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A resistance formula for coherent multi-barrier structures

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Using the Landauer formula and the quantum S-matrix scattering theory, we derive a resistance formula for multi-barrier structure under phase coherent transmission condition. This formula shows that when the transport is coherent, the potential wells of the structure are just like conductors contributing to the overall resistance. And because the resistance formula is derived based on the scattering theory, the barrier resistance will change with the number of scattering centres (i.e. the number of barriers) in the structure.

Keywords: resistance, phase coherent transmission, multi-barrier structure
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1. Introduction

Since Landauer derived several resistance formulae of a series of incoherent identical barriers in 1957,[1] a lot of work has been done to deal with the resistance of barriers. Sorbello[2] has given the resistance information of single barrier. Laikhtman *et al*[3] handled a single localized barrier. Godoy[4] showed that the one-channel Landauer diffusion coefficient could be obtained from a purely incoherent classical theory. Bagwell *et al*[5] discussed the condition that there were N scatters randomly distributed in one dimension between two reservoirs. And in 1995, Landauer[6] presented a review of analyses in this field and gave some simpler formations of those formulas contained in his 1957's paper. In addition, there have been published many other papers in this field. But most of the papers are concerned with the phase incoherent transmissions in single-barrier or multi-barrier structures. It is of interest to obtain more information about the resistance of multi-barrier structures when the transport is coherent. For example, when the transport is coherent, we need a general term to make the calculation of resistance as convenient as the Landauer formula in incoherent transport.[6] And we need to know whether the resistance in coherent multi-barrier structure follows the Ohm's law or it still has the same form as that in incoherent structure.

In this paper, we address all these questions. First, using the S-matrix theory, we calculate the resistance of a coherent double-barrier structure. The results show that the potential well in such a structure can make a contribution to the resistance. Then neglecting very small terms in transmission and reflection coefficients, we obtain a resistance formula of N-barrier structure (where N is a natural number) that has a similar formation to that for the double-barrier structure. Finally, we give out the explanations about the difference in resistance formula between the coherent structure and the incoherent structure.

2. Resistance in a coherent double-barrier structure

In Ref.[6] different kinds of resistance formulas are listed. The two-terminal resistance is expressed as

$$\Omega_2 = (h/2e^2)T^{-1}, \tag{1}$$

and the four-terminal resistance reads

$$\Omega_4 = (h/2e^2)(R/T), \tag{2}$$

where $h/2e^2 = \Delta$ is the quantum resistance, T is the transmission coefficient of barrier and R is the reflection coefficient. So the process of determining Ω_4 is as follows: we obtain a transmission amplitude t, and then T from $T = |t|^2$.[7] Finally, using $R = 1 - T$[6] and inserting T and R into the formula (2), we can obtain Ω_4.

In a two-barrier structure, if the transmission is coherent, using the quantum S-matrix scattering

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theory,[8] we can have the total transmission amplitude of the system[9]

$$t_T = t_1 \mathrm{e}^{\mathrm{i}\theta} t_2 [1 - r_1' r_2 \mathrm{e}^{\mathrm{i}2\theta_1}]^{-1}, \tag{3}$$

where t_n means the nth (where $n = 1, 2$) transmission amplitude of the barrier. r_1' is the reflection amplitude on the right-hand side of the first barrier, and r_2 is the reflection amplitude on the left-hand side of the second barrier. $\theta_m = kb_m$ belongs to the mth well (where $m = 1$), where b_m is the width of the mth well and $k = \sqrt{2m^*E}/\hbar$ is the wave-vector subject to the Schrödinger Equation.

From the expression (3), we have

$$T = |t_T|^2 = T_1 T_2 [1 + 2\sqrt{R_1 R_2}\cos(2\theta_1) + R_1 R_2]^{-1}, \tag{4}$$

and then

$$R = 1 - T = 1 - T_1 T_2 [1 + 2\sqrt{R_1 R_2}\cos(2\theta_1) + R_1 R_2]^{-1}, \tag{5}$$

where $r = r'$, and $t_n = \sqrt{T_n}\mathrm{e}^{\mathrm{i}\gamma_n}$[10] with the subscript n referring to the nth barrier, $\gamma_n = ka_n$, with a_n being the width of the nth barrier.

