Vol 16 No 2, February 2007
1009-1963/2007/16(02)/0420-09

Chinese Physics

© 2007 Chin. Phys. Soc.
and IOP Publishing Ltd

Entropy squeezing of a moving atom and control of noise of the quantum mechanical channel via the two-photon process*

Zhou Bing-Ju(周并举)[a], Liu Xiao-Juan(刘小娟)[a)b)†],
Zhou Qing-Ping(周清平)[b], and Liu Ming-Wei(刘明伟)[a]

[a] *Department of Physics, Hunan University of Science and Technology, Xiangtan* 411201, *China*

[b] *Anhui Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Hefei* 230031, *China*

(Received 22 May 2006; revised manuscript received 1 September 2006)

Based on the quantum information theory, we have investigated the entropy squeezing of a moving two-level atom interacting with the coherent field via the quantum mechanical channel of the two-photon process. The results are compared with those of atomic squeezing based on the Heisenberg uncertainty relation. The influences of the atomic motion and field-mode structure parameter on the atomic entropy squeezing and on the control of noise of the quantum mechanical channel via the two-photon process are examined. Our results show that the squeezed period, duration of optimal entropy squeezing of a two-level atom and the noise of the quantum mechanical channel can be controlled by appropriately choosing the atomic motion and the field-mode structure parameter, respectively. The quantum mechanical channel of two-photon process is an ideal channel for quantum information (atomic quantum state) transmission. Quantum information entropy is a remarkably accurate measure of the atomic squeezing.

Keywords: entropy squeezing, atomic motion and field-mode structure, quantum mechanical channel, two-photon process

PACC: 4250

1. Introduction

The squeezing of a two-level atom has been an attractive subject of theoretical and experimental studies in the quantum optics field over the past few years, which is not only because it has extensive potential applications in high-resolution spectroscopy,[1] the high-precision atomic fountain clocks,[2] high-precision spin polarization measurements,[3] generation of quantum controlled few-photon states,[4] control of quantum noise, atomic spin polarization measurements,[5] and generation of squeezed light[6] that has been recently applied to quantum information theory, for example in quantum teleportation,[7,8] cryptography,[9,10] and dense coding,[11] but also because it is an important quantum system of double state that can serve as a qubit for quantum information and quantum computation.[12] It is important to note that all these studies of atomic squeezing are based on the Heisenberg uncertainty relation (HUR), which is regarded as a standard limitation on measurements of quantum fluctuations. The HUR is formulated based on the variance of the system observable. However, the HUR cannot give us sufficient information on atomic squeezing in such cases as shown in Ref.[13]

There is an elegant entropic way of reformulating the HUR. Several authors,[14,15] have studied quantum uncertainty by using quantum entropy theory, and obtained an entropic uncertainty relation (EUR) for position and momentum, which can be free from the triviality of the HUR. The authors of Ref.[13] have presented EUR for a two-level atom and determined the entropy squeezing for the two-level atom by using the quantum information theory. More recently, much work on atomic information entropy squeezing in one-photon and nonlinear multiquantum Jaynes–Cumming (JC) model (JCM) has been reported.[16−18]

On the other hand, the two-photon JC model (TPJCM) has aroused much interest in recent years because of two-photon microwave maser running successfully and its property of the periodicity of quan-

*Project supported by the National Natural Science Foundation of China (Grant No 10374025), the Natural Science Foundation of Hunan Province, China (Grant No 05JJ30004) and the Scientific Research Fund of Hunan Provincial Education Department, China (Grant No 03c543)

†Corresponding author. E-mail: lxjdx@163.net

tum mechanical channels.[19] Therefore it is an important tool for transmitting quantum information correctly in the quantum communicate field.[20−23] In Refs.[18,24,25], the authors have studied the quantum reduced entropy of the two-photon process. Fang *et al*[19] have researched quantum mechanical channel and quantum mutual entropy in the TPJCM.

Even so, all these theoretical results are obtained only on condition that the atomic motion is neglected and the field-mode structure is not taken into account. Recently, the atomic spatial motion has been considered in order to achieve cooled atoms and supercooled atoms with the development of laser cooling and trapping atom technology.[26] Schlicher, and Joshi and Lawande[27] have examined the influences of the atomic motion and the field-mode structure on the atomic dynamics. Joshi and Lawande and Bartzzis[28] have analysed influences of the atomic motion and the field-mode structure on the squeezing of the field. Fang, and Zhu and Zhang[29] have studied the effects of the atomic motion and the field-mode structure on field entropy and Schrödinger-cat states in JCM. But their results show that the quantum effect due to the atomic motion cannot be underrated in experiment.

The fidelity[30] is a measure of distance between quantum states via the quantum mechanical channel and an important concept in the quantum information and quantum communication theories. In Ref.[31], the authors have studied the influence of entanglement states of atomic dipole–dipole interaction on the fidelity of quantum states.

However, less attention has been paid to the study on the squeezing of a moving atom interacting with the coherent field via the quantum mechanical channel of the two-photon process, and on the relation between the squeezing and fidelity of this channel for atomic quantum states, by using the quantum information entropy theory, which are to be studied in this paper. The obtained results are compared with those of atomic squeezing based on the HUR; the influences of the initial state of the system, atomic motion and field-mode structure on the atomic entropy squeezing are also examined in the paper. The results show that the quantum information entropy is a remarkably accurate measure of the atomic squeezing, and that the quantum mechanical channel of the two-photon process is an ideal quantum channel. Therefore, the obtained results may be important for the experimental realization of the preparation of squeezing atom and the control of quantum noise in quantum computation and quantum information.

