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Generation and propagation of subpicosecond pulse train*

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Higher-order nonlinear Schrödinger equation with the Hirota constraint conditions is considered, and an analytic solution, which can describe the modulational instability process, is presented. Based on the solution, a new pulse train without continuous wave (CW) background is generated in quadratures and the propagation of the pulse train is discussed in detail by simulating numerically. The results show that, unlike the propagation of the picosecond pulse train, under the effects of the higher-order terms, the pulse train cannot propagate along the fibre when the energy is very high; however, for some medium energy the pulse train can stably propagate. We also investigate the stability of the pulse train against violation of the Hirota conditions, and the results show that the pulse train can still propagate stably when the Hirota conditions are broken.

Keywords: modulational instability, subpicosecond pulse train, Hirota equation
PACC: 4265, 4265S

Since the propagation of optical pulses in dispersive fibres in the form of bright and dark solitons under certain conditions was first described in a seminal work by Hasegawa and Tappert,[1] optical solitons have been regarded as good information carriers and now have become particularly attractive for applications in long-distance and high-bit-rate optical communications due to their remarkable robustness against various perturbations. In the picosecond regime, the propagation of optical pulses in monomode optical fibres is well governed by the nonlinear Schrödinger (NLS) equation, in which only group velocity dispersion (GVD) and self-phase modulation (SPM) are considered. However, in the range of subpicosecond or femtosecond pulses, higher-order effects, such as the third-order dispersion (TOD), self-steepening and self-frequency shift due to the intra-pulse stimulated Raman scattering (SRS), become important.[2,3] Considering those effects, Kodama and Hasegawa[4] proposed a higher-order nonlinear Schrödinger (HNLS) equation in the form

$$q_z = \mathrm{i}(\alpha_1 q_{tt} + \alpha_2 |q|^2 q) + \alpha_3 q_{ttt} + \alpha_4 (|q|^2 q)_t + \alpha_5 q (|q|^2)_t, \tag{1}$$

where z and t are the propagation distance and retarded time, $q(z,t)$ is the slowly varying envelope of the electric field, and the parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4$, and α_5 are real constants related to the GVD, the SPM, the TOD, the self-steepening and the delayed nonlinear response effect, respectively. For picosecond light pulses, the last three terms on the right-hand side of Eq.(1) can be omitted, and Eq.(1) reduces to the standard NLS equation, which is completely integrable and has been studied extensively. However, for subpicosecond or femtosecond light pulses, whose duration is shorter than 100 fs, the last three terms on the right-hand side of Eq.(1) are important and should be retained.

In recent years, many authors have investigated the HNLS equation (1) from different points of view, and presented some exact solutions for special parameter relations, such as N-soliton solutions,[5−14]

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the dark soliton solutions,[15−19] combined solitary wave solutions,[20,21] and soliton solutions on the continuous-wave (CW) background.[22−25] To our knowledge, however, the problem of generation, compression and transmission of soliton trains in systems described by the HNLS equation including the higher-order terms was less considered. Recently, we proposed a new approach to generation, compression and transmission of soliton trains based on the soliton solutions on the CW background for NLS equation with distributed coefficients[26] and HNLS equation with exponentially distributed control system.[27] The results show that the pulse train generated by modulational instability can propagate a long distance undistortedly along fibres with exponentially distributed parameters, and the interaction between pulses in the pulse train is greatly reduced by using a suitable control system. However the counterpart for HNLS equation with constant coefficients was not considered.

In this paper, we first present the explicit, analytic solution describing the modulational instability process for the HNLS equation (1) with Hirota constraint conditions. Based on the solution, we will study in detail the generation and propagation of pulse trains under higher-order effects, and compare the propagation behaviour in both the presence and absence of the higher-order effects. Furthermore, we also investigate the stability of the pulse train against violation of the Hirota conditions. These results will be very useful for understanding more deeply the properties of optical transmission and realizing experimentally the undistorted transmission of femtosecond pulse trains in optical fibres.

