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Engineering three-dimensional maximally entangled states for two modes in a bimodal cavity*

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An alternative scheme is proposed for engineering three-dimensional maximally entangled states for two modes of a superconducting microwave cavity. In this scheme, an appropriately prepared four-level atom is sent through a bimodal cavity. During its passing through the cavity, the atom is coupled resonantly with two cavity modes simultaneously and addressed by a classical microwave pulse tuned to the required transition. Then the atomic states are detected to collapse two modes onto a three-dimensional maximally entangled state. The scheme is different from the previous one in which two nonlocal cavities are used. A comparison between them is also made.

Keywords: three-dimensional entanglement, bimodal cavity
PACC: 4250, 0365

Entanglement lies in the heart of the Einstein–Podolsky–Rosen (EPR) paradox,[1] Bell's inequalities,[2] and quantum nonlocality.[3] Furthermore, entanglement is a central resource for many applications of quantum information theory,[4−5] including quantum cryptography,[6] quantum dense coding,[7] and quantum teleportation.[8] Researches in quantum information processing are mostly based on two-dimensional systems. Recently, much attention has also been paid to multi-dimensional (more than two) quantum systems. It has been shown that violations of local realism by two entangled N-dimensional ($N \geq 3$) systems are stronger than that by two-dimensional systems.[9] The proof of Bell's theorem without the inequalities by Greenberger, Horne, and Zeilinger (GHZ) was extended to multiparticle multi-dimensional systems.[10−11] It has been demonstrated that quantum key distributions based on three-dimensional systems are more secure than those based on two-dimensional systems.[12−14] Protocols for quantum key distributions based on three-dimensional systems have also been proposed.[15−16] Experimental realization of multi-dimensional entanglement by parametric down-conversion has been reported.[17−18]

Cavity quantum electrodynamics (QED), with Rydberg atoms crossing superconducting microwave cavities, offers an almost ideal system for quantum information processing.[19] In the context of cavity QED, various schemes have been proposed for realizing multi-atom maximally entangled states,[20−22] multi-cavity maximally entangled states,[23−25] quantum teleportation,[26−29] and quantum logic gates.[30−31] Experimental demonstration of EPR states of two atoms,[32−33] GHZ states (two atoms and one cavity mode),[34] and quantum logic gates,[35] has been reported. Recently, Zou *et al*[36] have proposed a scheme for engineering three-dimensional entanglement between two three-level atoms by nonresonant interaction with cavity-assisted collisions. In the scheme, two three-level atoms are simultaneously sent into a nonresonant cavity. During the passage of the atoms through the cavity, one of the two atoms is addressed by a strong classical microwave pulse. In the scheme, the cavity field is virtually excited, no information is transferred from the atoms to the cavity, thus it is insensitive to cavity losses. Zheng[37] has proposed an alternative scheme for generating multi-dimensional entanglement between two or more multilevel atoms in a thermal cavity. In the scheme, two or more multilevel atoms are simultaneously coupled with the cavity mode in a nonresonant manner and driven by a classical field. The scheme requires that all the atoms are simultaneously addressed by another strong classical field tuned to the required transition. For generating N-dimensional entanglement between more than two

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atoms, $N+1$ atoms are required and a conditional measurement is needed. The scheme is insensitive to not only the cavity decay, but also the thermal field.

More recently, Zheng[38] has proposed a scheme for engineering three-dimensional maximally entangled states between two cavity modes. In the scheme, an appropriately prepared atom is sent through two nonlocal resonant cavities sequentially. In each cavity, the atom is addressed by a classical field tuned to the required transition, then the atomic states are detected to collapse two cavity modes onto a three-dimensional maximally entangled state, each mode contains no more than two photons. As far as we know, this is the first scheme for engineering such a type of entangled states for two cavity modes. In this paper, we propose an alternative scheme to prepare the same type of entangled state for two modes distributed in a single superconducting microwave cavity. In the scheme, an appropriately prepared atom is sent across a bimodal cavity. The atom is coupled resonantly with two cavity modes simultaneously and addressed by a strong classical microwave pulse. Then the three-dimensional maximally entangled state can be engineered by detecting the atom in a definite state. The obvious difference between the above two schemes is that, we use a bimodal cavity, in which the atom interacts simultaneously with two cavity modes, whereas in the scheme of Ref.[38], the atom interacts with each of two nonlocal cavity modes. Thus the present scheme requires a smaller number of operations than that required by the scheme of Ref.[38], which makes it easier to implement. Furthermore, we will show that the time required in the present scheme is shorter than that required by the scheme of Ref.[38], which is very important in view of decoherence.