2. The model and the atomic entropy squeezing

2.1. The model

The present cavity quantum electrodynamics experiments[32] are provided with an atomic beam passing along the axis of the rectangular or cylindrical cavity so that one can investigate the interaction of an atom in motion with different field modes. The system considered here consists of a two-level atom in motion interacting with a quantized field via the two-quantum transition processes. The effective Hamiltonian in the rotating-wave approximation[27] is

$$\begin{aligned} H = \ & \omega_0 S_z + \omega a^+ a \\ & + g[f(z)]^2 (S_+ a^2 + a^{+2} S_-), (\hbar = 1), \end{aligned} \quad (1)$$

where a_j^+ (a_j) is the photon creation (annihilation) operator of the field mode with a frequency w; S_z and $S_\pm$ are the atomic inversion and transition operators, respectively, ω_0 is the atomic transition frequency, g is the atom–field coupling constant, $f(z)$ is the shape function of the cavity field mode. We restrict our studies to the atomic motion along the z-axis so that the z-dependence of the field-mode function should be taken into account. The atomic motion can be involved in a usual way[33]

$$f(z) \to f(vt), \quad (2)$$

where υ denotes the atomic motion velocity. For the convenience of discussion, a TEM_{mnp} mode is defined as

$$f(z) = \sin\left(\frac{p\pi vt}{L}\right), \quad (3)$$

where p represents the number of half-wavelengths of a field mode inside a cavity with a length L.

For simplicity, we consider the two-photon resonant case, that is $\omega_0 = 2\omega$, and assume that the atom at the time $t = 0$ enters the cavity and leaves the cavity after passing through p half-wavelengths of an electric field mode. In an interaction picture and two-dimensional atomic space, the time evolution of the Dyson operator can be written as

$$U_I(t) = \begin{bmatrix} \hat{u}_{11} & -\mathrm{i}\hat{u}_{12} \\ -\mathrm{i}\hat{u}_{21} & \hat{u}_{22} \end{bmatrix}, \quad (4)$$

where

$$\begin{aligned}
\hat{u}_{11} &= \cos(\hat{A}g\theta(t)),\\
\hat{u}_{21} &= a^{+2}\frac{\sin(\hat{A}g\theta(t))}{\hat{A}},\\
\hat{u}_{22} &= \cos(\hat{B}g\theta(t)),\\
\hat{u}_{12} &= a^{2}\frac{\sin(\hat{B}g\theta(t))}{\hat{B}},\\
\hat{A} &= \sqrt{a^2a^{+2}}, \quad \hat{B} = \sqrt{a^{+2}a^2},\\
\theta(t) &= \int_0^t [f(vt'\mathrm{d}t')] = \int_0^t \left[\sin\left(\frac{p\pi vt'}{L}\right)\right]^2 \mathrm{d}t'\\
&= \frac{1}{2}\left[t - \frac{L}{2p\pi vt}\sin(2pgt)\right]. \qquad (5)
\end{aligned}$$

We consider at the time $t = 0$ the atom to be in a coherent superposition state of the excited state $\mid e\rangle$ and the ground state $\mid g\rangle$, i.e.

$$\mid \psi_A(0)\rangle = \cos(\theta/2) \mid \mathrm{e}\rangle + \exp(\mathrm{i}\varphi)\sin(\theta/2) \mid \mathrm{g}\rangle. \quad (6)$$

And the field is in the coherent state

$$\begin{aligned}
\mid \alpha > &= \sum_{n=0}^{\infty} F_{n,} \mid n >,\\
F_n &= \exp(-\bar{n})\frac{\alpha^n}{\sqrt{n!}} \qquad (7)
\end{aligned}$$

where $0 \ll \theta \ll \pi$ denotes the atomic distribution, $0 \ll \varphi \ll 2\pi$ is the phase of the atom dipole, $\alpha = \sqrt{\bar{n}}\exp(\mathrm{i}\beta)$, and $\bar{n}$ and β represent the initial average photon number and the phase of the coherent field, respectively.

The initial state of the system can be written as

$$\rho(0) = \rho_A(0) \otimes \rho_F(0), \qquad (8)$$

Let ξ_t^* denote the rise operator of the atomic time evolution, Π_t^* represents the quantum mechanical channel of the two-photon process, that is[33]

$$\xi_t^* : \quad A \to A \otimes F, \qquad (9)$$

$$\Pi_t^* : \quad A \to A. \qquad (10)$$

After the moving atom has interacted with the coherent field via the quantum mechanical channel of the two-photon process, the final state of the system can be given by[33]

$$\xi_t^*\rho_A = U_I(t)\rho_A \otimes \rho_F U_I^+(t). \qquad (11)$$

From the rise operator and quantum mechanical channel, the atomic final state can be rewritten as[32,33]

$$\Pi_t^*\rho_A = \mathrm{Tr}_F(U_I(t)\rho_A \otimes \rho_F U_I^+(t)) = \begin{bmatrix} \rho_{22} & \rho_{21} \\ \rho_{12} & \rho_{11} \end{bmatrix}, \qquad (12)$$

