Vol 16 No 2, February 2007
1009-1963/2007/16(02)/0463-05
Chinese Physics
© 2007 Chin. Phys. Soc.
and IOP Publishing Ltd

Chaos synchronization in injection-locked semiconductor lasers with optical feedback

Liu Yu-Jin(刘玉金)[a)b)†], Zhang Sheng-Hai(张胜海)[a)], and Qian Xing-Zhong(钱兴中)[a)]

[a)] *Department of Mathematics and Physics, Information and Engineering University, Zhengzhou* 450001, *China*

[b)] *Wanfang Institute of Science and Technology, Henan Polytechnic University, Jiaozuo* 454000, *China*

(Received 25 October 2005; revised manuscript received 1 September 2006)

Based on the rate equations, we have investigated three types of chaos synchronizations in injection-locked semiconductor lasers with optical feedback. Numerical simulation shows that the synchronization can be realized by the symmetric or asymmetric laser systems. Also, the influence of parameter mismatches on chaos synchronization is investigated, and the results imply that these two lasers can achieve good synchronization, with smaller tolerance of parameter mismatch existing.

Keywords: chaos synchronization, semiconductor laser, optical feedback, injection-locked
PACC: 4265T, 0545

1. Introduction

Chaos synchronization is of great interest because of its potential applications in secure communication.[1−3] It is used for investigating the chaos synchronization[4−7] and the chaotic optical communication[8−10] in semiconductor lasers. In the proposed configurations of chaos synchronization, two similar nonlinear systems that show chaotic dynamics are included, in which one is the transmitter system and the other is the receiver system. The transmitter must be a chaotic system, but the receiver itself may or may not be a chaotic system. When the chaotic output from the transmitter system is sent into the receiver, the two systems can realize synchronization with each other. Of course, all the parameters in these two laser systems must be identical or have little tolerance. The semiconductor laser is classified as class-B laser, it can exhibit excellent nonlinear dynamics including chaos by increasing the degree of freedom. Optical injection,[11,12] optical feedback,[13,14,15] and optoelectronic feedback[16,17] have been used to serve the purpose.

We are interested in studying the chaos synchronization of a hybrid system that is subjected to more than one perturbation.[18,19] In this paper we investigate chaos synchronization in injection-locked semiconductor lasers with optical feedback.[4,5,20−22] A semiconductor laser with an external-optical feedback is injection-locked by another semiconductor laser and used as a chaotic laser source. The other injection-locked semiconductor laser that has or has not an external-optical feedback loop is used as a receiver laser. We numerically examine unidirectional and bidirectional coupling chaos synchronizations of those configurations.

†E-mail: Liuyujin8@tom.com

2. The scheme for synchronization of semiconductor lasers

Figure 1 shows all of our schematic models of chaos synchronization systems. TL and RL denote the transmitter and the receiver lasers, and they are injected by external semiconductor lasers $\mathrm{IL_T}$ and $\mathrm{IL_R}$, respectively. In this paper, all of the semiconductor lasers have the same device parameter values.

To facilitate the investigation, we assume that constants A and D in the rate equations and also in Fig.1 can be set to zero or one. If there is external optical feedback loop in the receiver system we set constant A to one, otherwise it is set to zero; and if it is unidirectional coupling chaos synchronization we set constant D to zero, otherwise it is set to one.

The systems are described by the following set of rate equations of the complex fields $E_{\mathrm{T,R}}(t)$ and the carrier densities $n_{\mathrm{T,R}}(t)$.[18,23] The subscripts T and R denote the transmitter and receiver lasers, respectively. For the transmitter laser, we have

