \begin{pmatrix} \sin\dfrac{\theta_{\boldsymbol{k}}}{2} \\ -\cos\dfrac{\theta_{\boldsymbol{k}}}{2}\mathrm{e}^{\mathrm{i}\chi_{\boldsymbol{k}}} \end{pmatrix} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} \equiv u_{1\boldsymbol{k}}\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}, \quad (3a)$$

$$\Phi_2(\boldsymbol{r}) = \begin{pmatrix} \cos\dfrac{\theta_{\boldsymbol{k}}}{2}\mathrm{e}^{\mathrm{i}\chi_{\boldsymbol{k}}} \\ \sin\dfrac{\theta_{\boldsymbol{k}}}{2} \end{pmatrix} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} \equiv u_{2\boldsymbol{k}}\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}, \quad (3b)$$

where we have introduced $u_{n\boldsymbol{k}}$ $(n = 1, 2)$ to denote momentum-dependent spinors. The factors $\theta_{\boldsymbol{k}}$ and $\chi_{\boldsymbol{k}}$ in Eqs.(3a) and (3b) are determined by $\cos\theta_{\boldsymbol{k}} = h_0/\Delta_{\boldsymbol{k}}$ and $\cos\chi_{\boldsymbol{k}} = k_y(3k_x^2 - k_y^2)/k^3$ respectively. Note that although the SO coupling behaves like an effective magnetic field in $\boldsymbol{k}$-space, it does not destroy time-reversal symmetry, whereas the real Zeeman splitting h_0 can break this symmetry. The interplay between the SO coupling and Zeeman splitting will, as we show below, induce an intrinsic Hall effect for the electric current.

In the presence of a uniform electric field, by the introduction of a phase transformation $\bar{\Phi}(t,\boldsymbol{r}) = \exp[-\mathrm{i}e\boldsymbol{E}\cdot\boldsymbol{r}t]\,\Phi(\boldsymbol{r})$, the term $-e\boldsymbol{E}\cdot\boldsymbol{r}$ in Eq.(2) is absorbed and the $\boldsymbol{k}\cdot\boldsymbol{p}$ Hamiltonian is transformed as $\bar{H}(t) = H(\boldsymbol{p} - \boldsymbol{f}(t))$ with $\boldsymbol{f}(t) = -e\boldsymbol{E}t$. $\boldsymbol{f}(t)$ changes very slowly and can be treated as an adiabatic parameter. The time-dependent Schrödinger equation for $\bar{\Phi}(t,\boldsymbol{r})$ is

$$\mathrm{i}\frac{\partial}{\partial t}\bar{\Phi}(t,\boldsymbol{r}) = \bar{H}(t)\bar{\Phi}(t,\boldsymbol{r}). \quad (4)$$

Within the adiabatic approximation, the resulting instantaneous eigenvalue equation at time t is given by $\bar{H}(t)\bar{\Phi}_{n\boldsymbol{k}}(t) = \varepsilon_{n\boldsymbol{k}}(t)\bar{\Phi}_{n\boldsymbol{k}}(t)$, where $\varepsilon_{n\boldsymbol{k}}(t)$ are the instantaneous eigenenergies. The electric field is assumed to be homogeneous in the 2DHG system, therefore, for the static eigenstate $\Phi_{n\boldsymbol{k}}(\boldsymbol{r})$, its instantaneous eigenstate can also be written in the Bloch form as, $\bar{\Phi}_{n\boldsymbol{k}}(t,\boldsymbol{r}) = \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}u_{n\boldsymbol{k}}(t)$ where the function $u_{n\boldsymbol{k}}(t)$ is obtained from the static spinor $u_{n\boldsymbol{k}}$ via a shift in $\boldsymbol{k}$-space

$$u_{n\boldsymbol{k}}(t) = u_{n,\boldsymbol{k}-\boldsymbol{f}(t)}. \quad (5)$$

The solution of Eq.(4) in the adiabatic approximation is given by

$$\bar{\Psi}_{n\boldsymbol{k}}(t,\boldsymbol{r}) = \exp\left(\mathrm{i}\int_0^t \mathrm{d}t'[\varepsilon_{n\boldsymbol{k}}(t') + \gamma_{n\boldsymbol{k}}(t')]\right) \times \bar{\Phi}_{n\boldsymbol{k}}(t,\boldsymbol{r}) \quad (6)$$