Generally, the HNLS equation (1) is not integrable. However, to solve Eq.(1), we consider the special parametric choice of $\alpha_2 = 2\mu^2\alpha_1, \alpha_4 = 6\mu^2\alpha_3$, and $\alpha_4 + \alpha_5 = 0$, which are usually called the Hirota conditions, so that Eq. (1) becomes

$$q_z = \mathrm{i}(\alpha_1 q_{tt} + \alpha_2|q|^2 q) + \alpha_3 q_{ttt} + \alpha_4|q|^2 q_t, \qquad (2)$$

which is usually called the Hirota equation. Here the parameter μ is an arbitrary real constant. By employing Darboux transformation, one can obtain the exact solution for Eq.(2) in the form[24,25]

$$q = \mathrm{e}^{\mathrm{i}\varphi_c}\left(A_c + A_s\frac{2A_c\cos\varphi - A_s\cosh\theta + \mathrm{i}M\sinh\theta}{2A_c\cosh\theta - A_s\cos\varphi}\right), \qquad (3)$$

where

$$\theta = \mu^2 A_s M(\alpha_1 + 3\omega\alpha_3)z - t_0,$$
$$\varphi = \mu M t - \mu M[2\omega\alpha_1 + (3\omega^2 - 2\mu^2A_c^2 - \mu^2A_s^2)\alpha_3]z - \varphi_0,$$
$$\varphi_c = \omega t + \kappa z,$$

with $M = \sqrt{2A_c^2 - A_s^2}$, and $\kappa = (-\omega^2 + 2\mu A_c^2)\alpha_1 + \omega(-\omega^2 + 6\mu^2A_c^2)\alpha_3$. Here A_s and A_c with $4A_c^2 > A_s^2$, and ω, t_0 and φ_0 are arbitrary real constants. Especially, when A_s vanishes, the solution (3) reduces to the CW solution of the form $q_c = A_c\exp(\mathrm{i}\varphi_c)$ for Eq.(2). A simple analysis reveals that the solution (3) is periodic with period $L = 2\pi/(\mu M)$ in the temporal coordinate and aperiodic in the space variable. Therefore, it can be considered as describing the modulation instability process.[28] In order to understand the modulation instability process, we linearize the initial value of solution (3), which can be approximated by

$$q(0,t) \approx [\rho + \varepsilon\chi\cos(\mu M t - \varphi_0)]\mathrm{e}^{\mathrm{i}\omega t}, \qquad (4)$$

where $\rho = (2A_c^2 - A_s^2 - \mathrm{i}A_sM)/(2A_c)$ with $|\rho|^2 = A_c^2, \chi = A_sM(M - \mathrm{i}A_s)/(2A_c^2)$, and $\varepsilon = \exp(-t_0)$ is a small quantity for $t_0 > 0$ and the second-order terms in ε are neglected. By numerical simulations, we find that the solution of the initial value problem associated with Eq.(2) with initial condition (4) can be well described by the exact solution (3). This means that a small periodic perturbation of the CW solution may lead to the onset of instability. Based on this approximation, we can conveniently investigate the modulation instability process. Indeed, in practical applications, the modulation instability can be used to produce a train of optical solitons of the form (3).[25] However this soliton train cannot be directly applied to the description of established transmission

regimes in the optical fibre because of the presence of the background wave

$$q_B = \mathrm{e}^{\mathrm{i}\varphi_c}\left(A_c - A_s\frac{2A_c + A_s\cosh\theta - \mathrm{i}M\sinh\theta}{2A_c\cosh\theta + A_s}\right), \quad (5)$$

which is dependent on z only. To investigate a realistic situation, we subtract this background wave from formula (3), thus we can obtain a new train of pulses with zero background as follows:

$$q_T = \mathrm{e}^{\mathrm{i}\varphi_c}\frac{A_sM(M\cosh\theta + \mathrm{i}A_s\sinh\theta)(1+\cos\varphi)}{(2A_c\cosh\theta + A_s)(2A_c\cosh\theta - A_s\cos\varphi)}. \quad (6)$$

Note that the expression (6) is not an exact solution, which may be regarded as generating an array of raised-cosine pulses.[27] Our intention is to use the expression (6) as an initial approximation, and to investigate the propagations of the raised-cosine pulse (6) by direct simulations.