Inserting the expressions (4) and (5) into the formula (2), we obtain

$$\Omega_{4T} = \Delta\left[\frac{R_1 + 2\sqrt{R_1 R_2}\cos(2\theta_1) + R_2}{T_1 T_2}\right] = \Delta\left[\frac{R_1}{T_1 T_2} + \frac{2\sqrt{R_1 R_2}\cos(2\theta_1)}{T_1 T_2} + \frac{R_2}{T_1 T_2}\right]. \tag{6}$$

So the expression (6) is the formula of four-terminal resistance in the coherent double-barrier structure.

3. Resistance in a coherent multi-barrier structure

In a multi-barrier structure, in order to remain the coherence throughout the structure, carriers should not be scattered frequently by the barriers. In other words, the time when carriers stay in the well should be longer than in the barrier. And the carrier should not lose too much energy in a single barrier when the structure has a lot of barriers. In this case, the width of the barrier should be much shorter than the width of the well. And the resistance of the barrier exists in the centre of the barrier.[11] So in the next step we neglect the width of the barrier. And we consider all the wells to be symmetrical (the θ values of all wells are equal). In order to clearly express its physical significance, we still keep the subscripts of θ.

Using the S-matrix scattering theory, we have

$$t_T = t_1 \mathrm{e}^{\mathrm{i}\theta_1} t_2 \mathrm{e}^{\mathrm{i}\theta_2} t_3 [1 - r_1' r_2 \mathrm{e}^{\mathrm{i}2\theta_1} - r_2' r_3 \mathrm{e}^{\mathrm{i}\theta_2} - r_1' r_3 (t_2 t_2' - r_2 r_2') \mathrm{e}^{\mathrm{i}2(\theta_1+\theta_2)}]^{-1} \tag{7}$$

for the three-barrier structure;

$$\begin{aligned} t_T = \; & t_1 \mathrm{e}^{\mathrm{i}\theta_1} t_2 \mathrm{e}^{\mathrm{i}\theta_2} t_3 \mathrm{e}^{\mathrm{i}\theta_3} t_4 [1 - r_1' r_2 \mathrm{e}^{\mathrm{i}2\theta_1} - r_2' r_3 \mathrm{e}^{\mathrm{i}2\theta_2} - r_3' r_4 \mathrm{e}^{\mathrm{i}2\theta_3} - r_1' r_3 (t_2 t_2' - r_2 r_2') \mathrm{e}^{\mathrm{i}2(\theta_1+\theta_2)} \\ & - r_2' r_4 (t_3 t_3' - r_3 r_3') \mathrm{e}^{\mathrm{i}2(\theta_2+\theta_3)} - r_1' r_4 (t_2 t_2' - r_2 r_2')(t_3 t_3' - r_3 r_3') \mathrm{e}^{\mathrm{i}2(\theta_1+\theta_2+\theta_3)} + r_1' r_2 r_3' r_4 \mathrm{e}^{\mathrm{i}2(\theta_1+\theta_3)}]^{-1} \end{aligned} \tag{8}$$

for the four-barrier structure;