where $\gamma_{n\boldsymbol{k}}(t)$ is a consequence of the geometric effect and given by

$$\begin{aligned}\gamma_{n\boldsymbol{k}}(t) &= \mathrm{i}\int_0^t \mathrm{d}t' \left\langle \bar{\Phi}_{n\boldsymbol{k}}(t') \left| \frac{\partial}{\partial t'} \right| \bar{\Phi}_{n\boldsymbol{k}}(t') \right\rangle \\ &= \mathrm{i}\int_0^t \mathrm{d}t' u^*_{n\boldsymbol{k}}(t')\frac{\partial}{\partial t'}u_{n\boldsymbol{k}}(t'). \end{aligned} \quad (7)$$

When the vector $\boldsymbol{f}(t)$ is parallel to the reciprocal-lattice vector $\boldsymbol{G}$, then we have a period T for a closed loop in the momentum space.[27] The Berry phase is defined as $\int_0^T \mathrm{d}t\gamma_{\boldsymbol{k}}(t)$. The Berry connection defined in the parameter space is given by $\boldsymbol{a}_{n\boldsymbol{k}} = \langle u_{n\boldsymbol{k}}|\nabla_{\boldsymbol{k}}|u_{n\boldsymbol{k}}\rangle$. By using Eq.(5), the Berry phases for $\boldsymbol{f}(t)//\boldsymbol{G}$ can be written as a general form, $\Gamma_{n,f}(k) = \mathrm{i}\oint_{C(f)} \mathrm{d}\boldsymbol{k}\cdot\boldsymbol{a}_n\boldsymbol{k}$, where $C(f)$ is a closed loop on which $\boldsymbol{f}(t)$ moves. In general, the Berry phase depends on the details of $C(f)$.[28]

The above adiabatic description illustrates how the Berry phase and Berry connection arise from the perturbation of the electric field. However, we must go further to take into account the mixing between the adiabatic eigenstates. This is essential for the Berry curvature to occur in the transport equations. Given the zero-order adiabatic spinors $u_{nk}(t)$, the first-order wave functions are given in the time-dependent (non-degenerate) perturbation theory by

$$|\tilde{u}^{(1)}_{n\boldsymbol{k}}(t)\rangle = |u_{n\boldsymbol{k}}(t)\rangle + \sum_{n'\neq n} \frac{|u_{n'\boldsymbol{k}}(t)\rangle \left\langle u_{n'\boldsymbol{k}}(t)\right| \mathrm{i}\dfrac{\partial u_{n\boldsymbol{k}}(t)}{\partial t} \Big\rangle}{\varepsilon_{n'\boldsymbol{k}}(t) - \varepsilon_{n\boldsymbol{k}}(t)}, \quad (8)$$

which are the starting quantum states for further treatment.

The macroscopic currents are obtained by the quantum-mechanical and thermodynamical averages over the current operator. Using the expression (8)

for the first-order wave functions as a response to the electric field $\boldsymbol{E}$, the hole current $\boldsymbol{J}$ is given by

$$\begin{aligned}\boldsymbol{J} &= e\sum_n \int_{BZ} \frac{\mathrm{d}^2k}{(2\pi)^2} f_n \langle \tilde{u}_{n\boldsymbol{k}}(t)|\boldsymbol{v}|\tilde{u}_{n\boldsymbol{k}}(t)\rangle \\ &= \boldsymbol{J}_{\mathrm{N}} + \boldsymbol{J}_{\mathrm{H}}, \end{aligned} \tag{9}$$

where $\boldsymbol{v} = \partial H_0/\partial \boldsymbol{k}$ is the velocity operator in the Bloch states and f_n denotes Fermi-Dirac distribution function for the nth band under the electric field,

$$\boldsymbol{J}_{\mathrm{N}} = e\sum_n \int_{BZ} \frac{\mathrm{d}^2k}{(2\pi)^2} f_n \frac{\partial \varepsilon_{n\mathbf{k}}}{\partial \boldsymbol{k}} \tag{10}$$