Figure 1 displays the set-up for engineering three-dimensional entangled state for two modes of a bimodal cavity. The atom is produced in zone B by the excitation of a velocity-selected atomic beam effusing from oven O, F_1, F_2, and F_3 are three classical fields tuned to the transitions $|e\rangle \longleftrightarrow |g'\rangle$, $|e\rangle \longleftrightarrow |i\rangle$, and $|e\rangle \longleftrightarrow |g'\rangle$ respectively, the superconducting microwave cavity C sustains two non-degenerate resonant modes a and b, S denotes the classical microwave pulse tuned to the transition $|i\rangle \longleftrightarrow |e\rangle$ inside the cavity, and D_e ($D_{g'}$) denotes the atomic ionization detector for the state $|e\rangle$ ($|g'\rangle$).

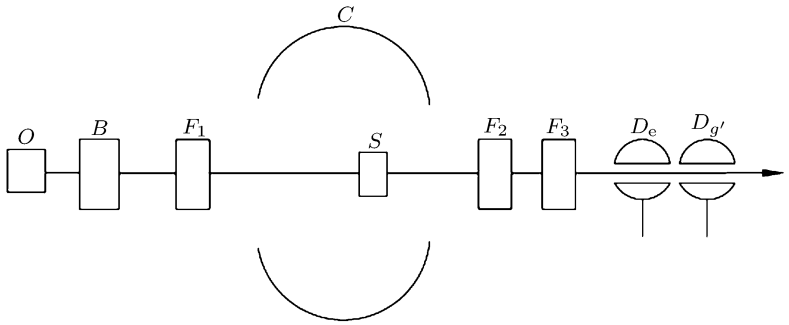


Fig.1. Scheme of set-up for engineering three-dimensional entangled state for two modes in a bimodal cavity.

The atom considered here has four states—$|e\rangle$, $|g\rangle$, $|i\rangle$, and $|g'\rangle$, with energies ω_e, ω_g, ω_i and $\omega_{g'}$ respectively ($\omega_e > \omega_g > \omega_i > \omega_{g'}$). Let us consider such an atom coupling to a two-mode cavity field, as sketched in Fig.2(a). The transition between states $|e\rangle$ and $|g\rangle$ is coupled to cavity mode 1, while the transition between states $|g\rangle$ and $|i\rangle$ is coupled to cavity mode 2. The transition between states $|x\rangle$ ($x = e, g, i$) and $|g'\rangle$ is highly detuned from both the cavity modes, respectively, thus the state $|g'\rangle$ is not affected by the atom–field interaction. The Hamiltonian for such a system is written as ($\hbar = 1$)[39−41]

$$H = H_0 + H_{\rm I}, \tag{1}$$

$$H_0 = \omega_e |e\rangle\langle e| + \omega_g |g\rangle\langle g| + \omega_i |i\rangle\langle i| + \omega_1 a_1^+ a_1 + \omega_2 a_2^+ a_2, \tag{2}$$

$$H_{\rm I} = g_1 \left(a_1^+ |g\rangle\langle e| + a_1 |e\rangle\langle g|\right) + g_2 \left(a_2^+ |i\rangle\langle g| + a_2 |g\rangle\langle i|\right), \tag{3}$$

where $a_1^+(a_2^+)$ and $a_1(a_2)$ are the creation and annihilation operators for the cavity mode 1(2) with frequency $\omega_1(\omega_2)$, g_1 (g_2) is the atom–field coupling constant for the transition $|e\rangle \longleftrightarrow |g\rangle(|g\rangle \longleftrightarrow |i\rangle)$. Supposing that both the cavity modes are resonant with the corresponding transitions

$$\omega_1 = \omega_e - \omega_g, \qquad \omega_2 = \omega_g - \omega_i, \tag{4}$$

we can see that the free Hamiltonian H_0 commutes with the interaction Hamiltonian $H_{\rm I}$. Thus we can define an interaction picture with the Hamiltonian

$$H_{\rm I}' = {\rm e}^{{\rm i}H_0 t} H_{\rm I} {\rm e}^{-{\rm i}H_0 t} = H_{\rm I}. \tag{5}$$

We assume that both the atom–field coupling constants are equal, i.e.