From the expression (6) we can directly obtain the full-width at half-maximum (FWHM) and the energy of each pulse in the pulse train (6) in the form

$$\tau(z) = \frac{2}{\mu M}\cos^{-1}\left[\frac{2(\sqrt{2}-1)A_c\cosh\theta + A_s}{2A_c\cosh\theta + (\sqrt{2}-1)A_s}\right], \quad (7)$$

and

$$E(z) = \int_0^L |q_T|^2\mathrm{d}t = \frac{2\pi M}{\mu}I(z), \quad (8)$$

where

$$I(z) = \frac{2A_c\cosh\theta - A_s}{2A_c\cosh\theta + A_s} - \frac{2A_c\cosh\theta - 2A_s}{\sqrt{4A_c^2\cosh^2\theta - A_s^2}}.$$

Figures 1(a), (b) and (c) present the intensity distribution in the pulse train (6) at different distances $z = 0, z = 3.5, z = 6.0$, and $z = 7.5$, and the evolutions of the FWHM given by the expression (7) and the energy given by the expression (8) for the different parameter values $\alpha_3 = 0, 0.02, 0.04$ and 0.06 respectively. From Fig.1, one can see that each pulse in the pulse train (6) narrows in width in succession (see Figs.1(a) and (b)), and the energy increases (see Fig.1(c)) until the propagation distance reaches the value $z = z_1$, which is given by

$$z_1 = t_0/[\mu^2 A_s M(\alpha_1 + 3\omega\alpha_3)].$$

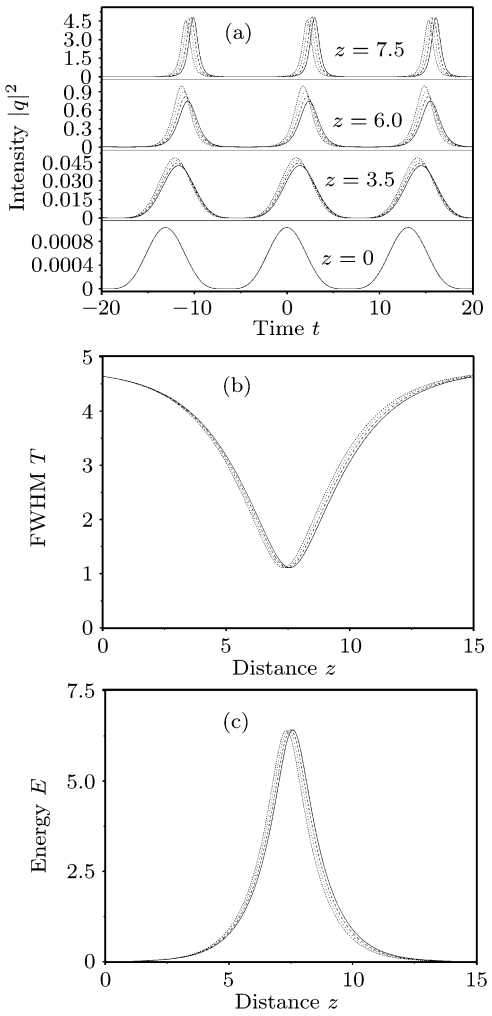


Fig.1. (a) The intensity distributions in the pulse train (6) at different values of distance $z = 0$, 3.5, 6.0, 7.5. (b) The evolution of the FWHM pulse width given by the expression (7), and (c) the energy given by the expression (8) for the different parameter values of α_3. Here, solid, dashed, dotted and short dashed curves correspond to $\alpha_3 = 0$, 0.02, 0.04 and 0.06, respectively. Other parameters are $A_s = 1.1$, $A_c = 0.6$, $\omega = 0.2$, $t_0 = 4$, $\varphi_0 = 0$, $\mu = 1$, and $\alpha_1 = 1$.

Figure 2 shows the dependence of the energy maximum point z_1 on the parameter α_3. Comparing the corresponding results with those obtained in Ref.[26] for the model without the higher-order terms ($\alpha_3 = 0$), we conclude that, under the action of the higher-order terms, the centre of each pulse gradually shifts from the initial position, and the energy maximum point z_1 decreases with the parameter α_3 increasing. Especially, when $\omega = 0$ the value z_1 is independent of α_3.