is the normal current along the direction of the electric field, and

$$\boldsymbol{J}_{\mathrm{H}} = e\sum_n \int_{BZ} \frac{\mathrm{d}^2k}{(2\pi)^2} f_n \left\{ \sum_{n'\neq n} \frac{\langle u_{n\boldsymbol{k}}(t)|\boldsymbol{v}|u_{n'\boldsymbol{k}}(t)\rangle \left\langle u_{n'\boldsymbol{k}}(t)\right| \mathrm{i}\frac{\partial u_{n\boldsymbol{k}}(t)}{\partial t}\right\rangle}{\varepsilon_{n'\boldsymbol{k}}(t) - \varepsilon_{n\boldsymbol{k}}(t)} + \mathrm{c.c.} \right\} \tag{11}$$

is the additional current contribution due to the mixing of the adiabatic wave functions. Its Hall properties of flow perpendicular to the electric field can be seen as follows. To further simplify the expression for $\boldsymbol{J}_{\mathrm{H}}$, notice that in Eq.(11) $\langle u_{n\boldsymbol{k}}|\boldsymbol{v}|u_{n'\boldsymbol{k}}\rangle/(\varepsilon_{n'\boldsymbol{k}} - \varepsilon_{n\boldsymbol{k}}) = \left\langle \frac{\partial u_{n\boldsymbol{k}}}{\partial \boldsymbol{k}}|u_{n'\boldsymbol{k}}\right\rangle$ and $\left|\frac{\partial u_{n\boldsymbol{k}}}{\partial t}\right\rangle = \frac{e\boldsymbol{E}}{\hbar}\cdot\left|\frac{\partial u_{n\boldsymbol{k}}}{\partial \boldsymbol{k}}\right\rangle$. By substituting these two equalities into Eq.(12), we obtain

$$\begin{aligned}\boldsymbol{J}_{\mathrm{H}} &= e\sum_n \int_{BZ} \frac{\mathrm{d}^2k}{(2\pi)^2} f_n \left\{ \left\langle \frac{\partial u_{n\boldsymbol{k}}(t)}{\partial \boldsymbol{k}}\right| \mathrm{i}\frac{\partial u_{n\boldsymbol{k}}(t)}{\partial t}\right\rangle + \mathrm{c.c.}\right\} \\ &= \frac{e^2}{\hbar}\sum_n \int_{BZ} \frac{\mathrm{d}^2k}{(2\pi)^2} f_n \left\{ \left\langle \frac{\partial u_{n\boldsymbol{k}}(t)}{\partial \boldsymbol{k}}\right| \boldsymbol{E}\cdot\left| \mathrm{i}\frac{\partial u_{n\boldsymbol{k}}(t)}{\partial \boldsymbol{k}}\right\rangle + \mathrm{c.c.}\right\} \\ &= \frac{e^2}{\hbar}\boldsymbol{E}\times\boldsymbol{z}\sum_n \int_{BZ} \frac{\mathrm{d}^2k}{(2\pi)^2} f_n \boldsymbol{\Omega}_{n,z}, \end{aligned} \tag{12}$$

where

$$\boldsymbol{\Omega}_n = \mathrm{i}\nabla_{\boldsymbol{k}}\times\boldsymbol{a}_n\boldsymbol{k} = \mathrm{i}\left\langle \frac{\partial u_{n\boldsymbol{k}}(t)}{\partial \boldsymbol{k}}\right| \times \left|\frac{\partial u_{n\boldsymbol{k}}(t)}{\partial \boldsymbol{k}}\right\rangle \tag{13}$$

is the usual Berry curvature[29,30] that is related to the Berry phase Γ_n by $\Gamma_n = \int_A \mathrm{d}\boldsymbol{S}\cdot\boldsymbol{\Omega}_n$, with A being the area enclosed by the adiabatic loop $C(f)$ in the wave-vector space. One can see from Eq.(12) that the current $\boldsymbol{J}_H$ is perpendicular to the external electric field and thus is a Hall-like current. Equation(12) can also be obtained from a derivation of the semiclassical equation of motion for the Bloch electrons.[31] The distribution function in Eq.(12) is formally obtained by a systematic solution of the Boltzmann equation. In the linear response approximation, $f_n = f_n^{(0)} + \delta f_n$, where $f_n^{(0)}$ is the equilibrium part and δf_n is proportional to the electric field. Then the total current has a form (to the first order in $\boldsymbol{E}$)