$$\begin{aligned} t_T = \; & t_1 \mathrm{e}^{\mathrm{i}\theta_1} t_2 \mathrm{e}^{\mathrm{i}\theta_2} t_3 \mathrm{e}^{\mathrm{i}\theta_3} t_4 \mathrm{e}^{\mathrm{i}\theta_4} t_5 [1 - r_1' r_2 \mathrm{e}^{\mathrm{i}2\theta_1} - r_2' r_3 \mathrm{e}^{\mathrm{i}2\theta_2} - r_3' r_4 \mathrm{e}^{\mathrm{i}2\theta_3} - r_4' r_5 \mathrm{e}^{\mathrm{i}2\theta_4} - r_1' r_3 (t_2 t_2' - r_2 r_2') \mathrm{e}^{\mathrm{i}2(\theta_1+\theta_2)} \\ & - r_2' r_4 (t_3 t_3' - r_3 r_3') \mathrm{e}^{\mathrm{i}2(\theta_2+\theta_3)} - r_3' r_5 (t_4 t_4' - r_4 r_4') \mathrm{e}^{\mathrm{i}2(\theta_3+\theta_4)} - r_1' r_4 (t_2 t_2' - r_2 r_2')(t_3 t_3' - r_3 r_3') \mathrm{e}^{\mathrm{i}2(\theta_1+\theta_2+\theta_3)} \\ & - r_2' r_5 (t_3 t_3' - r_3 r_3')(t_4 t_4' - r_4 r_4') \mathrm{e}^{\mathrm{i}2(\theta_2+\theta_3+\theta_4)} - r_1' r_5 (t_2 t_2' - r_2 r_2')(t_3 t_3' - r_3 r_3')(t_4 t_4' - r_4 r_4') \mathrm{e}^{\mathrm{i}2(\theta_1+\theta_2+\theta_3+\theta_4)} \\ & + r_1' r_2 r_3' r_4 \mathrm{e}^{\mathrm{i}2(\theta_1+\theta_3)} + r_2' r_3 r_4' r_5 \mathrm{e}^{\mathrm{i}2(\theta_2+\theta_4)} + r_1' r_2 r_4' r_5 \mathrm{e}^{\mathrm{i}2(\theta_1+\theta_4)} + r_1' r_2 r_3' (t_4 t_4' - r_4 r_4') r_5 \mathrm{e}^{\mathrm{i}2(\theta_1+\theta_3+\theta_4)} \\ & + r_1' (t_2 t_2' - r_2 r_2') r_3 r_4' r_5 \mathrm{e}^{\mathrm{i}2(\theta_1+\theta_2+\theta_4)}]^{-1} \end{aligned} \tag{9}$$

for the five-barrier structure.

If we use these equations to calculate T and R, we use the formula (2) to determine $\Omega_{4\mathrm{T}}$, which is very onerous work and the expression of resistance is too complex to find out its clear physical significance involved. Of course, the resistance could not keep its form as the expression (6). So it is necessary to deal with these equations by using some other methods.

Take the three-barrier structure for example. The expression (7) can be rewritten in a more concise form as $t_T = \lambda\Sigma$, where $\lambda = t_1\mathrm{e}^{\mathrm{i}\theta_1}t_2\mathrm{e}^{\mathrm{i}\theta_2}t_3$ and

$$\begin{aligned}\sum = \ & [1 - r_1'r_2\mathrm{e}^{\mathrm{i}2\theta_1} - r_2'r_3\mathrm{e}^{\mathrm{i}2\theta_2} \\ & -r_1'r_3(t_2t_2' - r_2r_2')\mathrm{e}^{\mathrm{i}2(\theta_1+\theta_2)}]^{-1}. \end{aligned} \quad (10)$$

When $M^{(n)} = \begin{bmatrix} M_{11}^{(n)} & M_{12}^{(n)} \\ M_{21}^{(n)} & M_{22}^{(n)} \end{bmatrix}$ and $S_n = \begin{bmatrix} r_n & t_n' \\ t_n & r_n' \end{bmatrix}$ are the transfer and scattering matrices of the nth barrier,[7] respectively, we have[12]

$$\begin{aligned} & M_{11}^{(n)} = 1/t_n,\ M_{21}^{(n)} - t_n/t_n, \\ & M_{12}^{(n)} = M_{21}^{(n)*} = -r_n'/t_n, \\ & M_{22}^{(n)} = M_{11}^{(n)*} = (t_nt_n' - r_nr_n')/t_n. \end{aligned} \quad (11)$$

where M^* is the conjugate of M.

From the expression (11), we have $-r_n' = t_n(r_n/t_n)^*$, so

$$\begin{aligned} t_2t_2' - r_2r_2' \ &= (t_2t_2^*t_2')/t_2^* + (r_2r_2^*t_2)/t_2^* \\ &= T_2^+t_2'/t_2^* + R_2t_2/t_2^*, \end{aligned} \quad (12)$$

where $T_2^+ = t_2t_2^*$. Because S is a unitary matrix,[13] we have $t = (\upsilon_\mathrm{L}/\upsilon_\mathrm{R})t'$, where υ_R refers to the velocity of the wave on the right of the barriers, and υ_L is that on the left. Inserting $t = (\upsilon_\mathrm{L}/\upsilon_\mathrm{R})t'$ into the expression (12), we have