In the following, we will take the pulse train (6) at the different values of distance z (which correspond to the different energy values) as the initial condition to trace their propagation by solving numerically Eq.(2).

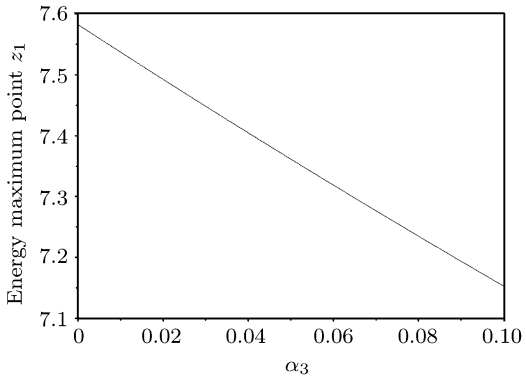


Fig.2. The energy maximum point z_1 versus the parameter α_3. Here the parameters are the same as those in Fig.1.

First, we consider the case without the higher-order effects, i.e. $\alpha_3 = 0$, which implies that $\alpha_4 = \alpha_5 = 0$ according to the Hirota conditions. The numerical simulations reveal that the pulse train generated by Eq.(6) can propagate stably along the fibre when the energy of each pulse in the pulse train is larger than some threshold value. Figure 3 presents the contour plots with over 1000 normalized propagation units, of the pulse train (6) at different values of distance $z = 6.3$, 6.4, 7.0, and 7.6 (where we take $\omega = 0$ for convenience), respectively. From Fig.3 we can see that the pulse train generated initially by Eq.(6) can stably propagate in the fibre when the energy of each pulse is larger than 2.8608 (corresponding to $z = 6.4$), and the larger the energy, the more stable the propagation of the pulse trains (6); especially, the propagation of the pulse train with the maximum energy is the best of all (see Fig.3(d)).

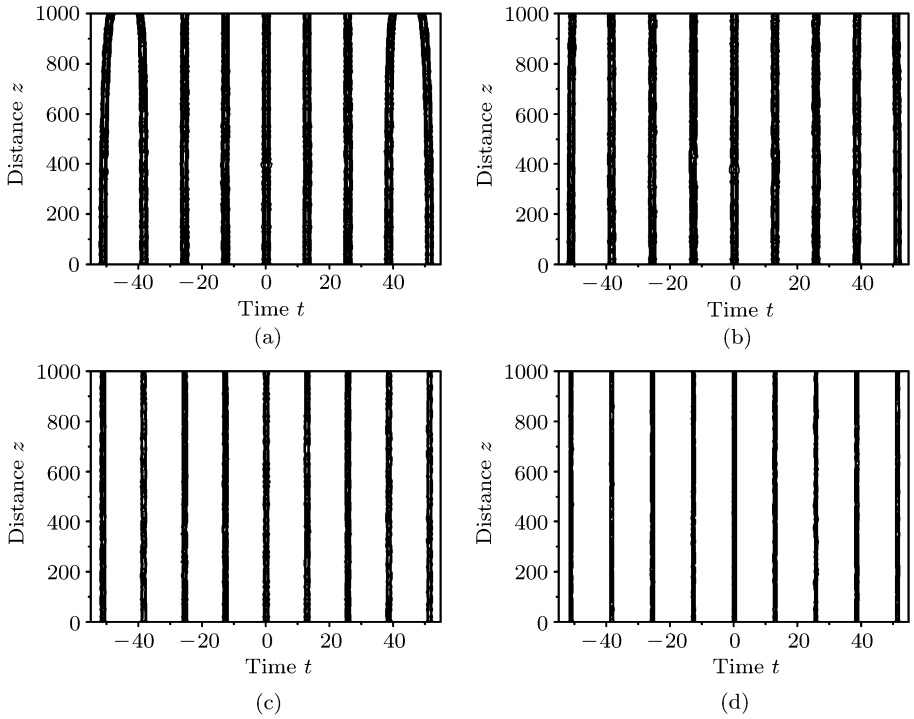


Fig.3. The contour plots showing the evolutions of four different input pulse trains of the form (6) without the higher-order effects corresponding to (a) $z = 6.3$; (b) $z = 6.4$; (c) $z = 7.0$; (d) $z = 7.6$ in Eq.(6), respectively. Here the other parameters are the same as those in Fig.1 except for $\omega = 0$.