$$\frac{\mathrm{d}E_{\mathrm{T}}(t)}{\mathrm{d}t} = \frac{1}{2}(1+\mathrm{i}\alpha)g_n[n_{\mathrm{T}}(t)-n_{\mathrm{th}}]E_{\mathrm{T}}(t) + \frac{k_{\mathrm{inj}}}{\tau_{\mathrm{in}}}E_{\mathrm{IL_T}}(t)\exp(\mathrm{i}2\pi\Delta f_{\mathrm{T}}t) + \frac{k_{\mathrm{T}}}{\tau_{\mathrm{in}}}E_{\mathrm{T}}(t-\tau)\exp(-\mathrm{i}2\pi f\tau) + D\frac{k_{\mathrm{RT}}}{\tau_{\mathrm{in}}}E_{\mathrm{T}}(t-\tau_c)\exp[-\mathrm{i}2\pi(f\tau_c+\Delta f_{\mathrm{RT}}t)], \quad (1)$$

$$\frac{\mathrm{d}n_{\mathrm{T}}(t)}{\mathrm{d}t} = J_{\mathrm{T}} - \frac{n_{\mathrm{T}}(t)}{\tau_s} - \left\{g_n[n_{\mathrm{T}}(t)-n_{\mathrm{th}}] + \frac{1}{\tau_{\mathrm{P}}}\right\}|E_{\mathrm{T}}(t)|^2, \quad (2)$$

and for the receiver laser, we have

$$\frac{\mathrm{d}E_{\mathrm{R}}(t)}{\mathrm{d}t} = \frac{1}{2}(1+\mathrm{i}\alpha)g_n[n_{\mathrm{R}}(t)-n_{\mathrm{th}}]E_{\mathrm{R}}(t) + \frac{k_{\mathrm{inj}}}{\tau_{\mathrm{in}}}E_{\mathrm{IL_R}}(t)\exp(\mathrm{i}2\pi\Delta f_{\mathrm{R}}t) + A\frac{k_{\mathrm{R}}}{\tau_{\mathrm{in}}}E_{\mathrm{R}}(t-\tau)\exp(-\mathrm{i}2\pi f\tau) + \frac{k_{\mathrm{TR}}}{\tau_{\mathrm{in}}}E_{\mathrm{R}}(t-\tau_c)\exp[-\mathrm{i}2\pi(f\tau_c+\Delta f_{\mathrm{TR}}t)], \quad (3)$$

$$\frac{\mathrm{d}n_{\mathrm{R}}(t)}{\mathrm{d}t} = J_{\mathrm{R}} - \frac{n_{\mathrm{R}}(t)}{\tau_s} - \left\{g_n[n_{\mathrm{R}}(t)-n_{\mathrm{th}}] + \frac{1}{\tau_{\mathrm{P}}}\right\}|E_{\mathrm{R}}(t)|^2, \quad (4)$$

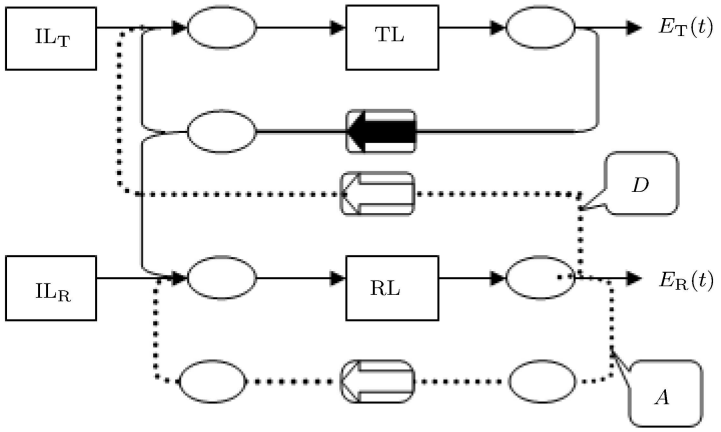


Fig.1. Schematic diagrams of three types of chaos synchronization systems.

In these equations, τ_c is the transmission time of light between the transmitter and the receiver, but τ_c is set to zero for simplifying the numerical simulations; $E_{\mathrm{IL_T}}(t)$ and $E_{\mathrm{IL_R}}(t)$ are the complex fields of the external injection lasers for the transmitter and the receiver lasers, respectively; Δf_{TR} and Δf_{RT} are the detuning between the transmitter and the receiver lasers, but they are also set to be zero for a complete chaos synchronization . Other parameters for the semiconductor laser can be found in Table 1.[18,23]

Table 1. Parameter values for semiconductor lasers used in our calculations.

