Vol 16 No 2, February 2007
1009-1963/2007/16(02)/0441-08

Chinese Physics

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# Phase-dependent inversionless gain in a four-level atomic system with a closed interaction loop*

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(Received 14 March 2006; revised manuscript received 9 August 2006)

A four-level atomic system with a closed interaction loop connected by two coherent driving fields and a microwave field is investigated. The results show that inversionless gain can be achieved on a higher frequency transition outside the closed interaction loop, and the gain behaviour can be modulated by the phase of the closed loop as well as the amplitude of the microwave field. The phase sensitivity property in such a scheme is similar to that in an analogous configuration with spontaneously generated coherence, but it is beyond the rigorous condition of near-degenerate levels with non-orthogonal dipole moments. Therefore this scheme is much more convenient in experimental realization.

**Keywords:** inversionless gain, microwave field, closed interaction loop, phase
**PACC:** 4250, 4210K, 4210M

## 1. Introduction

In the last decade, much attention has been paid to the studies of different kinds of coherence phenomena in atomic systems resulting from strong coherent fields.[1−14] One of these effects that has attracted more attention is lasing without inversion (LWI),[1−10] as it offers a new way to generate the laser oscillation in a high-frequency range where the fast spontaneous relaxation makes it difficult to achieve the population inversion. Up to now, various schemes for the realization of LWI have been proposed,[1−6] and the experimental observation of LWI has also been reported by several groups.[7−10] In the LWI an external driving field is necessary to create the atomic coherence that relaxes the population inversion condition for laser oscillation. Obviously, the LWI is more attractive in the frequency up-conversion regime, i.e. the generated laser field can have a frequency higher than that of the driving field.

Recently, another kind of coherence, spontaneously generated coherence (SGC), has aroused much interest. The SGC occurs in the case: a single excited state decaying to a near-degenerate lower doublet[15,16] or a near-degenerate excited doublet decaying to a common ground state.[17,18] In order to generate the SGC, the energy separation of the doublet must be small enough compared with the natural line width, and the two involved dipole matrix moments must be nonorthogonal.[15,16] The SGC can give rise to many remarkable phenomena, one of which is phase-dependent phenomenon during spontaneous emission, absorption and dispersion.[16,19−23] In Ref.[21], Wu *et al* proposed a four-level scheme with a near-degenerate intermediate doublet for achieving the phase control of the inversionless gain. The prominent feature of the model is the existence of the spontaneously generated coherence between the near-degenerate doublet. The SGC makes the system sensitive to the relative phase between the two intense fields, and makes it possible to realize the phase control of the inversionless gain. But, the existence of the SGC depends on the stringent condition that the dipole matrix moments for the two close states decaying to a common final state must be nonorthogonal. This condition is really difficult to meet in practical experiments. Notice that the phase-dependent

*Project supported by the Natural Science Foundation of Guangdong Province (Grant No 05301018), the Research and Development Fund of Shenzhen University, China (Grant No 200549), and the National Natural Science Foundation of China (Grant Nos 10334010 and 10404009).
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phenomena also can be found in multilevel systems containing a closed interaction loop.[24−28] In this paper, we propose a new scheme for achieving a phase-dependent inversionless gain based on a similar four-level atom to that in Ref.[21]. Instead of the SGC, we suppose that a microwave field $\Omega_{\rm m}$ is used to couple the two intermediate levels, thereby a closed interaction loop is formed. Inversionless gain can be achieved on a higher frequency transition outside the closed interaction loop. We find that the formation of the closed interaction loop makes the system dependent on the relative phase of the applied fields that form the closed loop, which is referred to as the phase of the closed interaction loop. The inversionless gain can be controlled by the phase of the closed interaction loop; also, it can be controlled by the amplitude of the microwave field. In such a scheme, the two involved dipole moments are no longer required to be nonorthogonal, and the frequency difference between the two intermediate levels can be much larger than spontaneous decay rates. All these advantages make it much more convenient in experimental realization.

## 2. Model and equations

A four-level atomic system under consideration is shown in Fig.1. The two intermediate levels $|3\rangle$ and $|2\rangle$, together with the ground level $|1\rangle$ form a V-type sub-configuration, $\gamma_2$ and $\gamma_3$ are the spontaneous decay rates from intermediate levels $|2\rangle$ and $|3\rangle$ to the ground level $|1\rangle$, respectively. Level $|4\rangle$ is a higher level and it decays to the ground level $|1\rangle$ with a rate $\gamma_4$. The experimental system for this atomic scheme can be realized, for example, by the Rb atom with $5S_{1/2}$, $5P_{1/2}$, $5P_{3/2}$ and $7P_{3/2}$ employed as the levels $|1\rangle$, $|2\rangle$, $|3\rangle$ and $|4\rangle$, respectively. Two coherent fields with carrier frequencies (Rabi frequencies) $\omega_{\rm c1}$ and $\omega_{\rm c2}$ ($\Omega_{\rm c1}$ and $\Omega_{\rm c2}$) drive transitions $|1\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |3\rangle$, respectively. The electric dipole forbidden transition $|2\rangle \leftrightarrow |3\rangle$ is coupled by a microwave field with the frequency (Lamor frequency) $\omega_{\rm m}$ ($\Omega_{\rm m}$) through an allowed magnetic dipole transition. A weak coherent field with the carrier frequency (Rabi frequency) $\omega_{\rm p}$ ($\Omega_{\rm p}$) is used to probe the inversionless gain on the high frequency transition $|1\rangle \leftrightarrow |4\rangle$. $\Lambda$ represents the rate of incoherent pumping from level $|1\rangle$ to level $|4\rangle$.