Next we investigate the propagation of the pulse train (6) in the presence of the higher-order effects, in which the parameter α_3 is not equal to zero. We trace the propagation of input pulse train (6) with the different energies, and the results show that the characteristic of the propagation of the pulse train (6) in the presence of the higher-order effects is different from that in the absence of the higher-order effects, i.e. the pulse train (6) cannot propagate a long distance in the fibre when the energy is very high. This results from the rapid increase of each pulse velocity in the presence of the higher-order effects as shown in Fig.4. Also, as seen from Fig.4(d), the pulse train near the maximum energy cannot propagate. However, for some medium energy the pulse train can stably propagate a long distance as shown in Fig.4(b). Comparing the corresponding results with those obtained in Ref.[27] for the model with exponentially distributed

control system, we conclude that the interaction between pulses in the pulse train is greatly reduced by using a suitable control system. Also, Fig.5 presents the contour plots with over 200 normalized propagation units, of the pulse train (6) at different values of distance $z = 5.9$, 6.0, 6.5, 6.8, 7.0, and 7.1. From Fig.5, we can see that over a moderate distance, such as 200 normalized distance units, the pulse train with the moderate energy can stably propagate along the fibre as shown in Figs. 5(b)-(f).

It should be noted that the above results are obtained under the Hirota conditions $\alpha_2 = 2\mu^2\alpha_1, \alpha_4 = 6\mu^2\alpha_3$, and $\alpha_4 + \alpha_5 = 0$, which impose strict relations on the TOD, the self-steepening and the delayed nonlinear response effect. However, these conditions may not be rigorously satisfied in real waveguides, therefore making a study of the deviation from Hirota conditions is necessary. Here, we consider four situations for the pulse train (6) at $z = 6.5$ as shown in Fig.6. The results show that the pulse train can still propagate stably over 200 normalized propagation units when the Hirota conditions are broken.

In summary, we have considered the Hirota equation and presented the explicit, analytic solution describing the modulational instability process, which can be used in practice to produce a train of ultrashort optical soliton pulses. Based on the solution, a new pulse train without the CW component has been generated in quadratures and the propagation of the pulse train has been discussed in detail by numerical simulation. The results show that, unlike the propagation of the picosecond pulse train, under the effects of the higher-order terms, the pulse train cannot propagate along the fibre when the energy is very high; however, for some medium energy the pulse train can stably propagate. Furthermore, we also have investigated the stability of the pulse train against the violation of the Hirota conditions, and the results show that the pulse train can still propagate stably over 200 normalized propagation units when the Hirota conditions are broken.