Symbol	Parameter	Value
α	Linewidth-enhancement factor	3.0
g_n	Gain coefficient	8.4×10^{-13} $\mathrm{m^3s^{-1}}$
n_{th}	Threshold carrier density	2.02×10^{24} $\mathrm{m^{-3}}$
J_{th}	Threshold current density	9.89×10^{32} $\mathrm{m^{-2}s^{-1}}$
$J_{\mathrm{T,R}}$	Bias injection current	1.3 J_{th}
f	Laser oscillation frequency	384 THz
τ_{in}	Round-trip time of light in the laser cavity	8.0 ps
τ_s	Carrier lifetime	2.04 ns
τ_{p}	Photon lifetime	1.93 ps
τ	Delay of light in the feedback loop of the laser	6.0 ns
$\Delta f_{\mathrm{T,R}}$	Detuning between the injection and the injected lasers in the transmitter (receiver) system	−4 GHz
k_{inj}	Injection coefficient of each system	Variable
$k_{\mathrm{T,R}}$	Feedback coefficient	Variable
$k_{\mathrm{TR,RT}}$	Optical injection rate between the transmitter and the receiver laser	Variable

3. Numerical results

3.1. Bidirectional coupling synchronization of symmetric system

In this simulation, constants A and D are both set to one. Figure 2(a) illustrates the time series of chaotic oscillations. To facilitate the comparison, the figure shows two traces, in which the upper trace is for the transmitter laser and the lower trace is for the receiver lasers. Obviously, the time series of transmitter laser is the same as that of receiver laser, which implies their good synchronization. Figure 2(b) shows the correlation between these two laser outputs, and we can find that all the outputs are in a diagonal line, which implies the excellent correlation between them and verifies their synchronization. According to the formula of correlation coefficient ρ,[24] we have

$$\rho = \frac{\langle [X(t) - \langle X(t)\rangle][Y(t) - \langle Y(t)\rangle]\rangle}{\langle |X(t) - \langle X(t)\rangle|^2\rangle^{1/2}\langle |Y(t) - \langle Y(t)\rangle|^2\rangle^{1/2}}\,, \quad (5)$$

where $X(t)$ and $Y(t)$, which are related to $E_{\mathrm{T}}(t)$ and $E_{\mathrm{R}}(t)$ in Eqs.(1)–(4), are the outputs of the transmitter and the receiver, respectively, and $\langle\cdot\rangle$ denotes the time average, the correlation coefficient is 1.00, and the two lasers show good bidirectional coupling chaos synchronization.

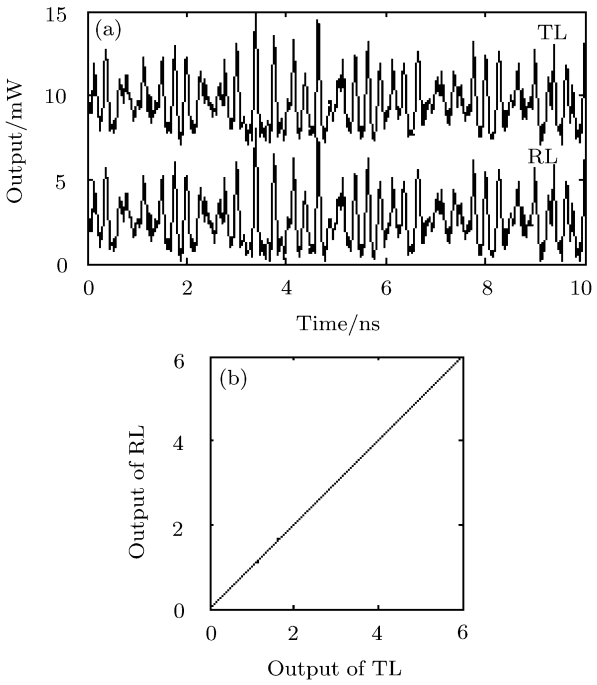


Fig.2. The bidirectional coupling chaos synchronization of symmetric system with parameters $k_{\mathrm{T,R}} = k_{\mathrm{TR,RT}} = 0.192$ and $k_{\mathrm{inj}} = 0.143$. (a) The time series of transmitter and receiver lasers, and (b) their correlation plot.