$$\begin{aligned}\boldsymbol{J} = \; & e\sum_n \int \frac{\mathrm{d}^2k}{(2\pi)^2}\delta f_n \frac{\partial \varepsilon_{n\boldsymbol{k}}}{\partial \boldsymbol{k}} + \frac{e^2}{\hbar}\boldsymbol{E} \\ & \times\boldsymbol{z}\sum_n \int \frac{\mathrm{d}^2k}{(2\pi)^2} f_n^{(0)} \boldsymbol{\Omega}_{n,z}. \end{aligned} \tag{14}$$

One can see that the first normal term on the right-hand side of Eq.(14) arises from the drift of the distribution function and thus is along the direction of the electric field, whereas the intrinsic (second) term of $\boldsymbol{J}$ is due to the first-order change of the electronic wave functions and behaves as being deflected by an effective magnetic field. Remarkably, the eigenstate mixing in Eq.(8) disappears in the transport equation, and the Hall current looks like an adiabatic response to the electric field. This is also why the single-band semiclassical approximation[31] gives the same expression as that in Eq.(12).

The subsequent Hall conductivity is given by

$$\sigma_{\rm H} = J_{{\rm H},x}/E_y = \frac{e^2}{\hbar}\sum_n \int \frac{{\rm d}^2k}{(2\pi)^2} f_n^{(0)} \boldsymbol{\Omega}_{n,z}. \quad (15)$$

For the present Hamiltonian (2), after a straightforward derivation, we obtain the expressions for Berry curvatures as follows:

$$\boldsymbol{\Omega}_{1,z} = \frac{9h_0\alpha^2k^4}{2\Delta_{\boldsymbol{k}}^3},\ \boldsymbol{\Omega}_{2,z} = -\boldsymbol{\Omega}_{1,z}. \quad (16)$$

When $h_0 = 0$, the time-reversal symmetry is recovered and the Berry curvature for both bands vanishes. At zero temperature, suppose that only the bottom band is occupied, and take its Fermi wave number up to infinity, then we can have the Hall conductivity

$$\sigma_{\rm H} = \frac{e^2}{2\pi h}\int_0^{2\pi}{\rm d}\theta \int_0^{\infty} k{\rm d}k \frac{9h_0\alpha^2k^4}{2\Delta_{\boldsymbol{k}}^3} = \frac{3}{2}\frac{e^2}{h}, \quad (17)$$

which is a product of the Berry phase $\Gamma_1 = 3\pi$ (in the zero-h_0 limit) and the quantum conductance $e^2/\hbar$.

Equation (17) corresponds to the ideal case of single-band occupation and infinite Fermi wave number. For the realistic consideration, we need to take into account the usual case that both of two bands are occupied. The effect of finite value of Fermi wave number should also be considered. In this case, the zero-temperature Hall conductivity is given by

$$\sigma_{\rm H} = \frac{3}{2}\frac{e^2}{h}\left(\frac{h_0}{\sqrt{h_0^2+\alpha^2k_{\rm f}^{(+)6}}} - \frac{h_0}{\sqrt{h_0^2+\alpha^2k_{\rm f}^{(-)6}}}\right) \quad (18)$$

where $k_{\rm f}^{(-)}$ and $k_{\rm f}^{(+)}$ are Fermi wave numbers for the bottom and top bands, respectively. To acquire the explicit expressions for the Fermi wave numbers, one needs to make use of their relation with the Fermi energy $\varepsilon_{\rm f}$:

$$\varepsilon_{\rm f} = \gamma k_{\rm f}^{(\pm)2} \pm \sqrt{h_0^2+\alpha^2k_{\rm f}^{(\pm)6}}. \quad (19)$$