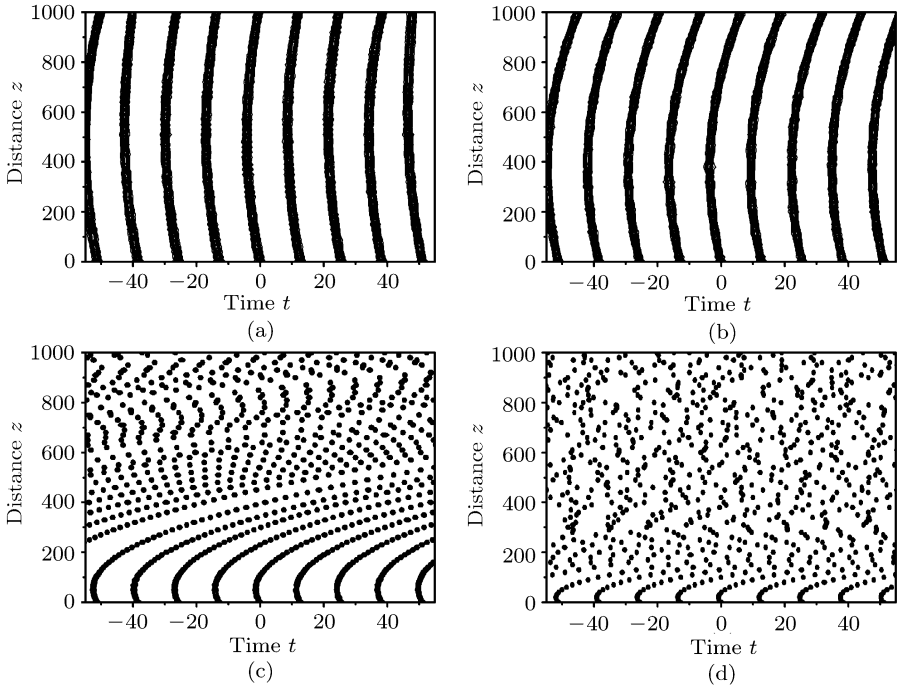


Fig.4. The contour plots showing the evolutions of four different input pulse trains of the form (6) in the presence of the higher-order effects corresponding to (a) $z = 6.3$; (b) $z = 6.4$; (c) $z = 7.0$; (d) $z = 7.6$ in Eq.(6), respectively. Here the parameters are $A_s = 1.1$, $A_c = 0.6$, $\omega = 0$, $t_0 = 4$, $\varphi_0 = 0$, $\mu = 1$, $\alpha_1 = 1$, $\alpha_3 = 0.01$.