In order to study the influence of parameter mismatches on chaos synchronization, we can change one of these parameters. Figure 3 shows the variation of the output power with the linewidth enhancement factor, in which δx is closely related to $E_{\mathrm{T}}(t)$ and $E_{\mathrm{R}}(t)$ in Eqs.(1)–(4) and represents the difference in output between the transmitter laser and the receiver laser, and $\delta\alpha$ denotes the tolerance on linewidth enhancement factor α between those two lasers. From the figure we can find that these two lasers can achieve a good chaos synchronization with each other when the tolerance on parameter mismatch is smaller. Obviously, the δx is in the range of $-0.15\times10^{-3} - 0.15\times10^{-3}$ when $\delta\alpha$ is in the range of $-0.08 - 0.07$, which indicates that the power output of the transmitter laser is almost equal to that of receiver laser. Of course, we also investigate the influence of the other parameters on chaos synchronization.

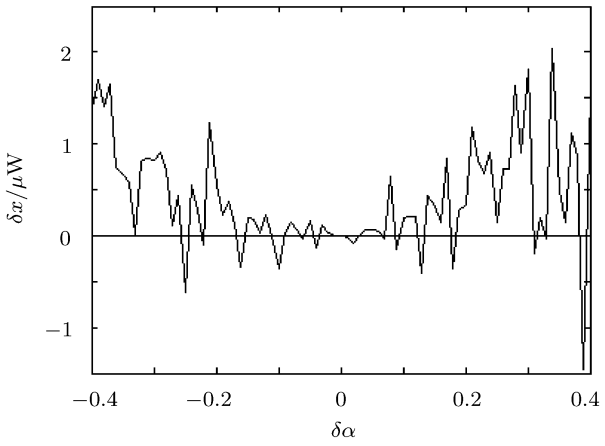


Fig.3. Variation of the output difference of two lasers with the linewidth enhancement factor.

3.2. Bidirectional coupling synchronization of asymmetric system

In the simulation of bidirectional coupling chaos synchronization of an asymmetric system, the value of constant A is set to zero and D is set to one. Figure 4 shows the results of the numerical simulations. Figure 4(a) illustrates the time series of the transmitter and receiver lasers in this configuration, which indicate that the outputs of the transmitter laser and receiver laser are almost the same. Figure 4(b) is the correlation plot of this configuration, and all of the output values do not form a diagonal line but a narrow belt in the diagonal direction, which also implies good synchronization between the transmitter and receiver lasers. Obviously, this synchronization is worse

than that of symmetric system. The correlation coefficient is calculated to be 0.9762 from Eq.(1), which indicates that these two different laser systems can also reach their good chaos synchronization by bidirectional coupling.