$$g_1 = g_2 = g, \tag{6}$$

which can be satisfied by choosing appropriate transitions in atomic systems,[42] thus the effective Hamiltonian for the system is written as

$$H_{\rm I}' = g\left(a_1^+ |g\rangle\langle e| + a_2^+ |i\rangle\langle g|\right) + {\rm H.c.} \tag{7}$$

Now we use the above system to engineer three-dimensional entanglement between two modes of the cavity. We assume that the cavity is initially in the two-mode vacuum state $|0,0\rangle_{\rm C}$. The atom, initially prepared in the state

$$a\,|g'\rangle + b\,|e\rangle\,, \quad a^2 + b^2 = 1, \tag{8}$$

by a classical field F_1, is sent through the cavity. The interaction is described by the Hamiltonian (7) that causes no effect on the state $|g'\rangle\,|0,0\rangle$. After an interaction time τ_1, the system combined by the atom and the cavity evolves into the state

$$\begin{aligned} |\,\Psi_1(\tau_1)\rangle_{\rm a-C} = \;& a\,|g'\rangle\,|0,0\rangle_{\rm C} + b\left\{\frac{1}{2}\left[\cos\left(\sqrt{2}g\tau_1\right)+1\right]|e\rangle\,|0,0\rangle_{\rm C}\right. \\ & \left. +\frac{1}{2}\left[\cos\left(\sqrt{2}g\tau_1\right)-1\right]|i\rangle\,|1,1\rangle_{\rm C} - \frac{\rm i}{\sqrt{2}}\sin\left(\sqrt{2}g\tau_1\right)|g\rangle\,|1,0\rangle_{\rm C}\right\}. \end{aligned} \tag{9}$$

If the velocity of the atom is adjusted approximately so that $\sqrt{2}g\tau_1 = \pi$ is fulfilled, we have

$$|\,\Psi_1\rangle_{\rm a-C} = a\,|g'\rangle\,|0,0\rangle_{\rm C} - b\,|i\rangle\,|1,1\rangle_{\rm C}\,. \tag{10}$$

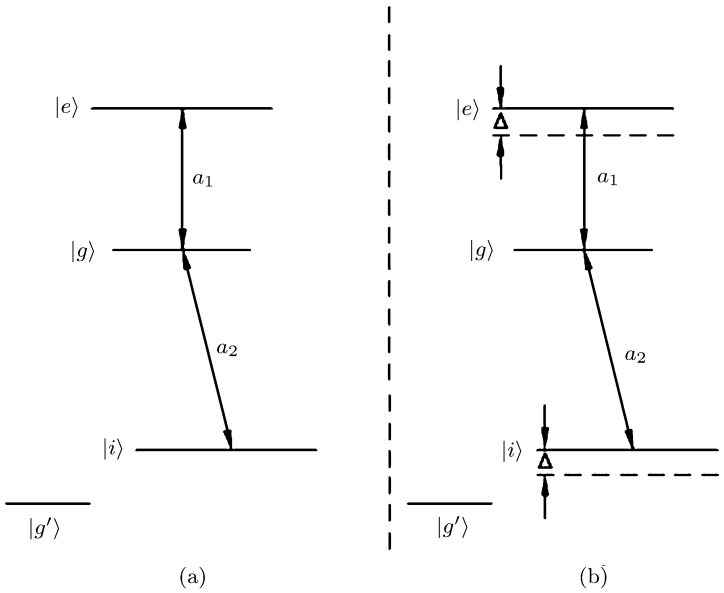


Fig.2. (a) Schematic diagram of energy levels of a four-level atom with the corresponding transitions. (b) Scheme for freezing the atom–field interaction through Stark effect.

Now we turn the atomic transitions $|e\rangle \longleftrightarrow |g\rangle$ and $|g\rangle \longleftrightarrow |i\rangle$ far off-resonant with the corresponding field modes by an amount of detuning Δ, separately, which can be achieved through the Stark effect[43] as sketched in Fig.2(b). Such a detuning can be set to be a large value so that the atom–field interaction is frozen. At the same time, the atom is addressed by a classical microwave pulse S tuned to the transition $|i\rangle \longrightarrow |e\rangle$,[36] thus we have

$$|\Psi_1'\rangle_{\mathrm{a-C}} = a\,|g'\rangle\,|0,0\rangle_{\mathrm{C}} - b\,|e\rangle\,|1,1\rangle_{\mathrm{C}}\,. \tag{11}$$