On the other hand, the Fermi energy $\varepsilon_{\rm f}$ is pinned by the hole density $n_{\rm h}$:

$$n_{\rm h} = \frac{1}{(2\pi)^2}\sum_{n=1}^{2}\int {\rm d}\boldsymbol{k} f_{n,\boldsymbol{k}}, \quad (20)$$

where $f_{n\boldsymbol{k}}$ is the Fermi–Dirac distribution function for heavy holes. In general, the Fermi level and wave vectors can be numerically obtained by combining Eqs.(19)–(20). At zero temperature Eq.(20) reduces to $n_{\rm h} = (k_{\rm f}^{(+)2} + k_{\rm f}^{(-)2})/4\pi$. One has an explicit expression for the Fermi level

$$\varepsilon_{\rm f} = \gamma\left(\frac{\gamma^2}{\alpha^2} - 4\pi n_{\rm h} - \delta\right), \quad (21)$$

where the parameter δ is determined by

$$\delta^2 = \left(4\pi n_{\rm h} - \frac{\gamma^2}{\alpha^2}\right)^2 - \left[\frac{\alpha^2}{\gamma^2}(4\pi n_{\rm h})^3 - 2\cdot(4\pi n_{\rm h})^2 + \frac{\gamma^2}{\alpha^2}(4\pi n_{\rm h}) - \frac{h_0^2}{\gamma^2}\right]. \quad (22)$$

After some straightforward manipulation, one can obtain the explicit expressions for $k_{\rm f}^{(\pm)}$:

$$k_{\rm f}^{(-)2} = \frac{1}{2}\left(4\pi n_{\rm h} + \sqrt{(4\pi n_{\rm h})^2 - 4\eta}\right), \quad (23a)$$

$$k_{\rm f}^{(+)2} = \frac{1}{2}\left(4\pi n_{\rm h} - \sqrt{(4\pi n_{\rm h})^2 - 4\eta}\right), \quad (23b)$$

where

$$\eta = k_{\rm f}^{(+)2}k_{\rm f}^{(-)2} = (4\pi n_{\rm h})^2 + \frac{2\gamma\varepsilon_{\rm f}}{\alpha^2} - \frac{\gamma^2}{\alpha^2}(4\pi n_{\rm h}). \quad (24)$$

Therefore, the behaviour of Hall conductivity in Eq.(18) can now be analysed in different parameter regimes by use of expressions for Fermi wave numbers $k_{\rm f}^{(\pm)}$ and Fermi energy $\varepsilon_{\rm f}$.

Figure 1 shows the Fermi wave numbers $k_{\rm f}^{(\pm)}$ as a function of α/γ for different values of Zeeman splitting ($h_0 = 0$ for panel(a) and $h_0 = 0.067\,{\rm meV}$ for panel (b)). The hole density is chosen to be $n_{\rm h} = 3\times10^{14}\,{\rm m}^{-2}$. Note that Winkler *et al*[24] have studied both theoretically and experimentally the magnitude of the Rashba spin orbit coupling in GaAs-based quantum well samples with heavy-hole densities of a few $10^{14}\,{\rm m}^{-2}$ and have found a typical value for the characteristic length scale α/γ of a few nanometres. Thus the variation in the value of α/γ shown in Fig.1 is experimentally accessible. Given the value of the effective Zeeman factor, $g^* = 2.2$ for bulk GaAs, the Zeeman splitting h_0 can be varied in a range of 0–0.30 meV, which is relatively small compared with the other energy scales. However, the situation is totally different if the Zeeman splitting is tuned by the magnetic impurity doping in the paramagnetic semiconductor hosts. Recent advance in diluted magnetic semiconductors[32] has made it possible to manipulate the kinetic exchange interaction of the free carriers in a controllable way. In the mean-field approximation, the kinetic exchange interaction splitting can be expressed as $h_0 = -N_0\beta x S B_S(\xi)$. Here, $N_0\beta$ is the exchange integral, x is the magnetic ion concentration, $B_S(\xi)$ is the standard Brillouin function[33] and

$\xi = g_S\mu_B SB/k_{\rm B}T$ with S and g_S being the total spin quantum number and the Landé factor of the magnetic ions respectively. For Mn doping in GaAs host, it has been acknowledged[34] that the Mn ions replace the cations, and are in the $3d^5$ configurations, thus leading to $S = 5/2$ and $g_S = 2$. The p–d exchange integral in (Ga, Mn)As has been measured[35] to be $N_0\beta \approx -1\,$eV. The exchange interaction h_0 can be changed in a wide range of 4–16 meV under a weak external magnetic field on the order of 10–100 mT at a typical low temperature $T = 1\,$K and Mn concentration $x = 0.05$.