$$t_2t_2' - r_2r_2' = (t_2/t_2^*) = [T_2^+(\upsilon_\mathrm{R}/\upsilon_\mathrm{L}) + R_2] = t_2/t_2^* = \mathrm{e}^{\mathrm{i}2\gamma_2}, \quad (13)$$

where $T_2^+(\upsilon_\mathrm{R}/\upsilon_\mathrm{L}) + R_2 = 1$[14] (note that $T_2^+(\upsilon_\mathrm{R}/\upsilon_\mathrm{L}) = T_2$) and $t_2 = \sqrt{T_2}\mathrm{e}^{\mathrm{i}\gamma_2}$ are used.

Inserting the expression (13) into the expression (7), we have

$$t_T = \lambda[1 - r_1'r_2\mathrm{e}^{\mathrm{i}2\theta_1} - r_2'r_3\mathrm{e}^{\mathrm{i}2\theta_2} - r_1'r_3\mathrm{e}^{\mathrm{i}2(\theta_1+\gamma_2+\theta_2)}]^{-1}. \quad (14)$$

Inserting $r = r'$ and $t_n = \sqrt{T_n}\mathrm{e}^{\mathrm{i}\gamma_n}$ into the expression (11), we can obtain $r_n = \mathrm{i}\sqrt{R_2}\mathrm{e}^{\mathrm{i}\gamma_n}$. Then the expression (14) can be rewritten as

$$t_T = \lambda[1 + \sqrt{R_1R_2}\mathrm{e}^{\mathrm{i}(2\theta_1+\gamma_1+\gamma_2)} + \sqrt{R_2R_3}\mathrm{e}^{\mathrm{i}(2\theta_2+\gamma_2+\gamma_3)} + \sqrt{R_1R_3}\mathrm{e}^{\mathrm{i}(2\theta_1+\gamma_1+2\gamma_2+\gamma_3+2\theta_2)}]^{-1}. \quad (15)$$

We neglect the width of the barrier, so $\gamma_n = 0$. Then expression (15) can be rewritten as

$$t_T = \lambda[1 + \sqrt{R_1R_2}\mathrm{e}^{\mathrm{i}(2\theta_1)} + \sqrt{R_2R_3}\mathrm{e}^{\mathrm{i}(2\theta_2)} + \sqrt{R_1R_3}\mathrm{e}^{\mathrm{i}(2\theta_1+2\theta_2)}]^{-1}. \quad (16)$$

and we have

$$\begin{aligned} T = \ & T_1T_2T_3[1 + 2\sqrt{R_1R_2}\cos(2\theta_1) + 2\sqrt{R_2R_3}\cos(2\theta_2) + 2\sqrt{R_1R_3}\cos(2\theta_1 + 2\theta_2) \\ & + R_1R_2 + R_2R_3 + R_1R_3 + 2\sqrt{R_1R_2R_2R_3}\cos(2\theta_1 - 2\theta_2) + 2\sqrt{R_1R_1R_2R_3}\cos(2\theta_2) \\ & + 2\sqrt{R_1R_2R_3R_3}\cos(2\theta_1)]^{-1} \\ = \ & T_1T_2T_3[1 + (1 + 2R_3)2\sqrt{R_1R_2}\cos(2\theta_1) + (1 + 2R_1)2\sqrt{R_2R_3}\cos(2\theta_2) + 2\sqrt{R_1R_2}\cos(2\theta_1 + 2\theta_2) \\ & + R_1R_2 + R_2R_3 + R_1R_3 + 2\sqrt{R_1R_2R_2R_3}\cos(2\theta_1 - 2\theta_2)]^{-1}, \end{aligned} \quad (17)$$

where $\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha - \beta)$ has been used. Now we focus on the terms in the brackets on the right-hand side of the expression (17). We consider all the wells to be symmetrical, so $2\sqrt{R_1R_2R_2R_3}\cos(2\theta_1 - 2\theta_2) = 2\sqrt{R_1R_2R_2R_3}$. In the brackets, terms such as $2\sqrt{R_1R_2}\cos(2\theta_1)$ are the first-order terms in the R_n expression. Other terms such as R_1R_2 and R_2R_3 are the second-order terms in the R_n expression. As is known, in the coherent structure, $T_n \approx 1$ and $R_n \ll 1$, where n refers to the nth

