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Coherence vortices of partially coherent beams in the far field*

Liu Pu-Sheng(刘普生)† and Lü Bai-da(吕百达)

Institute of Laser Physics and Chemistry, Sichuan University, Chengdu 610064, *China*

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Based on the propagation law of cross-spectral density function, studied in this paper are the coherence vortices of partially coherent, quasi-monochromatic singular beams with Gaussian envelope and Schell-model correlator in the far field, where our main attention is paid to the evolution of far-field coherence vortices into intensity vortices of fully coherent beams. The results show that, although there are usually no zeros of intensity in partially coherent beams with Gaussian envelope and Schell-model correlator, zeros of spectral degree of coherence exist. The coherence vortices of spectral degree of coherence depend on the relative coherence length, mode index and positions of pairs of points. If a point and mode index are kept fixed, the position of coherence vortices changes with the increase of the relative coherence length. For the low coherent case there is a circular phase dislocation. In the coherent limit coherence vortices become intensity vortices of fully coherent Laguerre–Gaussian beams.

Keywords: singular optics, coherence vortex in the far field, partially coherent beam, Schell-model correlator, Gaussian envelope

PACC: 4225, 4225k

1. Introduction

There has been much interest in singular optics, which has primarily been studied with fully coherent, quasi-monochromatic wave fields,[1−3] and it has recently been extended, on the one hand, to fully coherent, polychromatic wave fields,[4,5] and on the other hand, to the partially coherent, quasi-monochromatic wave fields.[6−9] The nature of phase singularities in partially coherent beams has been shown to be of coherence vortices of spectral degree of coherence. In Refs.[7,8] the coherence vortices were studied at the source plane for partially coherent beams consisting of a low-order Laguerre–Gaussian beams with Gaussian probability density and incoherent superposition of Hermite–Gaussian modes. The purpose of the present paper is to demonstrate the existence of coherence vortices of partially coherent beams in the far field, and to study the evolution of coherence vortices into intensity vortices of fully coherent beams. In Section 2 presented is the theoretical formulation, where the partially coherent singular beam with Gaussian envelope and Schell-model correlator is chosen as a typical example of partially coherent singular beams, and analysed are the far-field coherence vortices of such beam. Numerical calculation results and discussion are given in Section 3. Finally, Section 4 gives a brief summary of the main results obtained in this paper.

2. Coherence vortex of a type of partially coherent beams

Consider a fully coherent, quasi-monochromatic beam whose field at the source plane $z = 0$ has a Gaussian envelope with topological charge m[9,10]

$$E(\boldsymbol{\rho}_0, \varphi, 0) = E_0 \left(\frac{\rho_0}{w_0}\right)^m \exp\left(-\frac{\rho_0^2}{w_0^2}\right) \exp[-\mathrm{i}m\varphi_0] \exp(\mathrm{i}\beta), \tag{1}$$

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†Corresponding author. E-mail: p.s.liu@163.com

where E_0 is a constant, w_0 denotes the waist width, β is an arbitrary phase and ρ_0, φ_0 are radial and azimuthal coordinates in the cylindrical coordinate system. Equation (1) is essentially a circularly symmetric Laguerre–Gaussian mode with index m. If $\rho_0 = 0$, the amplitude vanishes, thus the phase becomes singular.

A partially coherent, quasi-monochromatic singular beam can be obtained from Eq.(1), and the cross-spectral density function of such beam is expressed as

$$W_0(\boldsymbol{\rho}_{10}, \boldsymbol{\rho}_{20}, 0) = < E^*(\boldsymbol{\rho}_{10}, \varphi_{10}, 0) E(\boldsymbol{\rho}_{20}, \varphi_{20}, 0) > \quad (2)$$

where $<>$ represents an ensemble average, $*$ denotes the complex conjugate. Assume that the statistical distribution of β corresponds to a Schell-model correlator[9,11]

$$C(\boldsymbol{\rho}_{10}, \boldsymbol{\rho}_{20}) = C(|\boldsymbol{\rho}_{10} - \boldsymbol{\rho}_{20}|) = \exp\left[-\frac{(\boldsymbol{\rho}_{10} - \boldsymbol{\rho}_{20})^2}{\sigma_0^2}\right], \quad (3)$$

where σ_0 is the coherence length. It follows from Eqs.(1)–(3) that the cross-spectral density of the partially coherent beam at the plane $z = 0$ is

$$W_0(\boldsymbol{\rho}_{10}, \boldsymbol{\rho}_{20}, 0) = E_0^2 \exp\left[-\frac{(\boldsymbol{\rho}_{10} - \boldsymbol{\rho}_{20})^2}{\sigma_0^2} - \frac{\rho_{10}^2 + \rho_{20}^2}{w_0^2}\right] \left(\frac{\rho_{10}\rho_{20}}{w_0^2}\right)^m \exp[\mathrm{i}m(\varphi_{01} - \varphi_{02})]. \quad (4)$$