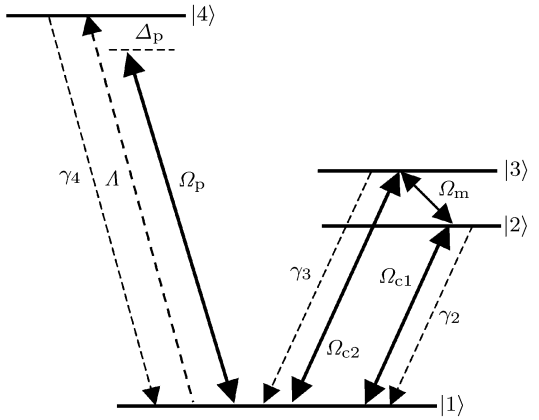



**Fig.1.** Schematic diagram of the four-level atomic system under consideration.

In the rotating-wave approximation and the electric-dipole approximation, the semi-classical Hamiltonian in an interaction picture is derived as

$$\begin{aligned} H_I = & -(\Delta_{\rm m}+\Delta_1)|1\rangle\langle 1| - \Delta_{\rm m}|2\rangle\langle 2| - (\Delta_{\rm m}+\Delta_1-\Delta_2)|3\rangle\langle 3| - (\Delta_{\rm m}+\Delta_1-\Delta_{\rm p})|4\rangle\langle 4| \\ & -[\Omega_{\rm c1}|2\rangle\langle 1|{\rm e}^{-{\rm i}\varphi_1} + \Omega_{\rm c2}|3\rangle\langle 1|{\rm e}^{-{\rm i}\varphi_2} + \Omega_{\rm m}|3\rangle\langle 2|{\rm e}^{-{\rm i}(\Delta\omega t+\varphi_{\rm m})} + \Omega_{\rm p}|4\rangle\langle 1|{\rm e}^{-{\rm i}\varphi_{\rm p}} + {\rm H.c.}], \end{aligned} \tag{1}$$

where $\Delta_1 = \omega_{21} - \omega_{\rm c1}$, $\Delta_2 = \omega_{31} - \omega_{\rm c2}$, $\Delta_{\rm m} = \omega_{32} - \omega_{\rm m}$, and $\Delta_{\rm p} = \omega_{41} - \omega_{\rm p}$ are atom-field detunings of the applied fields, respectively. $\varphi_1$, $\varphi_2$, $\varphi_{\rm m}$ and $\varphi_{\rm p}$ are the phases of the corresponding fields. $\Delta\omega = \omega_{\rm c1} + \omega_{\rm m} - \omega_{\rm c2} = \Delta_1 + \Delta_{\rm m} - \Delta_2$ is the multi-photon detuning. Obviously the Hamiltonian will be independent of time if $\Delta\omega = 0$. For simplicity, we assume that the carrier frequencies of the three fields satisfy the multi-photon resonance condition, i.e. $\Delta\omega = 0$, in all the following discussion.

The master equation of motion for the density operator can be written as

$$\frac{\partial\tilde{\rho}}{\partial t} = -\frac{\rm i}{\hbar}[H_I,\tilde{\rho}] - \sum_{j=2,3,4}\frac{\gamma_j}{2}\left(|j\rangle|j\rangle\tilde{\rho} - |1\rangle|1\rangle\tilde{\rho}_{jj}\right) - \frac{\Lambda}{2}\left(|1\rangle|1\rangle\tilde{\rho} - |4\rangle|4\rangle\tilde{\rho}_{11}\right). \tag{2}$$

With Eqs.(1) and (2) it is easy to derive the density-matrix equations of motion about $\tilde{\rho}_{ij}$. By redefining the atomic variables as $\tilde{\rho}_{41} = \rho_{41}\exp(-\mathrm{i}\varphi_\mathrm{p})$, $\tilde{\rho}_{31} = \rho_{31}\exp(-\mathrm{i}\varphi_2)$, $\tilde{\rho}_{21} = \rho_{21}\exp(-\mathrm{i}\varphi_1)$, $\tilde{\rho}_{32} = \rho_{32}\exp(-\mathrm{i}\varphi_\mathrm{m})$, $\tilde{\rho}_{42} = \rho_{42}\exp[-\mathrm{i}(\varphi_\mathrm{p}-\varphi_1)]$, $\tilde{\rho}_{43} = \rho_{43}\exp[-(\varphi_\mathrm{p}-\varphi_2)]$, and $\tilde{\rho}_{ii} = \rho_{ii}$, we obtain the equations for the redefined density matrix elements $\rho_{ij}$ from those for $\tilde{\rho}_{ij}$ as follows:

$$\dot{\rho}_{44} = -\gamma_4\rho_{44} + \Lambda\rho_{11} + \mathrm{i}\Omega_\mathrm{p}(\rho_{14}-\rho_{41}), \tag{3a}$$
$$\dot{\rho}_{33} = -\gamma_3\rho_{33} + \mathrm{i}\Omega_{\mathrm{c}2}(\rho_{13}-\rho_{31}) + \mathrm{i}\Omega_\mathrm{m}(\rho_{23}-\rho_{32}), \tag{3b}$$
$$\dot{\rho}_{22} = -\gamma_2\rho_{22} + \mathrm{i}\Omega_{\mathrm{c}1}(\rho_{12}-\rho_{21}) + \mathrm{i}\Omega_\mathrm{m}(\rho_{32}-\rho_{23}), \tag{3c}$$
$$\dot{\rho}_{43} = [-\gamma_{43} + \mathrm{i}(\Delta_2-\Delta_\mathrm{p})]\rho_{43} - \mathrm{i}\Omega_{\mathrm{c}2}\rho_{41} + \mathrm{i}\Omega_\mathrm{p}\rho_{13} - \mathrm{i}\Omega_\mathrm{m}\rho_{42}\exp(-\mathrm{i}\varphi), \tag{3d}$$
$$\dot{\rho}_{42} = [-\gamma_{42} + \mathrm{i}(\Delta_1-\Delta_\mathrm{p})]\rho_{42} - \mathrm{i}\Omega_{\mathrm{c}1}\rho_{41} + \mathrm{i}\Omega_\mathrm{p}\rho_{12} - \mathrm{i}\Omega_\mathrm{m}\exp(\mathrm{i}\varphi), \tag{3e}$$
$$\dot{\rho}_{41} = (-\gamma_{41} - \mathrm{i}\Delta_\mathrm{p})\rho_{41} - \mathrm{i}\Omega_{\mathrm{c}1}\rho_{42} - \mathrm{i}\Omega_{\mathrm{c}2}\rho_{43} + \mathrm{i}\Omega_\mathrm{p}(\rho_{11}-\rho_{44}), \tag{3f}$$
$$\dot{\rho}_{32} = [-\gamma_{32} + \mathrm{i}(\Delta_1-\Delta_2)]\rho_{32} + \mathrm{i}\Omega_\mathrm{m}(\rho_{22}-\rho_{33}) - \mathrm{i}\Omega_{\mathrm{c}1}\rho_{31}\exp(-\mathrm{i}\varphi) + \mathrm{i}\Omega_{\mathrm{c}2}\rho_{12}\exp(-\mathrm{i}\varphi), \tag{3g}$$
$$\dot{\rho}_{31} = (-\gamma_{31} - \mathrm{i}\Delta_2)\rho_{31} + \mathrm{i}\Omega_{\mathrm{c}2}(\rho_{11}-\rho_{33}) - \mathrm{i}\Omega_\mathrm{p}\rho_{34} - \mathrm{i}\Omega_{\mathrm{c}1}\rho_{32}\exp(\mathrm{i}\varphi) + \mathrm{i}\Omega_\mathrm{m}\rho_{21}\exp(\mathrm{i}\varphi), \tag{3h}$$
$$\dot{\rho}_{21} = (-\gamma_{21} - \mathrm{i}\Delta_1)\rho_{21} + \mathrm{i}\Omega_{\mathrm{c}1}(\rho_{11}-\rho_{22}) - \mathrm{i}\Omega_\mathrm{p}\rho_{24} - \mathrm{i}\Omega_{\mathrm{c}2}\rho_{23}\exp(-\mathrm{i}\varphi) + \mathrm{i}\Omega_\mathrm{m}\rho_{31}\exp(-\mathrm{i}\varphi), \tag{3i}$$
$$\rho_{11}+\rho_{22}+\rho_{33}+\rho_{44} = 1, \quad \rho_{ij} = \rho_{ji}^{*}. \tag{3j}$$

In the above equations, $\varphi = \varphi_2 - \varphi_1 - \varphi_\mathrm{m}$ is the relative phase of the three fields that form a closed interaction loop, and referred to as the phase of the closed interaction loop. $\gamma_{ij}$ are the coherence decay rates on the corresponding transition $i \to j$, in the radiative limit when there exists no dephasing collision, they are given by $\gamma_{43} = (\gamma_4+\gamma_3)/2$, $\gamma_{42} = (\gamma_4+\gamma_2)/2$, $\gamma_{41} = (\gamma_4+\Lambda)/2$, $\gamma_{32} = (\gamma_3+\gamma_2)/2$, $\gamma_{31} = (\gamma_3+\Lambda)/2$, and $\gamma_{21} = (\gamma_3+\Lambda)/2$.

As is well known, the susceptibility $\chi(\omega_\mathrm{p})$, which measures the response of the medium to the probe field, is proportional to the atomic coherence along the probe transition $\rho_{14}$ as follows:

$$\chi = \frac{N\mu_{14}^2}{2\varepsilon_0\hbar}\cdot\frac{\rho_{14}}{\Omega_\mathrm{p}}, \tag{4}$$

where $N$ is the atomic number density.

In the limit of a weak probe and steady-state condition, the solutions of Eqs.(3) for $\rho_{14}$ to the first order in the probe field and for other $\rho_{ij}$ to the zero order in the probe field are

$$\begin{aligned}\frac{\rho_{14}^{(1)}}{\Omega_\mathrm{p}} = \; & \frac{1}{a+\mathrm{i}b}\Big\{\left[\Delta_\mathrm{p}\Omega_{\mathrm{c}2} - \Omega_{\mathrm{c}1}\Omega_\mathrm{m}\cos\varphi + \mathrm{i}\left(\Omega_{\mathrm{c}2}\gamma_{42} + \Omega_{\mathrm{c}1}\Omega_\mathrm{m}\sin\varphi\right)\right]\rho_{31}^{(0)} \\ & + \left[\Delta_\mathrm{p}\Omega_{\mathrm{c}1} - \Omega_{\mathrm{c}2}\Omega_\mathrm{m}\cos\varphi + \mathrm{i}\left(\Omega_{\mathrm{c}1}\gamma_{43} - \Omega_{\mathrm{c}2}\Omega_\mathrm{m}\sin\varphi\right)\right]\rho_{21}^{(0)} \\ & + \left[\Delta_\mathrm{p}^2 - \Omega_\mathrm{m}^2 - \gamma_{42}\gamma_{43} + \mathrm{i}\Delta_\mathrm{p}\left(\gamma_{42}+\gamma_{43}\right)\right]\left(\rho_{44}^{(0)} - \rho_{11}^{(0)}\right)\Big\},\end{aligned} \tag{5}$$