barrier. So these second-order terms in the R_n expression can be neglected in the expression (17). And $(1+2R_3)2\sqrt{R_1R_2}\cos(2\theta_1) \approx 2\sqrt{R_1R_2}\cos(2\theta_1)$; $(1+2R_1)2\sqrt{R_2R_3}\cos(2\theta_2) \approx 2\sqrt{R_2R_3}\cos(2\theta_2)$. Note that the value of $2\sqrt{R_1R_2}\cos(2\theta_1)$ can change with the value of θ_1. Sometimes its value may be as small as the values of the second-order terms in the R_n expression, so we could not neglect $2\sqrt{R_1R_2}\cos(2\theta_1)$. Of course, $2\sqrt{R_2R_3}\cos(2\theta_2)$ and $2\sqrt{R_1R_3}\cos(2\theta_1+2\theta_2)$ are the same as $2\sqrt{R_1R_2}\cos(2\theta_1)$.

Then the expression (17) can be rewritten as

$$T = \frac{T_1T_2T_3}{1+2\sqrt{R_1R_2}\cos(2\theta_1)+2\sqrt{R_2R_3}\cos(2\theta_2)+2\sqrt{R_1R_3}\cos(2\theta_1+2\theta_2)}. \tag{18}$$

Now we must analyse R in the formula (2) in order to neglect the higher-order terms in the R_n expression just like T shown in the expression (18).

The expression (18) can be rewritten by the symbols as $T = T'/\Gamma$, where $T' = T_1T_2T_3$, and $\Gamma = 1 + 2\sqrt{R_1R_2}\cos(2\theta_1) + 2\sqrt{R_2R_3}\cos(2\theta_2) + 2\sqrt{R_1R_3}\cos(2\theta_1+2\theta_2)$.

Similarly, we denote R'/Γ by R for short. From $T+R = T'/\Gamma + R'/\Gamma = 1$, we have

$$\begin{aligned} R' = {} & \Gamma - T' = 2\sqrt{R_1R_2}\cos(2\theta_1) + 2\sqrt{R_2R_3}\cos(2\theta_2) + 2\sqrt{R_1R_3}\cos(2\theta_1+2\theta_2) \\ & + R_1 + R_2 + R_3 + R_1R_2R_3 - R_1R_2 - R_2R_3 - R_1R_3, \end{aligned} \tag{19}$$

where $T_n = 1 - R_n$ has been used.

In the expression (19), we neglect the higher-order terms in the R_n expression. Then we have

$$R' = 2\sqrt{R_1R_2}\cos(2\theta_1) + 2\sqrt{R_2R_3}\cos(2\theta_2) + 2\sqrt{R_1R_3}\cos(2\theta_1+2\theta_2) + R_1 + R_2 + R_3$$

and

$$R = R'/\Gamma = [2\sqrt{R_1R_2}\cos(2\theta_1) + 2\sqrt{R_2R_3}\cos(2\theta_2) + 2\sqrt{R_1R_3}\cos(2\theta_1+2\theta_2) + R_1 + R_2 + R_3]/\Gamma. \tag{20}$$

Inserting the expressions (18) and (20) into the formula (2), we obtain

$$\begin{aligned} \Omega_{4T} = {} & \Delta\left[\frac{R_1}{T_1T_2T_3} + \frac{R_2}{T_1T_2T_3} + \frac{R_3}{T_1T_2T_3} + \frac{2\sqrt{R_1R_2}\cos(2\theta_1)}{T_1T_2T_3} + \frac{2\sqrt{R_2R_3}\cos(2\theta_2)}{T_1T_2T_3}\right. \\ & \left. + \frac{2\sqrt{R_1R_3}\cos(2\theta_1+2\theta_2)}{T_1T_2T_3}\right]. \end{aligned} \tag{21}$$

So the expression (21) is the four-terminal resistance in the coherent three-barrier structure.