Equation (4) indicates that the cross-spectral density for such a type of partially coherent singular beams contains two vortices having topological charge $+m$ and $-m$.

In the framework of the paraxial approximation the propagation of the cross-spectral density of partially coherent beams obeys the following propagation law[12]

$$W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = \left(\frac{k}{2\pi z}\right)^2 \iint\limits_{\infty} W_0(\boldsymbol{\rho}_{10}, \boldsymbol{\rho}_{20}, 0) \exp\left\{-\frac{\mathrm{i}k}{2z}\left[(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_{10})^2 - (\boldsymbol{\rho}_2 - \boldsymbol{\rho}_{20})^2\right]\right\} d^2\rho_{10} d^2\rho_{20}, \quad (5)$$

where k is the wave number related to the wavelength λ by $k = 2\pi/\lambda$.

Substituting Eq.(4) into Eq.(5), converting the cylindrical coordinates into the Cartesian coordinates and making the far-field approximation, we obtain

$$W(x_1, y_1, x_2, y_2, z) = \left(\frac{E_0}{z w_0^m \lambda}\right)^2 \iint\limits_{\infty} [(x_{10} + \mathrm{i}y_{10})(x_{20} - \mathrm{i}y_{20})]^m F_x F_y \mathrm{d}x_{10}\mathrm{d}x_{20}\mathrm{d}y_{10}\mathrm{d}y_{20}, \quad (6)$$

where

$$F_x = \exp\left[g\left(x_{10}^2 + x_{20}^2\right) + \frac{2x_{10}x_{20}}{\sigma_0^2} + \frac{\mathrm{i}k(x_1 x_{10} - x_2 x_{20})}{z}\right], \quad (7)$$

$$F_y = \exp\left[g\left(y_{10}^2 + y_{20}^2\right) + \frac{2y_{10}y_{20}}{\sigma_0^2} + \frac{\mathrm{i}k(y_1 y_{10} - y_2 y_{20})}{z}\right], \quad (8)$$

and

$$g = -\frac{1}{\sigma_0^2} - \frac{1}{w_0^2}. \quad (9)$$

From Eq.(6) the spectral degree of coherence turns out to be

$$\mu(x_1, y_1, x_2, y_2, z) = \frac{W(x_1, y_1, x_2, y_2, z)}{[S(x_1, y_1, z) S(x_2, y_2, z)]^{1/2}}, \quad (10)$$

where $S(x_i, y_i, z) = W(x_i, y_i, x_i, y_i, z)$ denotes the spectral density which is often referred to as the intensity ($i = 1$, 2, unless otherwise stated).

The coherence vortex[7,8] means that at pairs of points (x_1, y_1) and (x_2, y_2) in the z plane,

$$\mu(x_1, y_1, x_2, y_2, z) = 0, \tag{11}$$

but the intensity $S(x, y, z)$ is nonzero. Equation (10) indicates that the coherence vortex of the partially coherent beams described by Eq.(4) depends on the coherence length σ_0, waist width w_0, or equally, relative coherence length $\alpha = \sigma_0/w_0$, mode index m and position coordinates (x_i, y_i) of pairs of points. Equation (11) can be written as

$$\begin{aligned} &\mathrm{Re}\{\mu(x_1, y_1, x_2, y_2, z)\} = 0, \\ &\mathrm{Im}\{\mu(x_1, y_1, x_2, y_2, z)\} = 0, \end{aligned} \tag{12}$$

where Re, Im denote the real and imaginary parts respectively. For a fixed point, e.g. (x_1, y_1), Eq.(12) has simultaneous solutions,[8] which determine coherence vortices of partially coherent beams.