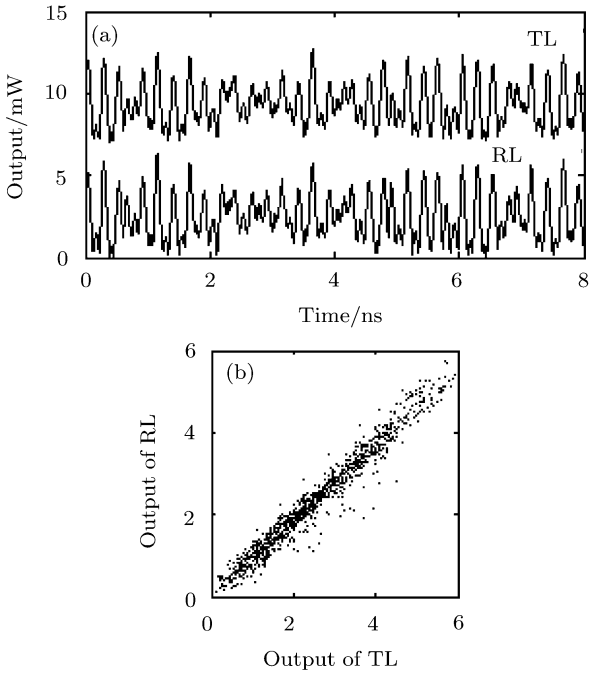


Fig.4. The bidirectional coupling synchronization of asymmetric system with $k_{\rm inj} = 0.18$ and $k_T = k_{\rm TR,RT} = 0.19$. (a) The chaotic time series of the transmitter and receiver lasers, and (b) the correlation plot.

3.3. Unidirectional coupling synchronization of symmetric system

For the unidirectional coupling, we set constants A and D to one and zero, respectively. Figure 5 illustrates the chaos synchronization of this configuration. Figure 5(a) illustrates the time series of these two lasers' chaotic oscillations: the upper trace for the power output of transmitter laser and the lower trace for that of the receiver. However, the same as Fig.4, Fig.5(a) shows good chaos synchronization but worse than Fig.2. Figure 5(b) also shows their correlation between these two laser outputs, and the coefficient is 0.9946, which also implies their good synchronization.

Similarly, we also study the influence of parameter mismatch on chaos synchronization. Figure 6 shows the variation of the output power with the tolerance on linewidth enhancement factor. When $\delta\alpha$ is in the range of $-0.01 - 0.02$, δx is in the range of $-0.2 - 0.2$ which is much smaller than other ranges of $\delta\alpha$, and these two lasers can still achieve synchronization. But comparing with Fig.2, we find that, to achieve a good synchronization, the allowed range of parameter mismatches for this configuration is smaller than that for the configuration of Fig.2(a). Furthermore, the value of δx in Fig.6 is much larger than that in Fig.2 in the whole parameter range. In other words, the effectiveness of bidirectional coupling synchronization of symmetric system is much better than the ones of the unidirectional coupling synchronization of symmetric system and the bidirectional coupling synchronization of asymmetric system.

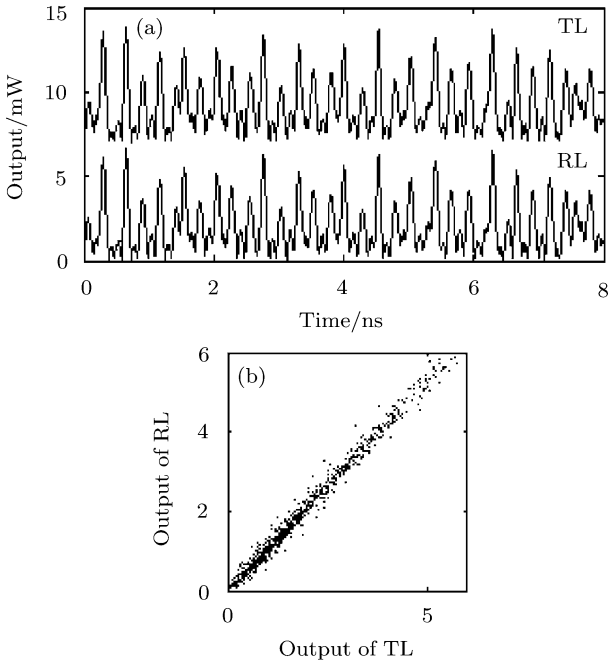


Fig.5. The unidirectional coupling synchronization of symmetric system with $k_{\rm inj} = 0.16$ and $k_{\rm T,R} = k_{\rm TR} = 0.12$. (a) The time series, and (b) their correlation plot.