Then we again turn the atomic transitions $|e\rangle \longleftrightarrow |g\rangle$ and $|g\rangle \longleftrightarrow |i\rangle$ resonant with the corresponding modes, respectively. After another interaction time τ_2, the whole system evolves into the state

$$\begin{aligned}|\Psi_2(\tau_2)\rangle_{\mathrm{a-C}} = \; & a\,|g'\rangle\,|0,0\rangle_{\mathrm{C}} - b\Big\{\frac{1}{2}\left[\cos(2g\tau_2)+1\right]|e\rangle\,|1,1\rangle_{\mathrm{C}} \\ & +\frac{1}{2}\left[\cos(2g\tau_2)-1\right]|i\rangle\,|2,2\rangle_{\mathrm{C}} - \frac{\mathrm{i}}{\sqrt{2}}\sin(2g\tau_2)\,|g\rangle\,|2,1\rangle_{\mathrm{C}}\Big\}. \end{aligned}\tag{12}$$

Choosing the interaction time in such a way that $2g\tau_2 = \dfrac{\pi}{2}$ is fulfilled, we have

$$|\Psi_2\rangle_{\mathrm{a-C}} = a\,|g'\rangle\,|0,0\rangle_{\mathrm{C}} - \frac{b}{2}\,|e\rangle\,|1,1\rangle_{\mathrm{C}} + \frac{b}{2}\,|i\rangle\,|2,2\rangle_{\mathrm{C}} + \frac{\mathrm{i}b}{\sqrt{2}}\,|g\rangle\,|2,1\rangle_{\mathrm{C}}\,. \tag{13}$$

After the atom exits from the cavity, it traverses a classical field F_2, which leads to the transformation[44]

$$|e\rangle \to \frac{1}{\sqrt{2}}\left(|e\rangle + |i\rangle\right), \quad |i\rangle \to \frac{1}{\sqrt{2}}\left(|i\rangle - |e\rangle\right), \tag{14}$$

then we have

$$\begin{aligned}|\Psi_2'\rangle_{\mathrm{a-C}} = \; & a\,|g'\rangle\,|0,0\rangle_{\mathrm{C}} - \frac{b}{2\sqrt{2}}[|e\rangle\left(|1,1\rangle_{\mathrm{C}} + |2,2\rangle_{\mathrm{C}}\right) \\ & + |i\rangle\left(|1,1\rangle_{\mathrm{C}} - |2,2\rangle_{\mathrm{C}}\right)] + \frac{\mathrm{i}b}{\sqrt{2}}\,|g\rangle\,|2,1\rangle_{\mathrm{C}}\,. \end{aligned}\tag{15}$$

Then the atom traverses another classical field F_3, which leads to the transformation

$$|e\rangle \to \frac{1}{\sqrt{2}}\left(|e\rangle + |g'\rangle\right), \quad |g'\rangle \to \frac{1}{\sqrt{2}}\left(|g'\rangle - |e\rangle\right), \tag{16}$$

thus we have

$$\begin{aligned}|\Psi_2''\rangle_{\mathrm{a-C}} = \; & |g'\rangle\left[\frac{a}{\sqrt{2}}\,|0,0\rangle_{\mathrm{C}} - \frac{b}{4}\left(|1,1\rangle_{\mathrm{C}} + |2,2\rangle_{\mathrm{C}}\right)\right] \\ & - |e\rangle\left[\frac{a}{\sqrt{2}}\,|0,0\rangle_{\mathrm{C}} + \frac{b}{4}\left(|1,1\rangle_{\mathrm{C}} + |2,2\rangle_{\mathrm{C}}\right)\right] \\ & + \frac{b}{2\sqrt{2}}\,|i\rangle\left(|1,1\rangle_{\mathrm{C}} - |2,2\rangle_{\mathrm{C}}\right) + \frac{\mathrm{i}b}{\sqrt{2}}\,|g\rangle\,|2,1\rangle_{\mathrm{C}}\,. \end{aligned}\tag{17}$$

The atom is finally detected by the field ionization detector D_e ($D_{g'}$), if the atom is found in state $|g'\rangle$, the cavity modes collapse onto the state

$$|\Psi\rangle_{\mathrm{C}_{g'}} = \frac{1}{N_{g'}}\left[\frac{a}{\sqrt{2}}\,|0,0\rangle_{\mathrm{C}} - \frac{b}{4}\left(|1,1\rangle_{\mathrm{C}} + |2,2\rangle_{\mathrm{C}}\right)\right]. \tag{18}$$