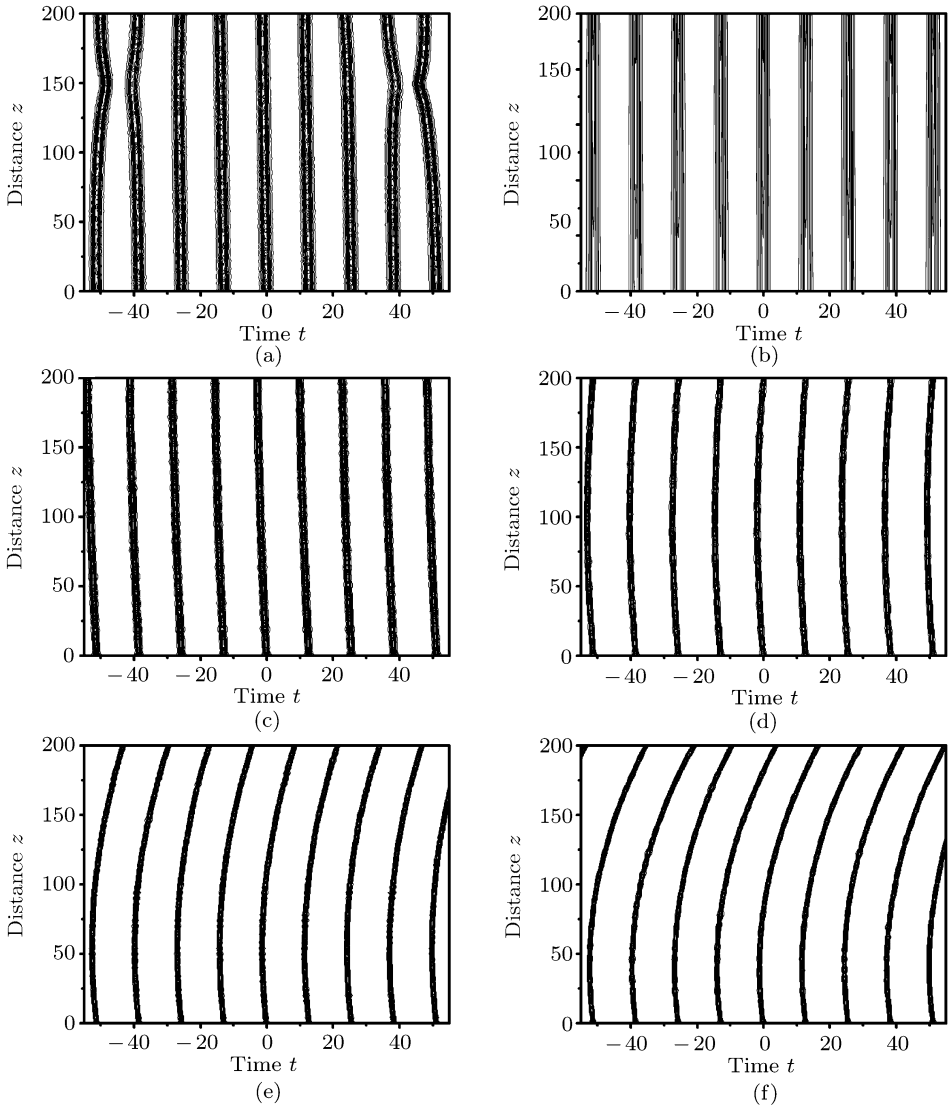


Fig.5. The contour plots showing the evolutions with over 200 normalized distance units, of six different input pulse trains of the form (6) in the presence of the higher-order effects corresponding to (a) $z = 5.9$; (b) $z = 6.0$; (c) $z = 6.5$; (d) $z = 6.8$; (e) $z = 7.0$; (f) $z = 7.1$ in Eq.(6), respectively. Here the parameters are the same as those in Fig.4.

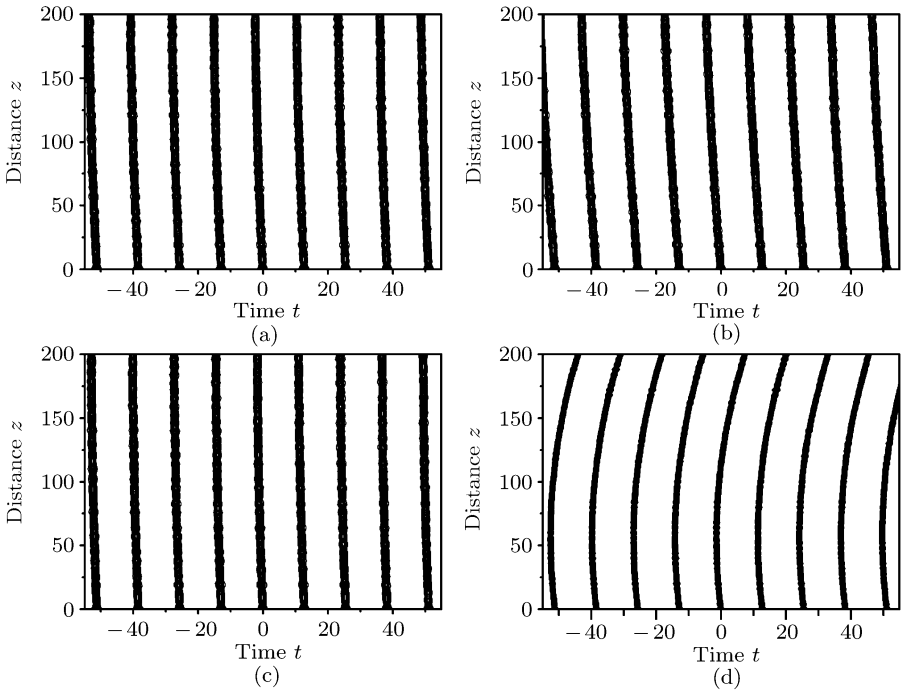


Fig.6. The contour plots of the input pulse train (6) in the presence of the higher-order effects corresponding to $z = 6.5$ in Eq.(6), when the Hirota conditions are broken by setting (a) $\alpha_1 = 1$, $\alpha_2 = 2$, $\alpha_3 = 0.011$, $\alpha_4 = 0.06$, $\alpha_5 = -0.06$; (b) $\alpha_1 = 1$, $\alpha_2 = 2$, $\alpha_3 = 0.01$, $\alpha_4 = 0.066$, $\alpha_5 = -0.06$; (c) $\alpha_1 = 1$, $\alpha_2 = 2$, $\alpha_3 = 0.01$, $\alpha_4 = 0.06$, $\alpha_5 = -0.066$; and (d) $\alpha_1 = 1.1$, $\alpha_2 = 1.8$, $\alpha_3 = 0.01$, $\alpha_4 = 0.06$, $\alpha_5 = -0.066$. Other parameters are the same as those in Fig.4.

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