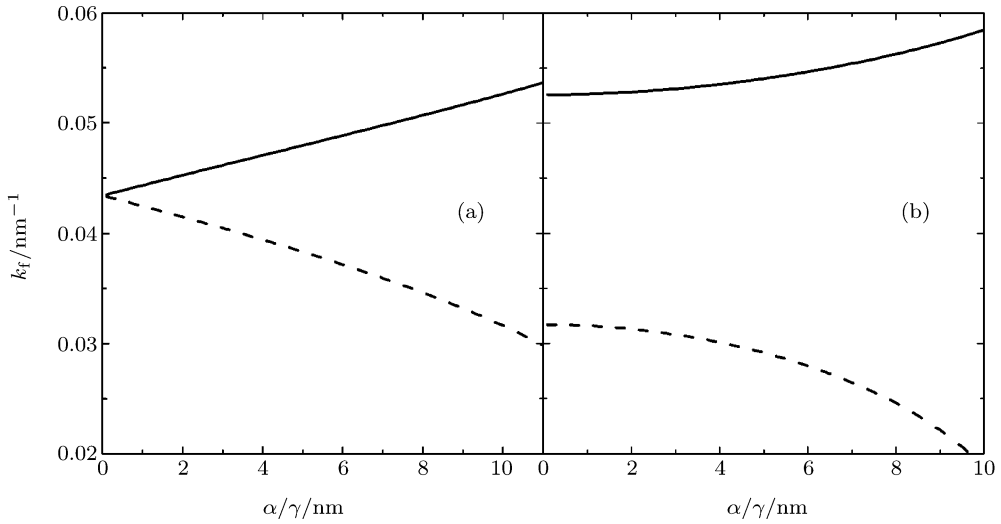


Fig.1. The Fermi wave numbers $k_{\rm f}^{(\mp)}$ of heavy holes as a function of the characteristic length α/γ of the Rashba coupling at a total hole density of $n_{\rm h} = 3 \times 10^{14} m^{-2}$ for $h_0 = 0$ (a) and 0.067 meV (b). The solid and dashed lines represent $k_{\rm f}^{(-)}$ and $k_{\rm f}^{(+)}$, respectively.

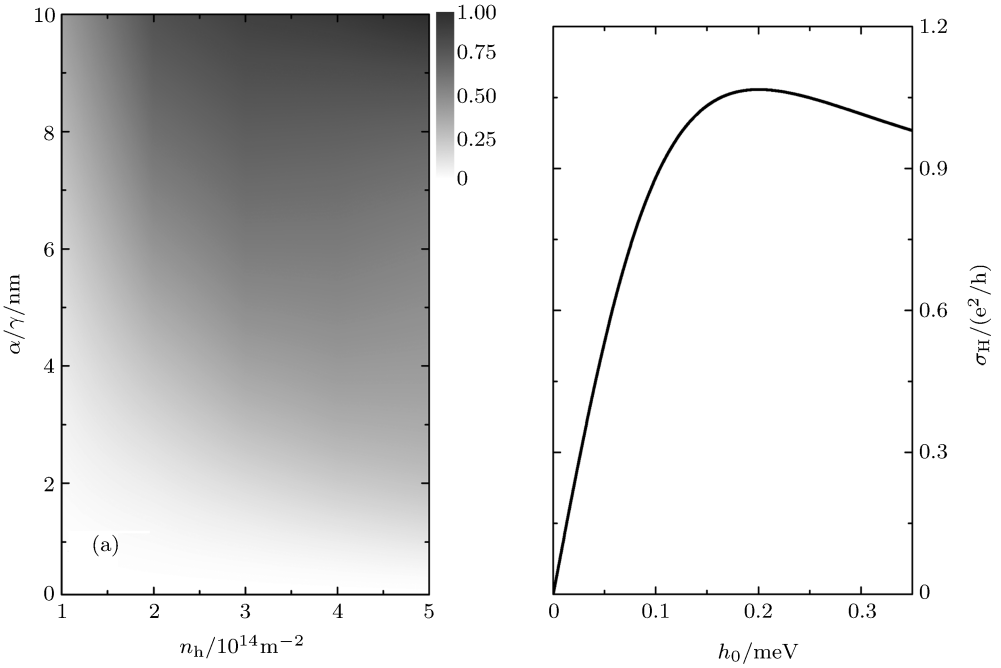


Fig.2. The contour plot of the Hall conductivity in the parameter space (n_h, α) (a), and Hall conductivity as a function of Zeeman splitting h_0 by fixing the values of $n_{\rm h} = 5 \times 10^{14}$ m^{-2} and $\alpha/\gamma = 5.0\,$nm (b).