Or, if the atom is found in state $|e\rangle$, two cavity modes are in the state

$$|\Psi\rangle_{\mathrm{C}_e} = \frac{1}{N_e}\left[\frac{a}{\sqrt{2}}\,|0,0\rangle_{\mathrm{C}} + \frac{b}{4}\left(|1,1\rangle_{\mathrm{C}} + |2,2\rangle_{\mathrm{C}}\right)\right]. \tag{19}$$

$N_{g'}$ and N_e are two normalization factors. Setting the parameters a and b of the classical field F_1 to be $\frac{1}{3}$ and $\frac{2\sqrt{2}}{3}$ respectively, we obtain the three-dimensional maximally entangled states for two cavity modes, i.e.

$$\frac{1}{\sqrt{3}}\left(|0,0\rangle_{\rm C}-|1,1\rangle_{\rm C}-|2,2\rangle_{\rm C}\right), \quad (|g'\rangle \quad \text{is} \quad \text{detected}), \tag{20}$$

and

$$\frac{1}{\sqrt{3}}\left(|0,0\rangle_{\rm C}+|1,1\rangle_{\rm C}+|2,2\rangle_{\rm C}\right), \quad (|e\rangle \quad \text{is} \quad \text{detected}), \tag{21}$$

respectively. The probability for each of the two states $|g'\rangle$ and $|e\rangle$ detected is $\frac{1}{6}$, thus the probability of success is $\frac{1}{3}$.

We now give a brief discussion on the experimental realization of the proposed scheme. To generate three-dimensional maximally entangled state as described above, we may consider a four-level Rydberg atom with states $|e\rangle$, $|g\rangle$, $|i\rangle$ and $|g'\rangle$, of which the radiative lifetimes are of the order of $T_{\rm at} = 30\,{\rm ms}$. In the experiment reported in Ref.[19], the photon lifetime in the bimodal cavity is $T_{\rm C} = \frac{Q}{2\pi\nu} \approx 10^{-3}\,{\rm s}$, thus the lifetime for the two-photon state should be $T_{\rm 2P} = \frac{T_{\rm C}}{2} = 5\times10^{-4}\,{\rm s}$. By using a velocity selector and applying a Stark field adjustment, we can make the atom resonant with the cavity field for the right amount of time.[34] Setting $g = 2\pi\times 25\,{\rm kHz}$, we have $\tau_1 = \frac{\pi}{\sqrt{2}g} \simeq 14\,\mu s$ and $\tau_2 = \frac{\pi}{4g} \simeq 5\,\mu{\rm s}$ in our scheme; Considering the travelling time of the atom,[45] we obtain that the total time required to complete the process is about $190\,\mu{\rm s}$ (this time is shorter than that of Ref.[38], in which the total time is about $350\,\mu{\rm s}$, with the same atom–field coupling constant.), much shorter than $T_{\rm at}$ and $T_{\rm 2P}$. Therefore, by employing the superconducting microwave cavity with an enough high-Q and Rydberg atom of long lifetime, there is sufficient time to complete the process for engineering three-dimensional maximally entangled state for two modes of a bimodal cavity.

In conclusion, we have proposed an alternative scheme for engineering three-dimensional maximally entangled states for two modes of a superconducting microwave cavity. The scheme is based on resonant interaction of two cavity modes with an atom. By comparison the present scheme is distinctively different from that of Ref.[38]. Similarly, based on resonant atom–field interaction (combined with Stark effect and classical addressing of an atom), the present scheme uses a bimodal cavity, in which an atom interacts simultaneously with two cavity modes, whereas in the scheme of Ref.[38], an atom interacts separately with two nonlocal cavity modes. Thus the present scheme requires a smaller number of operations, which makes it easier to implement than that of Ref.[38]. Moreover, it has been shown that the total time in our scheme, required to complete the process is much shorter than that required by the scheme in Ref.[38], thus the present scheme can be quickly completed, which is very important in the sense of decoherence. Based on the recent microwave cavity QED technique or the technique to be improved soon, the present scheme might be realizable, and it might have applications in testing fundamental principle of quantum mechanics and quantum information processing.

References

[1] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777

[2] Bell J S 1965 *Physics* **1** 195 (Long Island City, N.Y.)