3. Numerical calculation results and discussion

As a typical example, we consider partially coherent beams with $m = 1$. Numerical calculations were performed by using Eqs.(6)–(12). Figure 1 gives the intensity distributions of partially coherent beams at the plane $z = 20z_0$ ($z_0 = kw_0^2/2$—Rayleigh length, $w_0 = 100\lambda$) for selected values of (a) $\alpha = 0.5$, (b) $\alpha = 5$, (c) $\alpha = 15$, and (d) $\alpha = 50$. As can be seen, the far-field intensity distributions depend on the relative coherent length α. For the low coherent case of $\alpha = 0.5$ in Fig.1(a), the far-field intensity distribution shows a Gaussian-like shape. With the increase of α from 5 in Fig.1(b) to 15 in Fig.1(c) there appears a central dark spot at the $x = y = 0$, which, however, has nonzero value in intensity. It means that zeros of intensity do not exist usually for partially coherent beams described by Eq.(4). Only in the coherent limit of $\alpha = 50 \gg 1$ in Fig.1(d) will the intensity at the point $x = y = 0$ reach zero value.

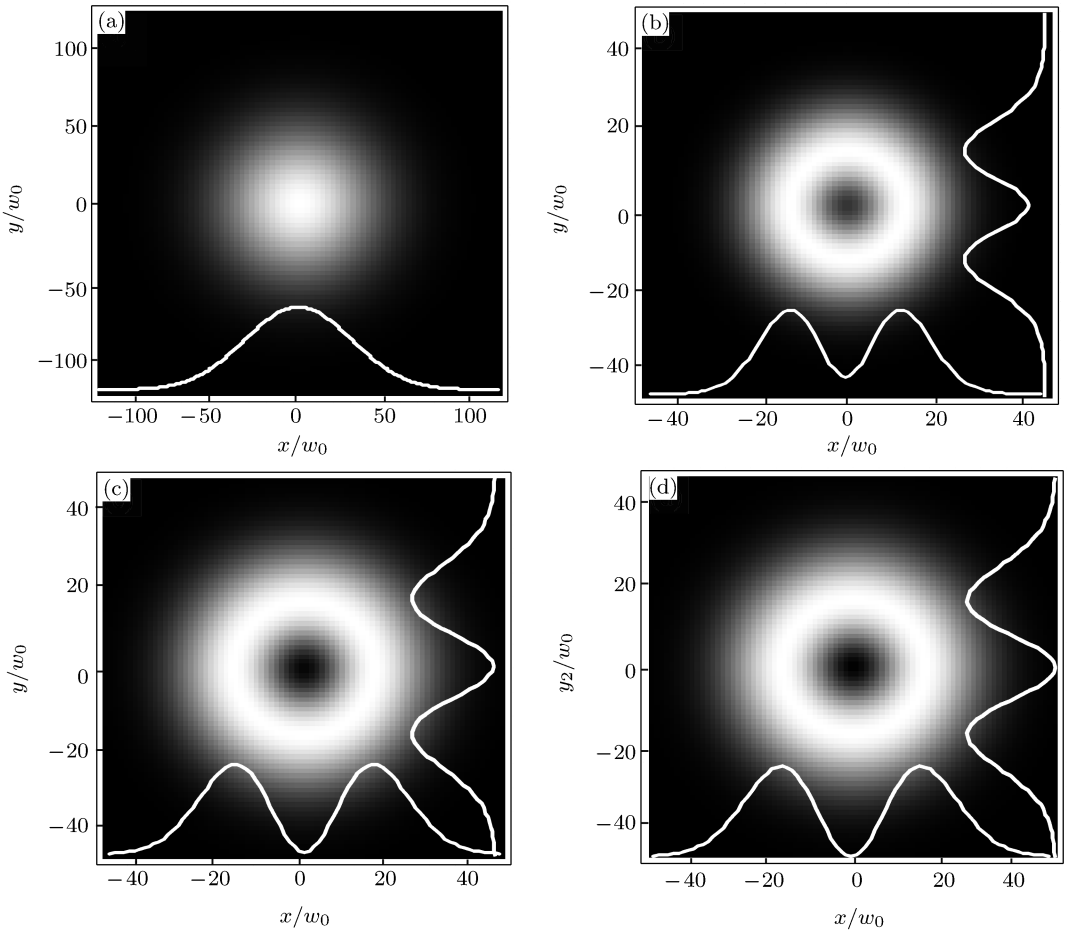
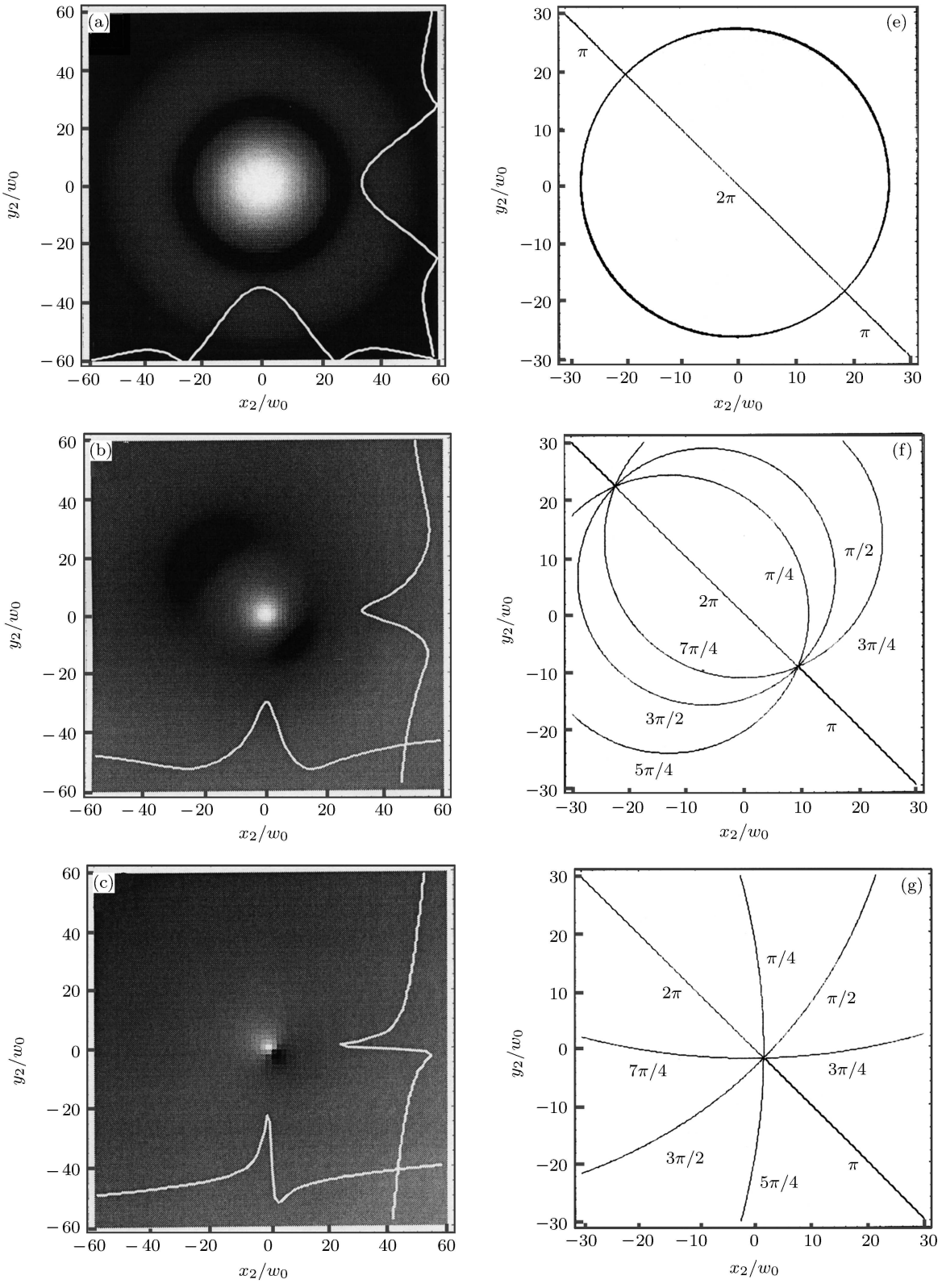


Fig.1. Intensity distributions at the plane $z = 20z_0$ of partially coherent beams for selected values of (a) $\alpha = 0.5$, (b) $\alpha = 5$, (c) $\alpha = 15$ and (d) $\alpha = 50$.