where

$$\begin{aligned}
\rho_{22} = &\sum_{n=0}^{\infty} |F_n|^2\{\cos(\theta/2)^2\cos(\sqrt{(n+1)(n+2)}g\theta(t))^2\\
&+\bar{n}^2/(n+1)(n+2)\sin(\theta/2)^2\sin(\sqrt{(n+1)(n+2)}g\theta(t))^2\\
&+\frac{1}{2}\frac{\bar{n}}{\sqrt{(n+1)(n+2)}}\sin(\theta)\sin(\varphi-2\beta)\sin(2\sqrt{(n+1)(n+2)}g\theta(t))\}, \qquad (13)
\end{aligned}$$

$$\begin{aligned}
\rho_{21} = \rho_{12}^* = &\frac{1}{2}\sin(\theta)\sum_{n=0}^{\infty}|F_n|^2\{\exp[\mathrm{i}(\varphi+4\beta)]\cos(\sqrt{(n+1)(n+2)}g\theta(t))\cos(\sqrt{(n)(n-1)}g\theta(t))\\
&+\exp(-\mathrm{i}\varphi)\sin(\sqrt{(n+1)(n+2)})g\theta(t))\sin(\sqrt{(n)(n-1)}g\theta(t))\}\\
&+\mathrm{i}\exp(\mathrm{i}2\beta)\{F_nF_{n-2}^*\cos(\theta/2)^2\cos(\sqrt{(n+1)(n+2)}g\theta(t))\sin(\sqrt{(n-1)(n)}g\theta(t))\\
&+F_{n+2}F_n^*\sin(\theta/2)^2\sin(\sqrt{(n+1)(n+2)}g\theta(t))\cos(\sqrt{(n+1)(n+2)}g\theta(t))\}, \qquad (14)
\end{aligned}$$

$$\rho_{11} = 1 - \rho_{22}. \qquad (15)$$

The quantum mutual entropy[19] shows the measure that how much information of atomic initial state ρ_A can be transmitted to final state $\Pi_t^*\rho_A$, being defined as follows:

$$S(\rho_A : \Pi_t^* \rho_A) = S(\sigma_E, \sigma_0) = \sum_{k=1}^{2} \lambda_k S(\Pi_t^* \sigma_E, \Pi_t^* \rho_A)$$
$$= \sum_{k=1}^{2} \lambda_k \mathrm{Tr} \Pi_t^* E_k (\ln \Pi_t^* E_k - \Pi_t^* \rho_A), \quad (16)$$

that is

$$S(\rho_A : \Pi_t^* \rho_A) = \sum_{k=1}^{2} \lambda_k (\nu_+^k (\ln \nu_+^k - \ln \mu_+) + \nu_-^k (\ln \nu_-^k - \ln \mu_-)), \quad (17)$$

where $\mu_\pm$, λ_k $\nu_+^k (k = 1, 2)$ are the eigenvalues of $\Pi_t^* \rho_A$, $\rho_A(0)$ and $\Pi_t^* E_k$ respectively. $E_1 = |0><0|; E_2 = |1><1|$ is the one-dimensional projection of the ρ_A.

The fidelity of the quantum mechanical channel for atomic quantum states can be defined as[31]

$$F(\rho_A(0), \Pi_t^* \rho_A(t))$$
$$= [\mathrm{Tr}(\sqrt{\rho_A(0)} \Pi_t^* \rho_A(t) \sqrt{\rho_A(0)})^{1/2}]^2, \quad (18)$$

where $\rho_A(0)$ is atomic initial state, $\Pi_t^* \rho_A(t)$ is atomic final state. $F(\rho_A(0), \Pi_t^* \rho_A(t))$ reflects the deviation of atomic initial state states from atomic final state.

If $F(\rho_A(0), \Pi_t^* \rho_A(t)) = 0$, showing that the information (atomic states) lacks the fidelity completely, the quantum mechanical channel becomes invalid. If $F(\rho_A(0), \Pi_t^* \rho_A(t)) = 1$, showing that the information transmission process is ideal, that is, atomic initial state and final state are identical, the quantum mechanical channel is good. Generally, $0 < F(\rho_A(0), \Pi_t^* \rho_A(t)) < 1$.

2.2. The atomic entropy squeezing

Here we determine the entropy squeezing of the two-level atom by using the quantum information entropy theory.[13] The information entropy of the Pauli operators S_α $(\alpha = x, y, z)$ for a two-level atom system is

$$H(S_\alpha) = -\sum_{i=1}^{2} P_i(S_\alpha) \ln P_i(S_\alpha), \alpha = x, y, z, \quad (19)$$

where $P_i(S_\alpha) = \langle \psi_{\alpha i} | \rho | \psi_{\alpha i} \rangle$ $(i = 1, 2)$, which are the probability distributions for two possible measurements of an operator S_α. $H(S_x), H(S_y), H(S_z)$ satisfy

$$H(S_x) + H(S_y) \geq 2 \ln 2 - H(S_z), \quad (20)$$

which may also be rewritten as

$$\delta H(S_x) \delta H(S_y) \geq \frac{4}{\delta H(S_z)} \quad , \quad (21)$$

where $\delta H(S_\alpha) = \exp[H(S_\alpha)]$.The squeezing of the atom is determined by using the EUR (21) named entropy squeezing, which has received little attention in the available publications. The fluctuation in the component $S(\alpha)$ $(\alpha \equiv x$ or $y)$ of the atomic dipole is said to be squeezed in entropy if the information entropy $H(S_\alpha)$ satisfies the following condition:

$$E(S_\alpha) = \delta H(S_\alpha) - \frac{2}{\sqrt{|\delta H(S_z)|}} < 0,$$
$$(\alpha \equiv x \ \ or \ \ y). \quad (22)$$