[3] Greenberger D M, Horne M, Shimony A and Zeilinger A 1990 *Am. J. Phys.* **58** 1131

[4] Michael K 2002 *Phys. Rep.* **369** 431

[5] Monroe C 2002 *Nature* **416** 238

[6] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661

[7] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881

[8] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895

[9] Kaszlikowski D, Gnaciski P, Zukowski M, Miklaszewski W and Zeilinger A 2000 *Phys. Rev. Lett.* **85** 4418

[10] Cabello A 2001 *Phys. Rev.* A **63** 022104

[11] Cerf N J, Massar S and Pironio S 2002 *Phys. Rev. Lett.* **89** 080402

[12] Bourennane M, Karlsson A, and Björk G 2001 *Phys. Rev.* A **64** 012306

[13] Bruβ D and Macchiavello C 2002 *Phys. Rev. Lett.* **88** 127901
[14] Cerf N J, Bourennane M, Karlsson A and Gisin N 2002 *Phys. Rev. Lett.* **88** 127902
[15] Kaszlikowski D, Chang K, Oi D K L, Kwek L C and Oh C H 2002 *preprint quant-ph*/0206170
[16] Durt T, Cerf N J, Gisin N and Zukowski M 2003 *Phys. Rev.* A **67** 012311
[17] Mair A, Vaziri A, Weihs G and Zeilinger A 2001 *Nature* **412** 313
[18] de Riedmatten H, Marcikic I, Zbinden H and Gisin N 2002 *Quant. Inf. and Comp.* **2** 425
[19] Raimond J M, Brune M and Haroche S 2001 *Rev. Mod. Phys.* **73** 565
[20] Zheng S B 1999 *J. Opt.* B: *Quantum Semiclass. Opt.* **1** 534
[21] Zheng S B 2001 *Phys. Rev. Lett.* **87** 230404
[22] Zhang J and Ye L 2004 *Chin. Phys.* **13** 1438
[23] Bergou J A and Hillery M 1997 *Phys. Rev.* A **55** 4585
[24] Song K H and Guo G C 1999 *Acta Phys. Sin.* **48** 661
[25] Zheng S B 2000 *Aust. J. Phys.* **53** 799
[26] Davidovich L, Zagury N, Brune M, Raimond J M and Haroche S 1994 *Phys. Rev.* A **50** R895
[27] Cirac J I and Parkins A S 1994 *Phys. Rev.* A **50** R4441
[28] Zheng S B 2004 *Phys. Rev.* A **69** 064302
[29] Zheng S B 2005 *Chin. Phys.* **14** 1825
[30] Sleator T and Weinfurter H 1995 *Phys. Rev. Lett.* **74** 4087
[31] Zheng S B 2005 *Phys. Rev.* A **71** 062335
[32] Hagley E, Maýtre X, Nogues G, Wunderlich C, Brune M, Raimond J M and Haroche S 1997 *Phys. Rev. Lett.* **79** 1
[33] Osnaghi S, Bertet P, Auffeves A, Maioli P, Brune M, Raimond J M, and Haroche S 2001 *Phys. Rev. Lett.* **87** 037902
[34] Rauschenbeutel A, Nogues G, Osnaghi S, Bertet P, Brune M, Raimond J M and Haroche S 2000 *Science* **288** 2024
[35] Rauschenbeutel A, Nogues G, Osnaghi S, Bertet P, Brune M, Raimond J M and Haroche S 1999 *Phys. Rev. Lett.* **83** 5166
[36] Zou X B, Pahlke K and Mathis W 2003 *Phys. Rev.* A **67** 044301
[37] Zheng S B 2003 *Phys. Rev.* A **68** 035801
[38] Zheng S B 2006 *Chin. Phys. Lett.* **23** 610
[39] Gou S C 1989 *Phys. Rev.* A **40** 5116
[40] Wu Y 1996 *Phys. Rev.* A **54** 1586
[41] Wu Y and Yang X X 1997 *Phys. Rev.* A **56** 2443
[42] Biswas A and Agarwal G S 2004 *Phys. Rev.* A **70** 022323
[43] Yamaguchi F, Milman P, Brune M, Raimond J M and Haroche S 2002 *Phys. Rev.* A **66** 010302
[44] Guo G C and Zheng S B 1996 *Phys. Lett.* A **223** 332
[45] Gerry C C 1996 *Phys. Rev.* A **54** R2529