Figure 2 summarizes the behaviour of the Hall conductivity at zero temperature. Panel (a) is a contour plot of this quantity in the parameter space ($n_{\rm h}$, α). The Zeeman splitting has been fixed to be $h_0 = 0.027\,{\rm meV}$. One can see that the Hall conductivity can increase with the value of hole density or Rashba coefficient increasing. The cause is that the large spin–orbit coupling and hole density at finite Zeeman splitting will prominently unbalance the hole population in the two bands, which in turn, due to the opposite signs of Berry curvatures [see Eq.(16)], enlarges the Hall effect as shown in panel (a). Panel(b) shows the Hall conductivity as a function of Zeeman splitting h_0 by fixing the values of $n_{\rm h} = 5 \times 10^{14}\,{\rm m}^{-2}$ and $\alpha/\gamma = 5.0\,{\rm nm}$. It reveals that the Hall conductivity first increases with the Zeeman splitting increasing. When the energy of Zeeman splitting increases to a value comparable with the spin-orbit splitting energy, then the Hall conductivity begins to decrease due to the competition mechanism between these two typical energies. This competition is featured by the fact that the spin-orbit coupling in the (x, y) plane tends to deflect the particles from the driving electric force, while the Zeeman splitting along the z-axis tends to polarize the carrier spins, thus eliminate the deflection degrees of freedom during the motion of carriers. It should be addressed that although the temperature dependence of the present Hall conductivity is only reflected by the Fermi distribution function in Eq.(3), it does not mean that this Hall effect is insensitive to thermal excitation. To ensure that the Hall effect is not completely overshadowed by thermal excitation, the Zeeman splitting h_0 and the spin-orbit interaction must exceed the energy broadening due to impurity scattering.

Finally, some comments are made on the validity of the model we employed in this work. The Hamiltonian Eq.(1) does not take into account the possible contribution from the light holes. Thus the effectiveness of the above model Hamiltonian is limited to the case of small wave numbers and low hole densities,[26] for which the Zeeman splitting between the heavy and light holes enables the carriers to solely occupy the heavy-hole eigenstates. One does not need to worry about the possible occurrence of the mixing between the heavy and light holes due to the presence of the external perturbation. A simple derivation gives the result that this mixing is absent in the linear order in Zeeman splitting. On the other hand, the neglect of the light holes in the Hamiltonian is consistent with an obvious fact that, as indicated by the hole dispersion, the lower dispersion branch is not bounded from below for large wave numbers and has positive eigenenergies only for $k \leq \hbar^2/2m\alpha$ (for $h_0 = 0$) with a maximum at $k = \hbar^2/3m\alpha$. The unbounded decrease of the single-particle energies with increasing wave number for $k > \hbar^2/3m\alpha$ is clearly an unphysical feature of the model. Therefore the consideration of heavy-hole densities in the present work has been carefully taken into account to enable the k-value variations to hold true.

In summary, we have studied the AHE of heavy holes in III–V semiconductor quantum wells in the presence of spin-orbit coupling of the Rashba type due to structure inversion asymmetry. Based on the first-order perturbation of wave function, the expression for the Hall conductivity the same as that from the semiclassical equation of motion of the Bloch particles is derived. It has been shown that the Hall conductivity depends on the hole density, the Rasha SO coupling strength, and the Zeeman splitting. In particular, the amplitude of Hall conductivity has been found to be balanced by a competition between the Zeeman splitting and the SO splitting.