We recall the HUR

$$\Delta S(x) \Delta S(y) \geq |\langle S_Z \rangle|/2. \quad (23)$$

Based on the HUR the fluctuation in the component S_α of the atomic dipole is said to be squeezed if $S(\alpha)$ satisfies the condition

$$V(S_\alpha) = \Delta S(\alpha) - \sqrt{|\langle S_Z \rangle|/2} < 0,$$
$$(\alpha \equiv x \ \ \mathrm{or} \ \ y), \quad (24)$$

where $\Delta S_\alpha = [\langle S_\alpha^2 \rangle - \langle S_\alpha \rangle^2]^{1/2}$. Next, we calculate the atomic entropy squeezing in the considered system. By using the reduced atomic density operator $\rho_A(t)$, we obtain the information entropies of the atomic operators S_x, S_y, S_z as follows:

$$H(S_x) = -\left[\frac{1}{2} + \mathrm{Re}(\rho_{21}(t))\right] \ln\left[\frac{1}{2} + \mathrm{Re}(\rho_{21}(t))\right] - \left[\frac{1}{2} - \mathrm{Re}(\rho_{21}(t))\right] \ln\left[\frac{1}{2} - \mathrm{Re}(\rho_{21}(t))\right], \tag{25a}$$

$$H(S_y) = -\left[\frac{1}{2} + \mathrm{Im}(\rho_{21}(t))\right] \ln\left[\frac{1}{2} + \mathrm{Im}(\rho_{21}(t))\right] - \left[\frac{1}{2} - \mathrm{Im}(\rho_{21}(t))\right] \ln\left[\frac{1}{2} - \mathrm{Im}(\rho_{21}(t))\right], \tag{25b}$$

$$H(S_z) = -\rho_{22}(t) \ln \rho_{22}(t) - \rho_{11} \ln \rho_{11}(t), \tag{25c}$$

where, $\rho_{ij}(t)$ is given by the expressions (13)–(15).

Obviously, the formula (22) directly relates the quantum information with quantum fluctuations, and contains all order statistical moments; while the condition (24) only contains the second-order statistical moments. Meaningful information can be retrieved from the formula (22) in the case of $< Sz >= 0$ since the right-hand side of expression (15) is always nonzero whereas the inequality (23) is trivially satisfied.

Next, examine some numerical results about the atomic information entropy squeezing.

3. Numerical results and discussion

On the basis of the analytical solution presented in the previous section, we numerically examine the properties of the atomic information entropy squeezing. For comparison we also consider the atomic variance squeezing in the different initial states of the system.

3.1. Comparing atomic entropy squeezing with variance squeezing

In Figs.1(a)–(d) we show the time evolution of the entropy squeezing $E(S_x)$, $E(S_y)$ as well as the variance squeezing $V(S_x)$ and $V(S_y)$, while the atomic inversion is shown in Figs.2(a) and (b) for the atom initially in coherent superposition state with $\theta = \pi/2$, $\varphi = 0$ and the field in the coherent state with the average photon number $\bar{n} = 36$ and the phase $\beta = 0$ of the coherent fields, field-mode structure parameter $p = 1$.