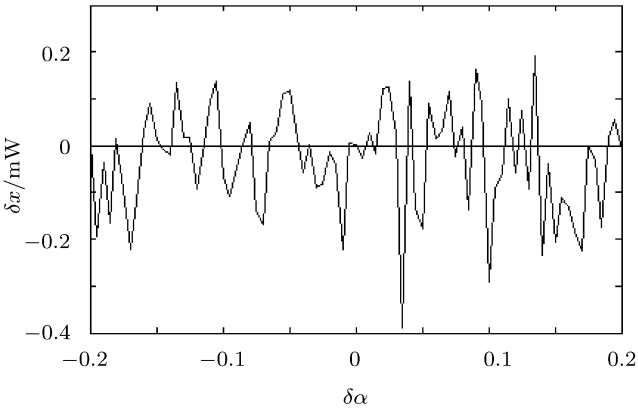


Fig.6. Variation of the output power with the linewidth enhancement factor.

4. Conclusion

Based on the rate equations, we have numerically investigated three types of chaos synchronizations in injection-locked semiconductor lasers with optical feedback in the case of parameter matches. We find that the bidirectional coupling in a symmetric system has the best effectiveness, and it can achieve a complete chaos synchronization. Furthermore, the bidirectional coupling in an asymmetric semiconductor laser system can reach a good chaos synchronization, and the unidirectional coupling in the symmetric system can also achieve a chaos synchronization, but this scheme is worse than the others. However, we hope that our investigations are beneficial to the applications of semiconductor in secure communications.

References

[1] VanWiggeren G D and Roy R 1998 *Science* **279** 1198
[2] Goedgebuer J P, Larger L and Porte H 1998 *Phys. Rev. Lett.* **80** 2249
[3] Zhang S H and Shen K 2004 *Chin. Phys.* **13** 1215
[4] Murakami A and Ohtsubo J 2002 *Phys. Rev.* A **65** 033826
[5] Murakami A 2003 *IEEE J. Quantum Electron.* **39** 438
[6] Tang S and Liu J M 2003 *IEEE J. Quantum Electron.* **39** 708
[7] Liu Y, Takiguchi Y, Davis P, Aida T and Saito S 2002 *Appl. Phys. Lett.* **80** 4306
[8] Fischer I, Liu Y and Davis P 2000 *Phys. Rev.* A **62** 011801(1-4)
[9] Kusumoto K and Ohtsubo J 2002 *Opt. Lett.* **27** 989
[10] Ohtsubo J 2002 *IEEE J. Quantum Electron.* **38** 1141
[11] Hwang S K and Liu J M 2000 *Opt. Commun.* **183** 195
[12] Simpson T B, Liu J M, Gavrielides A, Kovanis V and Alsing P M 1994 *Appl. Phys. Lett.* **64** 3539
[13] Mukai T and Otsuka K 1985 *Phys. Rev. Lett.* **55** 1711
[14] Mork J, Tromborg B and Mark J 1992 *IEEE J. Quantum Electron.* **28** 93
[15] Gu C M and Shen K 1998 *Acta Phys. Sin.* **47** 732
[16] Tang S and Liu J M 2001 *IEEE J. Quantum Electron.* **37** 329
[17] Lin F Y and Liu J M 2002 *Appl. Phys. Lett.* **81** 3128
[18] Takiguchi Y, Ohyagi K and Ohtsubo J 2003 *Opt. Lett.* **28** 319
[19] Lin F Y and Liu J M 2003 *Opt. Commun.* **28** 319
[20] Locquet A, Rogister F, Sciamanna M, Megret P and Blondel M 2001 *Phys. Rev.* E **64** 045203(R)
[21] Koryukin I V and Mandel P 2002 *Phys. Rev.* E **65** 026201
[22] Saboureau P, Foing J P and Schanne P 1997 *IEEE J. Quantum Electron.* **33** 1582
[23] Uchida A, Shibasaki N, Nogawa S and Yoshimori S 2004 *Phys. Rev.* E **69** 056201
[24] Tang S and Liu J M 2001 *Opt. Lett.* **26** 596