To study coherence vortices of partially coherent beams in the far field and the evolution of coherence vortices into intensity vortices of fully coherent beams, a point $x_1 = -0.5w_0$, $y_1 = 0.5w_0$ at the plane $z = 20z_0$ is fixed. In Fig.2 we plot the spectral degree of coherence (absolute value) and corresponding phase contour lines of partially coherent beams for selected values of (a) $\alpha = 0.5$, (b) $\alpha = 5$, (c) $\alpha = 15$, and (d) $\alpha = 50$.



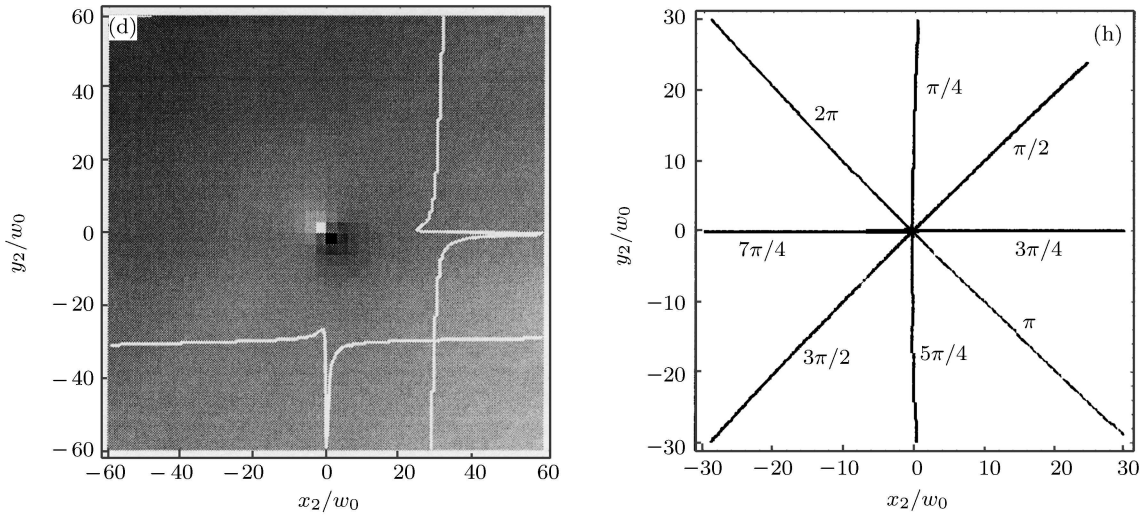


Fig.2. Degrees of spatial coherence (absolute value) (a)–(d) and phase contour lines (e)–(h) of partially coherent beams for selected values of $\alpha = 0.5$ (a) and (e); $\alpha = 5$ (b) and (f); $\alpha = 15$ (c) and (g); $\alpha = 50$ (d) and (h).

From Fig.2 we see that zeros of spectral degree of coherence are typically presented in partially coherent beams even for the low coherent case of $\alpha = 0.5$ in Fig.2(a), which shows a circular phase dislocation in Fig.2(e) characterized by a π phase jump across the circular boundary of radius $\rho = 26.3w_0$ where $\mu = 0$. With α increasing from 5 to 15 in Figs.2(b), (f) and (c), (g), the upper vortex on the diagonal axis, which makes an angle of 135° with the x_2-axis, moves from the point ($x_2 = -22.3w_0$, $y_2 = 22.3w_0$) to the point ($x_2 = -114.9w_0$, $y_2 = 114.9w_0$), whereas the lower vortex moves from the point ($x_2 = 9.3w_0$, $y_2 = -9.3w_0$) to the point ($x_2 = 1.7w_0$, $y_2 = -1.7w_0$). Finally, in the coherent limit in Figs.2 (d) and (h), the point of zero of μ coincides with that of intensity, i.e. $x_2 = y_2 = 0$. The lower coherence vortex becomes the vortex of intensity and the upper coherence vortex moves towards infinity.

4. Concluding remarks

In this paper we have studied the far-field coherence vortices of partially coherent, quasi-monochromatic singular beams with Gaussian envelope and Schell-model correlator. The far-field coherence vortices of such beams depend on the relative coherence length, mode index and positions of pairs of points. If a point and mode index are kept fixed, for the low coherent case there exists a circular phase dislocation. With increasing α the two vortices take place, and one vortex moves towards to infinity, whereas another moves to the origin in the coherent limit. As a result, the coherence vortex evolves into the intensity vortex of fully coherent Laguerre-Gaussian beams.

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