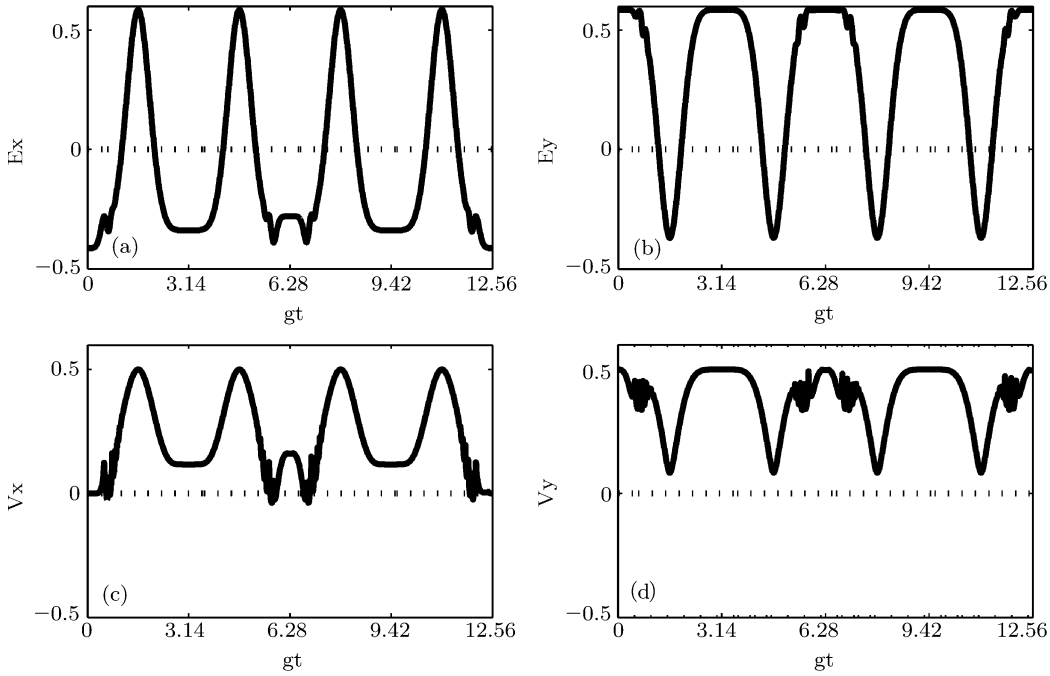


Fig.1. The time evolution of the squeezing factors. The atom is initially in a coherent superposition state with $\theta = \pi/2$, and $\varphi = 0$, and the field is in a coherent state with an initial average photon number $\bar{n} = 36$, phase $\beta = 0$ and field-mode structure parameter $p = 1$. Information entropy squeezing factors: (a) $E(S_x)$ and (b) $E(S_y)$. Variance squeezing factors: (c) $V(S_x)$ and (d) $V(S_y)$.

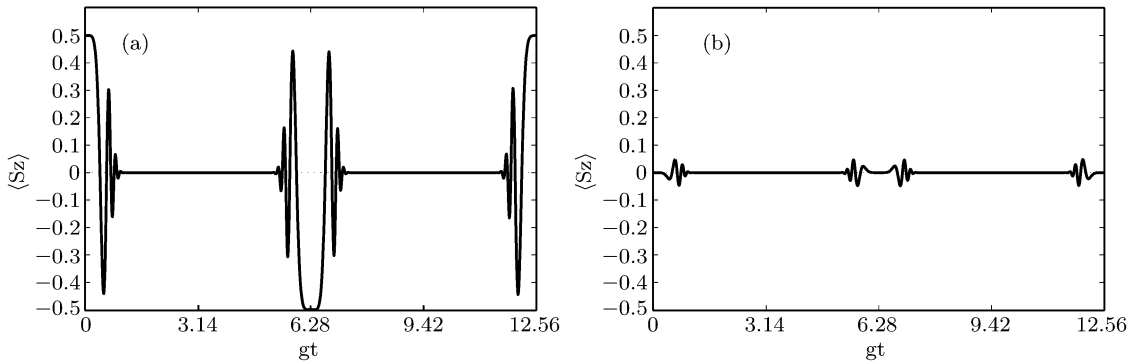


Fig.2. The time evolution of the atomic inversion. The field is initially in a coherent state with an average photon number $\bar{n} = 36$ and phase $\beta = 0$, and field-mode structure parameter $p = 1$. (a) is for the atom being initially in the excited state with $\theta = 0$; (b) for the atom initially in the coherent superposition state with $\theta = \pi/2$.

It is interesting to find that there is an alternative information entropy squeezing in the atomic polarization components S_x and S_y as illustrated in Fig.1(a) and Fig.1(b),whereas neither $V(S_x)$ nor $V(S_y)$ displays variance squeezing, (cf. Fig.1(c) and Fig.1(d)). This can be quantitatively interpreted by using the result in Ref.[25]. At the time $t = t_{\rm R}/4 = (2k+1)\pi/2g$ $(k = 0, 1, \ldots)$, i.e. a quarter of the revival time of the inversion $< S_z >$, the atomic reduced entropy tends to 0, and the atom and the coherent fields are disentangled; therefore the atom has achieved its pure state[25]

$$| \psi_A(t_{\rm R}/4)\rangle \simeq (|+\rangle + {\rm i}|-\rangle)/\sqrt{2}, \tag{26}$$

which is just eigenstate of the atomic Pauli operator S_y. From the expression(13), we obtain information entropies of atomic Pauli operators S_x, S_y, and S_z at this time and they are, respectively, $H(S_y) = 0.0$, $H(S_x) = H(S_z) = \ln 2$. Correspondingly, $\delta H(S_y) = 1$, and $\delta H(S_x) = \delta H(S_z) = 2$. By using these results the entropy squeezing factors are calculated to be $E(S_y) \simeq -0.414 < 0$ and $E(S_x) = 0.586 > 0$. This shows that the operator S_y exhibits optimal entropy squeezing at $t = t_{\rm R}/4$. This analytical result is in accord with the numerical calculation as shown in Fig.3(b). The interpretation of Fig.3(a) is similar to that of Fig.3(b). The difference between the results of variance squeezing and those of information entropy squeezing can be interpreted as follows. Due to the influence of the initial atomic coherence, the amplitude of the atomic inversion $\langle S_z \rangle$ evolves to very small values, close to zero as shown in Fig.2(b). Therefore the definition of the variance based on the HUR is almost no longer valid, which means that the information entropy is a measure sensitive to the atomic squeezing.