where

$$a = -\Delta_\mathrm{p}^3 + \Delta_\mathrm{p}(\Omega_{\mathrm{c}1}^2 + \Omega_{\mathrm{c}2}^2 + \Omega_\mathrm{m}^2 + \gamma_{41}\gamma_{42} + \gamma_{41}\gamma_{43} + \gamma_{42}\gamma_{43}) - 2\Omega_{\mathrm{c}1}\Omega_{\mathrm{c}2}\Omega_\mathrm{m}\cos\varphi,$$
$$b = -\Delta_\mathrm{p}^2(\gamma_{41}+\gamma_{42}+\gamma_{43}) + (\Omega_{\mathrm{c}1}^2\gamma_{43} + \Omega_{\mathrm{c}2}^2\gamma_4 + \Omega_\mathrm{m}^2\gamma_{41}) + \gamma_{41}\gamma_{42}\gamma_{43}.$$

The gain coefficient is determined by the imaginary part of the susceptibility as follows:

$$\begin{aligned}\mathrm{Im}\left(\chi\right) = \; & \frac{N\mu_{14}^2}{2\varepsilon_0\hbar}\cdot\frac{1}{a^2+b^2}\Big\{\left[a\left(\Delta_\mathrm{p}\Omega_{\mathrm{c}2} - \Omega_{\mathrm{c}1}\Omega_\mathrm{m}\cos\varphi\right) + b\left(\Omega_{\mathrm{c}2}\gamma_{42} + \Omega_{\mathrm{c}1}\Omega_\mathrm{m}\sin\varphi\right)\right]\mathrm{Im}\left(\rho_{31}^{(0)}\right) \\ & + \left[a\left(\Omega_{\mathrm{c}1}\Omega_\mathrm{m}\sin\varphi + \Omega_{\mathrm{c}2}\gamma_{42}\right) - b\left(\Delta_\mathrm{p}\Omega_{\mathrm{c}2} - \Omega_{\mathrm{c}1}\Omega_\mathrm{m}\cos\varphi\right)\right]\mathrm{Re}\left(\rho_{31}^{(0)}\right) \\ & + \left[a\left(\Delta_\mathrm{p}\Omega_{\mathrm{c}1} - \Omega_{\mathrm{c}2}\Omega_\mathrm{m}\cos\varphi\right) + b\left(\Omega_{\mathrm{c}1}\gamma_{43} - \Omega_{\mathrm{c}2}\Omega_\mathrm{m}\sin\varphi\right)\right]\mathrm{Im}\left(\rho_{21}^{(0)}\right) \\ & + \left[a\left(\Omega_{\mathrm{c}1}\gamma_{43} - \Omega_{\mathrm{c}2}\Omega_\mathrm{m}\sin\varphi\right) - b\left(\Delta_\mathrm{p}\Omega_{\mathrm{c}1} - \Omega_{\mathrm{c}2}\Omega_\mathrm{m}\cos\varphi\right)\right]\mathrm{Re}\left(\rho_{21}^{(0)}\right) \\ & + \left[a\Delta_\mathrm{p}\left(\gamma_{42}+\gamma_{43}\right) - b\left(\Delta_\mathrm{p}^2 - \Omega_\mathrm{m}^2 - \gamma_{42}\gamma_{43}\right)\right]\left(\rho_{44}^{(0)} - \rho_{11}^{(0)}\right)\Big\}.\end{aligned} \tag{6}$$

In the zeroth order of the probe field, the atomic coherences $\rho_{31}^{(0)}$ and $\rho_{21}^{(0)}$ are expressed as

$$\begin{aligned}
\mathrm{Im}\left(\rho_{31}^{(0)}\right) = & \frac{1}{D}\left[\left(\Omega_{\mathrm{c1}}^2\Omega_{\mathrm{c2}}+\Omega_{\mathrm{c1}}\Omega_{\mathrm{m}}\gamma_{32}\sin\varphi\right)\left(\rho_{11}^{(0)}-\rho_{22}^{(0)}\right)\right. \\
& \left.+\left(\Omega_{\mathrm{c1}}\Omega_{\mathrm{m}}\gamma_{21}\sin\varphi-\Omega_{\mathrm{c2}}\Omega_{\mathrm{m}}^2\right)\left(\rho_{22}^{(0)}-\rho_{33}^{(0)}\right)-\left(\Omega_{\mathrm{c2}}^3+\Omega_{\mathrm{c2}}\gamma_{32}\gamma_{21}\right)\left(\rho_{11}^{(0)}-\rho_{33}^{(0)}\right)\right], \qquad (7)\\
\mathrm{Re}\left(\rho_{31}^{(0)}\right) = & \frac{1}{D}\left[\Omega_{\mathrm{c1}}\Omega_{\mathrm{m}}\gamma_{32}\cos\varphi\left(\rho_{11}^{(0)}-\rho_{22}^{(0)}\right)+\Omega_{\mathrm{c1}}\Omega_{\mathrm{m}}\gamma_{21}\cos\varphi\left(\rho_{22}^{(0)}-\rho_{33}^{(0)}\right)\right], \qquad (8)\\
\mathrm{Im}\left(\rho_{21}^{(0)}\right) = & \frac{1}{D}\left[\left(\Omega_{\mathrm{c1}}\Omega_{\mathrm{c2}}^2-\Omega_{\mathrm{c2}}\Omega_{\mathrm{m}}\gamma_{32}\sin\varphi\right)\left(\rho_{11}^{(0)}-\rho_{33}^{(0)}\right)\right. \\
& \left.+\left(\Omega_{\mathrm{c1}}\Omega_{\mathrm{m}}^2+\Omega_{\mathrm{c2}}\Omega_{\mathrm{m}}\gamma_{31}\sin\varphi\right)\left(\rho_{22}^{(0)}-\rho_{33}^{(0)}\right)-\left(\Omega_{\mathrm{c1}}^3+\Omega_{\mathrm{c1}}\gamma_{32}\gamma_{31}\right)\left(\rho_{11}^{(0)}-\rho_{22}^{(0)}\right)\right], \qquad (9)\\
\mathrm{Re}\left(\rho_{21}^{(0)}\right) = & \frac{1}{D}\left[\Omega_{\mathrm{c2}}\Omega_{\mathrm{m}}\gamma_{32}\cos\varphi\left(\rho_{11}^{(0)}-\rho_{33}^{(0)}\right)-\Omega_{\mathrm{c2}}\Omega_{\mathrm{m}}\gamma_{31}\cos\varphi\left(\rho_{22}^{(0)}-\rho_{33}^{(0)}\right)\right], \qquad (10)
\end{aligned}$$