Following the same steps in the three-barrier structure (from the expression (17) to the expression (18) and from the expression (19) to the expression (20)), for a four-barrier structure we obtain

$$\begin{aligned} \Omega_{4T} = {} & \Delta\left[\frac{R_1}{T_1T_2T_3T_4} + \frac{R_2}{T_1T_2T_3T_4} + \frac{R_3}{T_1T_2T_3T_4} + \frac{R_4}{T_1T_2T_3T_4} + \frac{2\sqrt{R_1R_2}\cos(2\theta_1)}{T_1T_2T_3T_4} + \frac{2\sqrt{R_2R_3}\cos(2\theta_2)}{T_1T_2T_3T_4}\right. \\ & + \frac{2\sqrt{R_3R_4}\cos(2\theta_3)}{T_1T_2T_3T_4} + \frac{2\sqrt{R_1R_3}\cos(2\theta_1+2\theta_2)}{T_1T_2T_3T_4} + \frac{2\sqrt{R_2R_4}\cos(2\theta_2+2\theta_3)}{T_1T_2T_3T_4} \\ & \left. + \frac{2\sqrt{R_1R_4}\cos(2\theta_1+2\theta_2+2\theta_3)}{T_1T_2T_3T_4}\right]. \end{aligned} \tag{22}$$

And for a five-barrier structure, we have

$$\begin{aligned} \Omega_{4T} = {} & \Delta\left[\frac{R_1}{T_1T_2T_3T_4T_5} + \frac{R_2}{T_1T_2T_3T_4T_5} + \frac{R_3}{T_1T_2T_3T_4T_5} + \frac{R_4}{T_1T_2T_3T_4T_5} + \frac{R_5}{T_1T_2T_3T_4T_5} + \frac{2\sqrt{R_1R_2}\cos(2\theta_1)}{T_1T_2T_3T_4T_5}\right. \\ & + \frac{2\sqrt{R_2R_3}\cos(2\theta_2)}{T_1T_2T_3T_4T_5} + \frac{2\sqrt{R_3R_4}\cos(2\theta_3)}{T_1T_2T_3T_4T_5} + \frac{2\sqrt{R_4R_5}\cos(2\theta_4)}{T_1T_2T_3T_4T_5} + \frac{2\sqrt{R_1R_3}\cos(2\theta_1+2\theta_2)}{T_1T_2T_3T_4T_5} \\ & + \frac{2\sqrt{R_2R_4}\cos(2\theta_2+2\theta_3)}{T_1T_2T_3T_4T_5} + \frac{2\sqrt{R_3R_5}\cos(2\theta_3+2\theta_4)}{T_1T_2T_3T_4T_5} + \frac{2\sqrt{R_1R_4}\cos(2\theta_1+2\theta_2+2\theta_3)}{T_1T_2T_3T_4T_5} \\ & \left. + \frac{2\sqrt{R_2R_5}\cos(2\theta_2+2\theta_3+2\theta_4)}{T_1T_2T_3T_4T_5} + \frac{2\sqrt{R_1R_5}\cos(2\theta_1+2\theta_2+2\theta_3+2\theta_4)}{T_1T_2T_3T_4T_5}\right]. \end{aligned} \tag{23}$$

From the expressions (21), (22) and (23), we find that the four-terminal resistance follows a simple pattern. Actually, because $R_n \ll 1$, the higher-order terms in the R_n expression are neglected. The first-order terms follow a simple pattern in the calculations of T and R with the increase of the number of barrier. When using T and R to calculate the resistance, the expression with this simple pattern turns to the resistance.

Following the same pattern as the above, we obtain for an N-barrier structure (N is a natural number) the four-terminal resistance formula

$$\Omega_{4T} = \Delta \left(\prod_{a=1}^{N} T_a \right)^{-1} \left[\sum_{b=1}^{N} R_b + 2 \sum_{c=1}^{N-1} \sqrt{R_c} \sum_{d=c+1}^{N} \sqrt{R_d} \cos \left(2 \sum_{e=c}^{d-1} \theta_e \right) \right], \tag{24}$$

where T_a, R_b, R_c and R_d belong to the ath, bth, cth and dth barrier, respectively. θ_e belongs to the eth well. Now the expression (24) is the final result as expected.