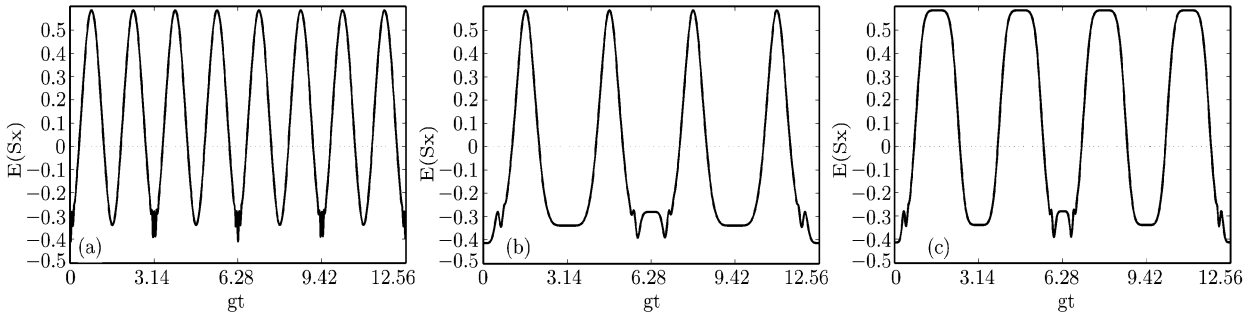


Fig.3. The time evolutions of the information entropy squeezing factors $E(S_x)$. The atom is initially in the coherent superposition with $\theta = \pi/2$ and $\varphi = 0$, the field is in the coherent state with an initial average photon number $\bar{n} = 36$ and phase $\beta = \pi/4$. (a) is for the atomic motion being neglected; (b) for the atom in motion at a velocity $v = gL/\pi$ and $p = 1$; (c) for the atom in motion at a velocity $v = gL/\pi$ and $p = 2$.

3.2. Influences of atomic motion and field-mode structure parameter on entropy squeezing

Furthermore, we discuss the influences of atomic motion and field-mode structure parameter on the entropy squeezing. The atom is initially in the coherent superposition with $\theta = \pi/2$, and $\varphi = 0$, the field in the coherent state with an initial average photon number $\bar{n} = 36$ and phase $\beta = \pi/4$. Figure 3(a) presents the time evolution of $E(S_x)$ when the atomic motion is not taken into account, this corresponds to the result obtained in Ref.[34]. It can be seen that if the atomic motion is neglected, the $E(S_x)$ evolves with $\pi/2$. Figures 3(b) and (c) illustrate the time evolution of $E(S_x)$ when we take atom speed as $v = gL/\pi$, but field-mode structure parameter $p = 1$ and $p = 2$ respectively, and S_x exhibits optimal entropy squeezing at time $t_o = m\pi$ ($m = 1, 2, 3, \ldots$) nearly, where t_o denotes the optimal entropy squeezing time.

From these figures we can conclude as follows: (1) Atomic motion leads the evolution period T of the entropy squeezing to be the double of the evolution period in the case of atomic motion being neglected.[34] that is $T = \pi$ (see Figs.3(a)–(b)). (2) The field-mode structure parameter p leads the optimum squeezing time t_o to extend (Fig.3(b)). (3) The increase of the field-mode structure parameter p causes the range of optimal entropy squeezing time to decrease (Fig.3(c)).

Physically, all these features may be attributed to the change in the atom–field interaction time due to atomic motion. Actually, the change in the atom–field interaction time is due to the difference in the entropy squeezing factor $E(S_x)$ between the case in which the atomic motion is neglected and the case in which the atomic motion is taken into account resulting from the time factor. The time factor is the scaled time gt when the atomic motion is neglected, i.e. $g\theta(t)$, which is $g\theta(t)=\dfrac{1}{2g}\left[gt-\dfrac{1}{2p}\sin(2pgt)\right]$ when the atomic motion is taken into account. This is made up of two parts: one is linear part $1/2gt$, and the other is nonlinear part $\dfrac{1}{4p}\sin(2pgt)$; its value is very small($\leq 1/4p$), so $g\theta(t)\approx 1/2gt$, this just leads the period of squeezing factors to a twofold increase, but the nonlinear part determines the range of t_o. It is shown that the range of t_o can be controlled by choosing the parameters of atomic motion and field-mode structure.

3.3. Influences of atomic motion and field-mode structure on control of noise of the quantum mechanical channel

Finally, we examine the relation between atomic entropy squeezing and control of noise of the quantum mechanical channel, in the same case as the above one.

For the convenience of comparison, using the results in our paper,[35] Figs.4(a) and (b) illustrate the time evolutions of quantum mutual entropy $S(\rho_A : \Pi_t^*\rho_A)$ that expresses the ability to transmit the information of atomic initial state ρ_A to final state $\Pi_t^*\rho_A$, when we take atom speed as $v = gL/\pi$, and field-mode structure parameters $p = 1$ and $p = 2$ respectively.