where $D=\Omega_{\mathrm{c1}}^2\gamma_{21}+\Omega_{\mathrm{c2}}^2\gamma_{31}+\Omega_{\mathrm{m}}^2\gamma_{32}+\gamma_{31}\gamma_{32}\gamma_{21}$.

Here we do not present the analytical solutions of $\rho_{ii}^{(0)}$ ($i=1,\ 2,\ 3,\ 4$) as they are complicated expressions and it is hard to draw physical information from them.

The expressions (6)–(10) are the basis for the numerical plots in Section 3. From the expressions (6)–(10) it can be found that the gain coefficient is dependent on the phase $\varphi$ of the closed interaction loop, and the dependence of the probe gain on the phase $\varphi$ exists only in the presence of the microwave field $\Omega_{\mathrm{m}}$.

To obtain a further insight into the physical origin of the phase-dependence of the gain coefficient, we take a look at the dressed state picture. In the dressed state picture, the three levels $|1\rangle$, $|2\rangle$ and $|3\rangle$ in the bare state involved in the interaction loop of the three fields $\Omega_{\mathrm{c1}}$, $\Omega_{\mathrm{c2}}$ and $\Omega_{\mathrm{m}}$ can be replaced by three dressed states $|a\rangle$, $|b\rangle$ and $|c\rangle$, whose eigenvalues under the full-resonance condition ($\Delta_1=\Delta_2=\Delta_{\mathrm{m}}=0$) are determined by the following equation:

$$\begin{vmatrix}
-\lambda & \Omega_{\mathrm{c1}}\exp(\mathrm{i}\varphi_1) & \Omega_{\mathrm{c2}}\exp(\mathrm{i}\varphi_2)\\
\Omega_{\mathrm{c1}}\exp(-\mathrm{i}\varphi_1) & -\lambda & \Omega_{\mathrm{m}}\exp(\mathrm{i}\varphi_{\mathrm{m}})\\
\Omega_{\mathrm{c2}}\exp(-\mathrm{i}\varphi_2) & \Omega_{\mathrm{m}}\exp(-\mathrm{i}\varphi_{\mathrm{m}}) & -\lambda
\end{vmatrix}=0. \qquad (11)$$

Further calculation simplifies it into

$$\lambda^3-\lambda\left(\Omega_{\mathrm{c1}}^2+\Omega_{\mathrm{c2}}^2+\Omega_{\mathrm{m}}^2\right)-2\Omega_{\mathrm{c1}}\Omega_{\mathrm{c2}}\Omega_{\mathrm{m}}\cos\varphi=0. \qquad (12)$$

It shows that the eigenvalues of the dressed states are dependent on the phase $\varphi$: for different values of phase $\varphi$, the eigenvalues are different. Obviously, the weak probe scanning through transitions between the dressed states and the state $|4\rangle$ at different frequencies for different values of phase $\varphi$ can bring on different profiles of the gain spectrum.

## 3. Numerical results and analysis

In this section, we show how the inversionless gain is affected by the phase of the closed interaction loop and the strength of the microwave field. From the expression (3a), it is easy to find that in the limit of a weak probe, we have $\Lambda\rho_{11}\cong\gamma_4\rho_{44}$. So, as long as $\Lambda<\gamma_4$, no population inversion occurs on transition $|1\rangle\leftrightarrow|4\rangle$. Note that all the discussion in the following is on condition that $\Lambda<\gamma_4$, so that the population inversion cannot be established on the probe transition. In our notation, $\mathrm{Im}(\chi)>0$ means that the probe field is amplified; otherwise it is absorbed. For simplicity in the following calculations we set $\gamma_4=\gamma_3=\gamma_2=\gamma$ and other parameters $\Lambda$, $\Omega_{\mathrm{c1}}$, $\Omega_{\mathrm{c2}}$, $\Omega_{\mathrm{m}}$ and $\Delta_{\mathrm{p}}$ are scaled by $\gamma$.

First, we consider a simple case in which $\Omega_{\mathrm{c1}}=$

$\Omega_{c2} = \Omega_m = \Omega$. From Eq.(12) we can derive the eigenvalues of the dressed states as follows: for $\varphi = 0$, then $\lambda_a = 2\Omega$, $\lambda_b = \lambda_c = -\Omega$; for $\varphi = \pi$, then $\lambda_a = -2\Omega$, $\lambda_b = \lambda_c = \Omega$; for $\varphi = \pi/2$, then $\lambda_a = \sqrt{3}\Omega$, $\lambda_b = 0$, $\lambda_c = -\sqrt{3}\Omega$. It can be predicted that due to the transitions between the dressed states and the state $|4\rangle$, there should exist two gain or absorption peaks in the spectrum for $\varphi = 0$ and $\varphi = \pi$; and there should exist three peaks in the spectrum for $\varphi = \pi/2$.