References

[1] Prinz G A 1998 *Science* **282** 1660
Wolf S A, Awshalom D D, Buhrman R A, Daughton J M, von Molnars, Roukes M L, Chtchelkanova A Y and Treger D M 2001 *Science* **294** 1488

[2] Awschalom D D, Sarmarth N and Loss D 2002 *Semiconductor Spintronics and Quantum Computation* (Berlin: Springer)

[3] Zutic I, Fabian J and Sarma S D 2004 *Rev. Mod. Phys.* **76** 323

[4] Pan W and Shen W Z 2004 *Acta Phys. Sin.* **53** 1501 (in Chinese)

[5] Zhang S Y 2003 *Acta Phys. Sin.* **52** 2290 (in Chinese)

[6] Yuan X L and Shi Y 2000 *Acta Phys. Sin.* **49** 2037 (in Chinese)

[7] Wu F and Wang T H 2003 *Acta Phys. Sin.* **52** 0696 (in Chinese)

[8] Jungwirth T, Niu Q and MacDonald A H 2002 *Phys. Rev. Lett.* **88** 207208

[9] Culcer D, MacDonald A H and Niu Q 2003 *Phys. Rev.* B **68** 045327

[10] Fang Z, Nagaosa N, Takahashi K S, Asamitsu A, Mathieu R, Ogasawara T, Yamada H, Kawasaki M, Tokura Y and Terakura K, 2003 *Science* **302** 92
Yao Y, Kleinman L, MacDonald A H, Sinova J, Jungwirth

T, Wang D, Wang E and Niu Q 2004 *Phys. Rev. Lett.* **92** 037204
[11] Dyakonov M I and Perel V I 1971 *JETP* **33** 1053
[12] Hirsch J E 1999 *Phys. Rev. Lett.* **83** 1834
[13] Zhang S 2000 *Phys. Rev. Lett.* **85** 393
[14] Murakami S, Nagaosa N and Zhang S C 2003 *Science* **301** 1348
[15] Sinova J, Culcer D, Niu Q, Sinitsyn N A, Jungwirth T and MacDonald A H 2004 *Phys. Rev. Lett.* **92** 126603
[16] Kato Y K, Myers R C, Gossard A C and Awschalom D D 2004 *Science* **306** 1910
[17] Wunderlich J, Kaestner B, Sinova J and Jungwirth T 2005 *Phys. Rev. Lett.* **94** 047204
[18] Crépieux A and Bruno P 2001 *Phys. Rev.* B **64** 0114416
[19] Karplus R and Luttinger J M 1954 *Phys. Rev.* **95** 1154
[20] Berger L 1970 *Phys. Rev.* B **2** 4559
[21] Smit J 1955 *Physica* (Amsterdam) **21** 877
Smit J 1958 *Physica* **24** 39
[22] Lee W L, Watauchi S, Miller V L, Cava R J and Ong N P 2004 *Science* **303** 1647
[23] Gerchikov L G and Subashiev A V 1992 *Sov. Phys. Semicond.* **26** 73
[24] Winkler R 2002 *Phys. Rev.* B **62** 4245
Winkler R, Noh H, Tutuc E and Shayegan M 2002 *Phys. Rev.* B **65** 155303
[25] Pala M G, Governale M, Konig J and Zulicke U 2004 *Europhys. Lett.* **65** 850
Pala M G, Governale M, Konig J, Zulicke U and Iannacone G 2004 *Phys. Rev.* B **69** 045304
[26] Schliemann J and Loss D 2005 *Phys. Rev.* B **71** 085308
[27] Zak J 1989 *Phys. Rev. Lett.* **62** 2747
[28] Goryo J and Kohmoto M 2002 *Phys. Rev.* B **66** 085118
[29] Berry M V 1984 *Proc. R. Soc. London,* Ser. A **392** 45
[30] Simon B 1983 *Phys. Rev. Lett.* **51** 2167
[31] Sundaram G and Niu Q 1999 *Phys. Rev.* B **59** 14915
[32] Ohno H 1998 *Science* **281** 951
[33] Kittel C 1996 *Introduction to Solid State Physics* (Wiley, New York).
[34] Dietl T, Ohno H, Matsukura F, Cibert J and Ferrand D 2000 *Science* **287** 1019
[35] Bhattacharjee A K and Benoit C à la Guillaume 2000 *Solid State Commun.* **113** 17