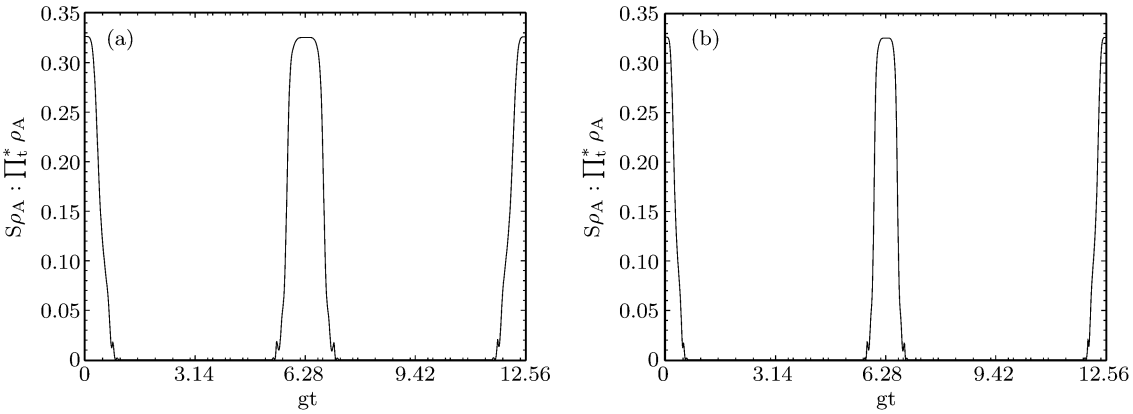


Fig.4. The time evolutions of the quantum mutual entropy. The atom is initially in the coherent superposition with $\theta = \pi/2$ and $\varphi = 0$; the field is in the coherent state with an initial average photon number $\bar{n} = 36$ and phase $\beta = \pi/4$. (a) the atom being in motion at a velocity $v = gL/\pi$ and $p = 1$; (b) the atom in motion at velocity $v = gL/\pi$ and $p = 2$.

Obviously, when the revival time of the atomic inversion $t = t_{\rm R} = n\pi, n = 2, 4, \ldots$, quantum mutual entropy $S(\rho_A : \Pi_t^* \rho_A)$ takes its maximal value. The quantum mechanical channel of the two-photon process is open nearly at $t_{\rm R}$,[35] that is, $t_{\rm R}$ reflects the open time of the quantum mechanical channel and both the atom and the field (environment) being disentangled.[35]

An important phenomenon has been found that $t_{\rm R}$ is contained just in atomic optimal entropy squeezing time $t_{\rm o} = m\pi (m = 1, 2, 3, \ldots)$, when $m = 2, 4, 8, \ldots$, $t_{\rm o} = t_{\rm R}$ (see Figs.3(b) and (c) and Fig.4).

This point can be quantitatively interpreted by using the expressions (15) and (18) after a straightforward calculation. Therefore we can obtain the fidelity of the quantum mechanical channel for atomic quantum states when $\theta = \pi/2$, $\varphi = 0$ and $\beta = \pi/4$ at $t = t_{\rm R}$ as follows:

$$\begin{aligned} & F(\rho_A(0), \Pi_t^* \rho_A(t_R)) \\ = \; & [{\rm Tr}(\sqrt{\rho_A(0)} \Pi_t^* \rho_A(t_{\rm R}) \sqrt{\rho_A(0)})^{1/2}]^2 = 1, \quad (27) \end{aligned}$$

which means that all the information of atomic initial state can be transmitted to its final state via this channel correctly,[35] in the above case. From an information entropy squeezing point of view, this comes from the fact that $t_{\rm o}$ represents just the time when the quantum fluctuation (or quantum noise) of the atomic states due to coupling the field (environment) is minimum via the quantum mechanical channel of two-photon process, which means that the atom and the field (environment) are disentangled in an information transmission process, too. Therefore $t_{\rm R}$ is contained in $t_{\rm o}$ certainly, this is just the physical interpretation that the quantum mechanical channel of two-photon process is an ideal quantum channel because $F(\rho_A(0), \Pi_t^* \rho_A(t)) = 1$ from quantum information point of view.

Once time $t_{\rm o}$ is controlled, we can control the noise of quantum mechanical channel of two-photon process in time $t_{\rm o}$ by choosing the parameters of atomic motion and field-mode structure. These results are important for controlling the quantum noise in quantum computation and quantum information.

4. Conclusions

In conclusion, based on quantum information theory, we have investigated the entropy squeezing of a moving two-level atom interacting with the coherent field and the control of noise of the quantum mechanical channel via a two-photon process. We can conclude, first, that quantum information entropy is a remarkably accurate measure of atomic squeezing and it is free from the triviality of the variance squeezing originating from the HUR. In particular, when an atom is in the eigenstate of the operator S_x or S_y, the entropy squeezing is quite a good measure. Second, the period T of the entropy squeezing is determined by choosing the parameters of atomic motion. Third, the duration of the optimal information entropy squeezing can be determine by choosing the field-mode structure parameter p. Finally, open time $t_{\rm R}$ of the quantum mechanical channel is just included in optimal entropy squeezing time $t_{\rm o}$ of atom. The noise in a quantum mechanical channel also can be controlled by choosing the field-mode structure parameter p. The quantum mechanical channel of two-photon process is an ideal channel of quantum information transmission, for its fidelity for atomic quantum states is equal to 1. Our results are important for the experimental realization of the atomic squeezing and the control of quantum noise in quantum computation and quantum information.