In Fig.2, with $\Omega_{c1} = \Omega_{c2} = \Omega_m = \Omega$, we plot probe gain (or absorption) $\mathrm{Im}(\chi)$ versus probe detuning $\Delta_p$ for three values of $\varphi$. The solid, dotted, and dashed curves correspond to three different values of $\Omega$: $3\gamma$, $6\gamma$ and $9\gamma$, respectively. It is found that the probe gain is indeed quite sensitive to the phase of the closed interaction loop. With a different $\varphi$, we obtain a different gain profile.

When $\varphi = 0$ ($\varphi = \pi$) [shown in Figs.2(a) and (b)], a gain peak and an absorption peak appear at $\Delta_p = -2\Omega$ ($\Delta_p = 2\Omega$) and $\Delta_p = \Omega$ ($\Delta_p = -\Omega$), respectively. Obviously, they result from the transitions $|a\rangle \leftrightarrow |4\rangle$ and $|b\rangle$, $|c\rangle \leftrightarrow |4\rangle$, respectively. The frequency interval between the gain and the absorption peaks is $3\Omega$, which is the energy spacing between the dressed-state sublevels.

Comparing Fig.2(a) with Fig.2(b), it can be found that we can obtain the spectrum for $\varphi = \pi$ just by replacing $\Delta_p$ with $-\Delta_p$ from the spectrum for $\varphi = 0$, and the relation $\mathrm{Im}(\chi)|_{\Delta_p,\varphi=0} = \mathrm{Im}(\chi)|_{-\Delta_p,\varphi=\pi}$ also can be derived from the expressions (6)–(10). This phenomenon can be understood by the fact that the eigenvalues of the dressed states for $\varphi = 0$ and $\varphi = \pi$ are just negative.

From Figs.2(a) and (b), it also can be found that on condition that $\Omega_{c1} = \Omega_{c2} = \Omega_m = \Omega$, when $\varphi = 0$ or $\varphi = \pi$, the amplitudes of the gain (absorption) peaks cannot be affected by the strengths of the applied fields. With different values of $\Omega$, the gain (absorption) peaks are located at different frequencies $\Delta_p = \mp 2\Omega$ ($\Delta_p = \pm\Omega$), but with the same amplitudes.

When $\varphi = \pi/2$ [shown in Fig.2(c)], the spectrum exhibits quite a different profile. As predicted above, there are three absorption peaks located at $\Delta_p = -\sqrt{3}\Omega$, 0, and $\sqrt{3}\Omega$, separately, which result from the transitions between the dressed states $|a\rangle$, $|b\rangle$, $|c\rangle$ and the state $|4\rangle$. Between the absorption peaks, there exist two symmetric regions of probe gain. With the enhancing of the driving fields, the gain region becomes wider and flatter, and the amplitude of the gain decreases.

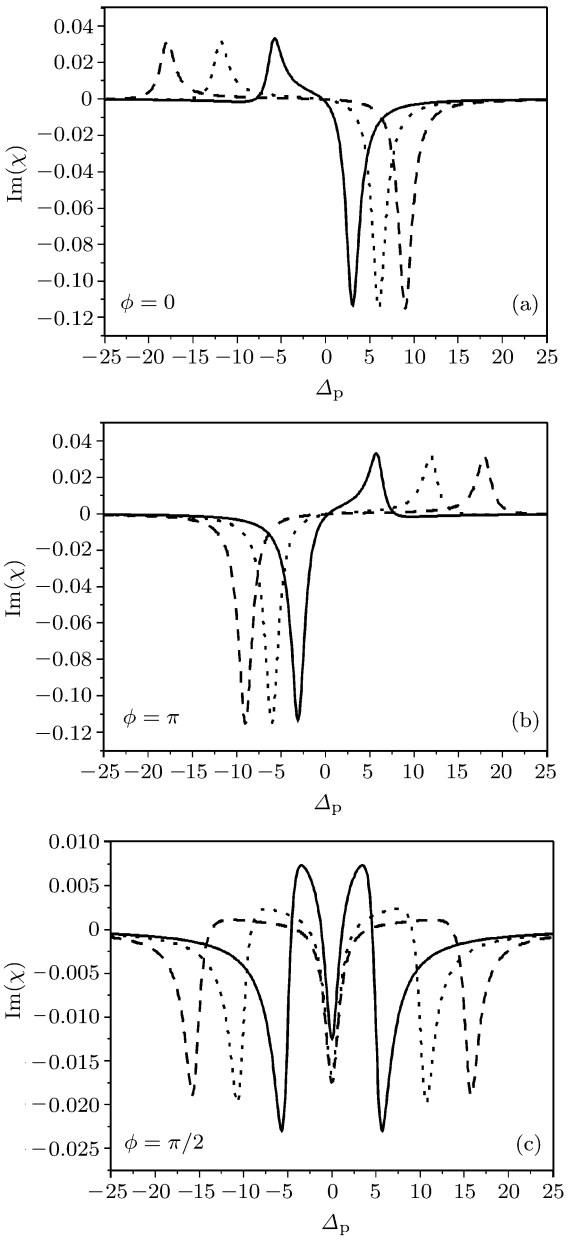



**Fig.2.** Probe gain (or absorption) $\mathrm{Im}(\chi)$ in units of $N\mu_{14}^2/2\varepsilon_0\hbar$ versus probe detuning $\Delta_p$ for different values of $\varphi$, (a) $\varphi = 0$, (b) $\varphi = \pi$, (c) $\varphi = \pi/2$, under the condition of $\Omega_{c1} = \Omega_{c2} = \Omega_m = \Omega$. The solid, dotted, and dashed curves correspond to three different values of $\Omega$: $3\gamma$, $6\gamma$ and $9\gamma$, respectively. Other parameters are chosen as $\Delta_1 = \Delta_2 = \Delta_m = 0$, $\gamma_2 = \gamma_3 = \gamma_4 = \gamma$, and $\Lambda = 0.8\gamma$.