4. Discussion

In Ref.[6], Landauer showed that when the transport was incoherent, the four-terminal resistance formula of N-barrier structure was expressed as

$$\Omega_{4T} = \Delta \sum_{n=1}^{N} (R_n/T_n). \tag{25}$$

Comparing with the coherent transport (the expression (24)), we can find some differences. In order to clearly point them out, we take the five-barrier structure for example. Then the expression (25) can be rewritten as

$$\begin{aligned} \Omega_{4T} = \ & \Delta(R_1/T_1 + R_2/T_2 \\ & + R_3/T_3 + R_4/T_4 + R_5/T_5) \end{aligned} \tag{26}$$

In the expression (26), because the transport is incoherent, the terms are the resistances of barriers. Comparing with the expression (23) shows that the first difference is that the terms in the expression (26) have changed into the terms in the expression (23) such as $\Delta[R_1/T_1T_2T_3T_4T_5]$. It is clear that $\Delta[R_1/T_1T_2T_3T_4T_5]$ is the resistance of the first barrier when the transport is coherent. This means that under the same environmental condition, if the system is coherent, a conductor's resistance changes with barrier's number. And the more the barrier's number is, the larger the barrier's resistance is. But this is different from our general macrocosm where the resistance is constant no matter how many conductors the structure has. Actually the reason is clear. In Landauer's theory, the conductance of system is derived based on the scattering theory. And as is known, the overall coherent transmission coefficient changes with the number of scattering centres (here barriers are scattering centres). So the resistances are different. Actually, in the coherent structure, $T \approx 1$. So the difference between $\Delta(R_1/T_1)$ and $\Delta[R_1/T_1T_2T_3T_4T_5]$ is very small.

The second difference is that in the expression (23), there are several terms, such as $\Delta \left[\dfrac{2\sqrt{R_1R_2}\cos(2\theta_1)}{T_1T_2T_3T_4T_5} \right]$, which are not the barrier's resistance. These terms contain the information from the well (θ_1) and from the barriers ($\sqrt{R_1}$ and $\sqrt{R_2}$). So these terms belong to the well between two barriers. We can consider these terms as the resistances of the wells on the grounds of the following facts:

In the reservoirs, the electrons move randomly. When they pass through the first barrier, they connect with each other, just like a line. But the condition in the well is different. If the system is not coherent, when electrons arrive at the boundary of the well, they turn disconnected. So the well serves as another reservoir. And these two barriers are independent. So the total resistance is the sum of two barriers' resistances. If the system is coherent, when the electrons reach the boundary of the well, they do not turn disconnected. So the well is just like a conductor between two barriers and it will have its own resistance. The system amounts to three conductors being connected in series.

And we notice that there are some differences between the wells' resistances. The 10th term in the expression (23), $\Delta \left[\dfrac{2\sqrt{R_1R_3}\cos(2\theta_1 + 2\theta_2)}{T_1T_2T_3T_4T_5} \right]$, is

different from the 6th term, $\Delta\left[\frac{2\sqrt{R_1R_2}\cos(2\theta_1)}{T_1T_2T_3T_4T_5}\right]$. $\Delta\left[\frac{2\sqrt{R_1R_3}\cos(2\theta_1+2\theta_2)}{T_1T_2T_3T_4T_5}\right]$ means that the first and second wells are just like a single well between the first and third barriers. In the multi-barrier structure, the barriers and wells are connected in series. So it is clear that the expression (26) follows Ohm's law. But in the expression (23), besides the single barrier's resistance (from the 1st to 5th terms in expression (23)) and the single well's resistance (from the 6th to 9th terms), there are many other terms (from the 10th to the last terms). If the expression (23) is still considered to follow the Ohm's law, it accounts to several other conductors being added to the structure. And the more number of potential wells the structure has, the more additional conductors are added.

5. Conclusions

We have derived the resistance formula for an N-barrier structure subject to neglecting the higher-order terms in the R_n expression. The above discussion indicates that because the resistance formula is derived based on the scattering theory, the barrier's resistance will change with the number of scattering centres (barriers). In the phase coherent transmission, the potential well is like a conductor that can make a contribution to the overall resistance. And when the transport is coherent, it accounts to some additional conductors being added to the multi-barrier structure. And the more number of potential wells the structure has, the more additional conductors are added.

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