References

[1] Kitagawa M and Ueda M 1993 *Phys. Rev.* A **47** 5138

[2] Wineland D J, Bollinger J J and Itano W M 1994 *Phys. Rev.* A **50** 67
Sørensen A and mølmer K 1999 *e-print quantph* 199003044

[3] Sørensen J L, Hald J and Polzik E S 1998 *Phys. Rev. Lett.* **80** 3487

[4] Saito H and Ueda M 1997 *Phys. Rev. Lett.* **79** 3869
Saito H and Ueda M 1999 *Phys. Rev. Lett.* **59** 3059

[5] Wu Y 2000 *Phys. Rev.* A **61** 033803
Kuzmich A, Mølmer K and Polaik E S 1997 *Phys. Rev. Lett.* **79** 4782

[6] Kuang L M and Chen X 1994 *Phys. Rev.* A **50** 4228
Walls D F and Zoller P 1981 *Phys. Rev. Lett.* **47** 709
Zhu K C 1996 *J. Mod. Opt.* **43** 323
Kuang L M, Wang F B and Zhou Y G 1994 *J. Mod. Opt.* **47** 1307

[7] Wu Y and Yang X X 1997 *Phys. Rev. Lett.* **78** 3086
Zhou L and Kuang L M 2002 *Phys. Lett.* A **302** 273
Braunstein S L and Kimble H J 1998 *Phys. Rev. Lett.* **80** 869

Milburn G J and Braunstein S L 1999 *Phys. Rev.* A **60** 937
[8] Zheng S B 2005 *Chin. Phys.* **14** 1825
Furusawa A, Sorensen J, Braunstein S L, Fuchs C A, Kimble H J and Polzik E S 1998 *Science* **282** 706
[9] Ralph T C 2000 *Phys. Rev.* A **61** 010303R
[10] Hillery M 2000 *Phys. Rev.* A **61** 022309
[11] Ban M 2000 *J. Opt.* B: *Quantum Semiclass. Opt.* **1** L9
Ban M 2000 *J. Opt.* B: *Quantum Semiclass. Opt.* **2** 786
[12] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392
Zheng S B 2004 *Chin. Phys.* **13** 1707
[13] Fang M F, Zhou P and Swain S 2000 *J. opt.* **47**1043
[14] Hirschman I I 1957 *Am. J Math.* **79** 152
[15] Bialynicki B I and Mycielski J 1975 *Commun. Math. Phys.* **44** 129
Beckner W 1975 *Ann. Math.* **102** 159
Deutsch D 1983 *Phys. Rev. Lett.* **50** 631
[16] Fang M F, Zhou P and Swain S 2000 *Chin. Phys. Lett.* **17** 885
[17] Mahmoud Abdel-Aty, Aballa M Sebawe and Obada A S F 2002 *J. Opt.* B **4** 13
[18] Li C X and Fang M F 2003 *Chin. Phys.* **12** 294
[19] Fang M F and Liu X 2000 *Acta Phys. Sin.* **49** 0405 (in chinese)
Sun W H, Du S D and Chen X S 2006 *Chin. Phys.* **15** 375
[20] Bemett C H and Divincenzo D P 2000 *Nature* **404** 247
Zeng K and Fang M F 2005 *Chin. Phys.* **14** 2009
Ekert A K 1992 *Phys. Rev. Lett.* **67** 661
Deutsch D and Jozsa R 1992 *Proc. R. Soc.* London A **493** 553
Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wootters W 1993 *Phys. Rev. lett.* **70** 1895
[21] Cavers C M and Schumaker B L 1985 *Phys. Rev.* A **31** 3068
[22] Shen Y R 1984 *The principles of Nonlinear Optics* (New York: Wiley) Chapter 10
[23] YANG X X and Wu Y 1999 *Commun. Theor. Phys.* **32** 189
[24] Wang C Z and Fang M F 2002 *Chin. Phys.* **51** 1989
Tohya H 2001 *Phys. Rev.* A **63** 022305
[25] Liu X J and Fang M F 2003 *Chin. Phys.* **12** 0971
[26] Wu Y and Yang X 1997 *Phys. Rev.* A **56** 2443
[27] Schlicher R R 1989 *Opt. Commun.* **70** 97
Joshi A and Lawande S V 1990 *Phys. Rev.* A **42** 1752
[28] Joshi A and Lawande S V 1992 *Int. J. Mod. Phys.* B **6** 3539
Bartzzis V 1992 *Physica* A **180** 428
[29] Fang M F 1998 *Physica* A **259** 193
Zhu A D and Zhang S 2004 *Chin. Phys.* **18** 1276
[30] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Chapter 11) (Cambridge: Cambridge University Press)
[31] Liu T K, Wang J S, Liu X J and Zhan M S 2000 *Acta Phys. Sin.* **49** 0708 (in Chinese)
Liu T K, Wang J S, Liu X J and Zhan M S 1999 *Acta Phys. Sin.* **48** 2051 (in Chinese)
[32] Wu Y 1996 *Phys. Rev.* A **54** 1586
Meschede D, Walther H and Muller G 1985 *Phys. Rev.* A **54** 551
Rempe G, Walther H and Klein N 1987 *Phys. Rev. Lett.* **58** 353
[33] Sargent III M, Scully M O and Lamb W E Jr 1974 *Laser Physics* (Reading, MA: Addison-Wesley)
[34] Liu X J, Zhou B J, Fang M F and Zhou Q P2006 *Acta Phys. Sin.* **55** 0704 (in Chinese)
[35] Liu X J, Fang M F and Zhou Q P 2005 *Acta Phys. Sin.* **54** 0703 (in Chinese)