To obtain a deeper insight into the dependence of the probe gain on the phase $\varphi$, in Fig.3 we display the probe gain versus phase $\varphi$ at two pairs of special detunings $\Delta_p = \pm 2\Omega$ and $\Delta_p = \pm\Omega$. It

can be found from Fig.3(a) (where solid and dashed curves correspond to $\Delta_{\rm p} = 2\Omega$ and $-2\Omega$, respectively.) that when $\Delta_{\rm p} = 2\Omega$ we can obtain probe gain by setting $\varphi = (2k+1)\pi$, $(k = 0, \pm1, \pm2, \ldots)$; and when $\Delta_{\rm p} = -2\Omega$ we can obtain probe gain by setting $\varphi = 2k\pi$. It can be found from Fig.3(b) (where solid and dashed curves correspond to $\Delta_{\rm p} = \Omega$ and $-\Omega$, respectively.) that when $\Delta_{\rm p} = \Omega$ strong absorption occurs at $\varphi = 2k\pi$, and the probe gain can be obtained by tuning the phase of the closed-loop to the wide region of $2k\pi < \varphi < 2(k+1)\pi$. When $\Delta_{\rm p} = -\Omega$, strong absorption occur at $\varphi = (2k+1)\pi$, and the probe gain can be obtained by tuning the phase of the closed-loop to the wide region of $(2k+1)\pi < \varphi < (2k+3)\pi$. It can be seen that with a different phase $\varphi$, the amplitude of the probe gain is also different; we can modulate the amplitude of the probe gain by controlling the phase $\varphi$.

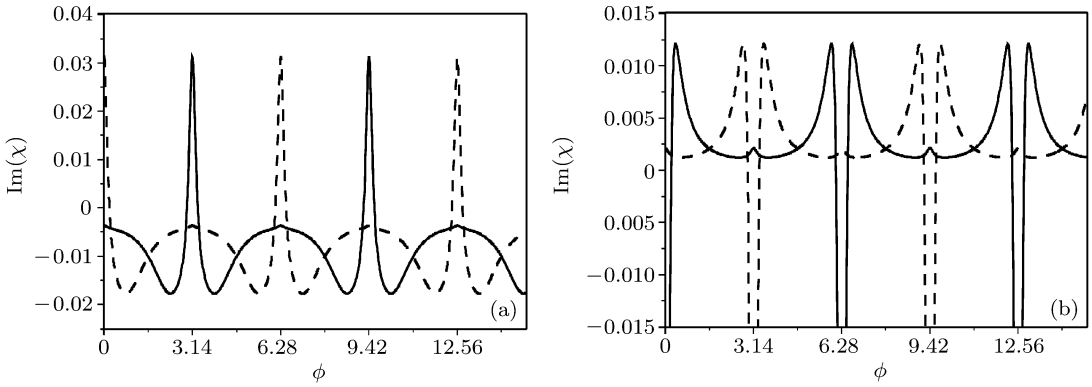



**Fig.3.** The probe gain (or absorption) $\mathrm{Im}(\chi)$ in units of $N\mu_{14}^2/2\varepsilon_0\hbar$ versus phase $\varphi$ at two pairs of special detunings: (a) $\Delta_{\rm p} = 2\Omega$ (solid curve) and $\Delta_{\rm p} = -2\Omega$ (dashed curve), (b) $\Delta_{\rm p} = \Omega$ (solid curve) and $\Delta_{\rm p} = -\Omega$ (dashed curve). $\Omega_{\rm c1} = \Omega_{\rm c2} = \Omega_{\rm m} = 6\gamma$ and other parameters are the same as those in Fig.2.

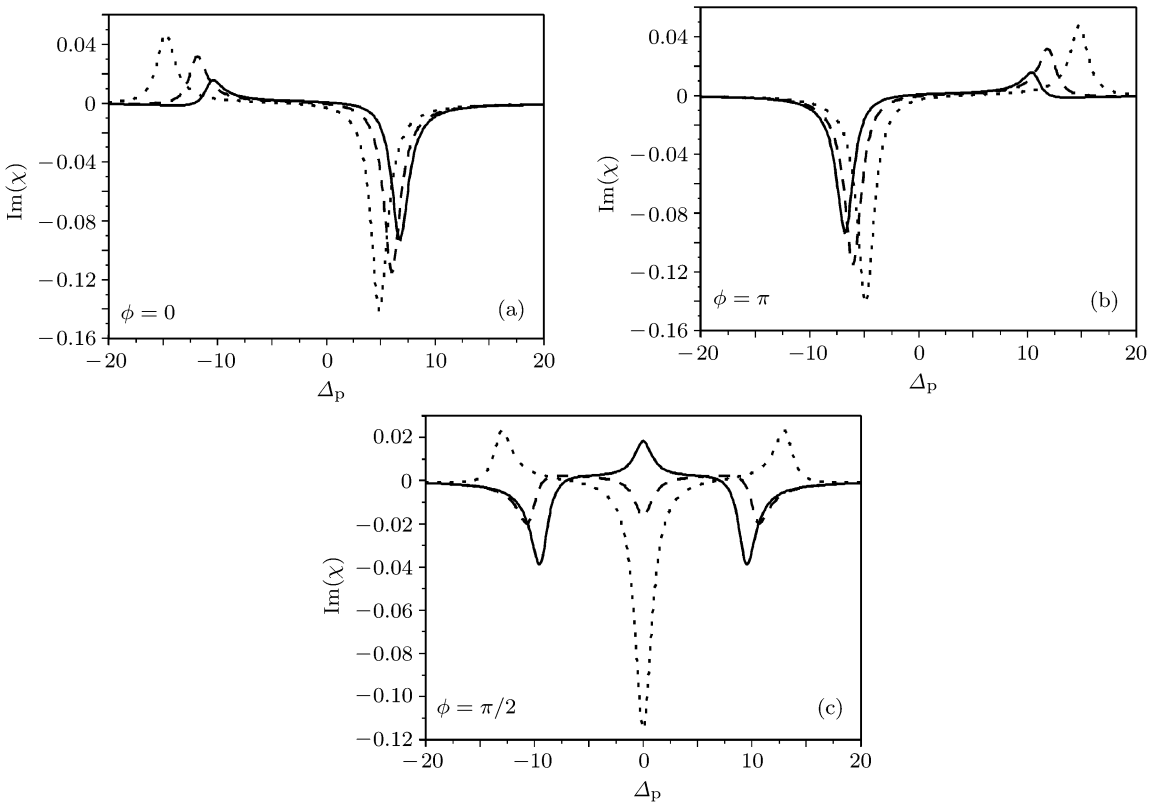



**Fig.4.** Probe gain (or absorption) $\mathrm{Im}(\chi)$ in units of $N\mu_{14}^2/2\varepsilon_0\hbar$ versus probe detuning $\Delta_{\rm p}$ for different values of $\Omega_{\rm m}$. The parameters are $\Omega_{\rm c1} = \Omega_{\rm c2} = 6\gamma$, $\Omega_{\rm m} = 4\gamma$ (solid curve), $6\gamma$ (dashed curve), $10\gamma$ (dotted curve). Other parameters are the same as those in Fig.2.

The above discussion is conducted in the simple case where $\Omega_{c1} = \Omega_{c2} = \Omega_m = \Omega$. Now, we keep $\Omega_{c1} = \Omega_{c2}$ and examine the influence of the variation in the microwave field $\Omega_m$ on the probe spectrum. In Fig.4, with $\Omega_{c1} = \Omega_{c2} = 6\gamma$, we plot probe gain (or absorption) $\mathrm{Im}(\chi)$ versus probe detuning $\Delta_p$ for three different values of $\Omega_m$. The solid, dashed and dotted curves correspond to $\Omega = 4\gamma$, $6\gamma$ and $10\gamma$, respectively. First we consider the case for $\varphi = 0$ or $\pi$ (shown in Figs.4(a) and (b)). It is shown that corresponding to the eigenvalues of the dressed states, the gain and the absorption peaks are located at $\Delta_p = \mp(1/2)(\Omega_m + \sqrt{8\Omega_{c1,c2}^2 + \Omega_m^2})$ and $\Delta_p = \mp(1/2)(\Omega_m - \sqrt{8\Omega_{c1,c2}^2 + \Omega_m^2})$, respectively. It can be seen that not only the frequency positions, but also the amplitudes of the two peaks are affected by the variation of the amplitude of the microwave field $\Omega_m$. With the enhancement of the microwave field, the gain peak shifts toward the far sidebands, of which the amplitude experiences a variation correspondingly. As is known, the inversionless gain is usually related to the appearance of a population inversion between dressed states.[29] We can deduce that the population distribution in the dressed states being inevitably affected by the variation of the strength of the microwave field results in the variation of the gain amplitude.

Then we consider the case for $\varphi = \pi/2$ (shown in Fig.4(c)). It shows that when $\varphi = \pi/2$, the three gain or absorption peaks are located at $\Delta_p = 0$, $\mp\sqrt{2\Omega_{c1,c2}^2 + \Omega_m^2}$, which correspond to the three eigenvalues of the dressed states. It can be found that the line profile is dramatically modified by the variation of $\Omega_m$. When $\Omega_m < \Omega_{c1,c2}$, the gain peak appears at $\Delta_p = 0$ in a resonant region (denoted as the solid curve); when $\Omega_m = \Omega_{c1,c2}$, the gain peak at $\Delta_p = 0$ is replaced by an absorption peak, and there are developed two symmetric gain regions at the wings of the resonant region $\Delta_p = 0$ (denoted as the dashed curve). When $\Omega_m > \Omega_{c1,c2}$, the absorption peak at $\Delta_p = 0$ is enhanced, and the two symmetric gain regions evolve into two gain peaks and shift toward the far sidebands (denoted as the dotted curve).

## 4. Conclusions

We have investigated the gain properties of a four-level atomic system with a closed interaction loop under the full-resonance condition. We find that the formation of the closed interaction loop makes the system depend on the phase of the closed-loop. The probe gain on the higher frequency transition outside the closed-loop can be manipulated by the phase of the closed-loop, and it also can be modulated by the amplitude of the microwave field. The phase sensitivity property in such a scheme is similar to that in an analogous configuration with SGC,[21] but it is beyond the non-orthogonal condition of the two dipole matrix elements and the near-degenerate condition of the two intermediate levels. Therefore this scheme is very convenient and feasible for the experimental realization.

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