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Energy-optimal control model for train movements*

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In this paper, we propose a new cellular automaton (CA) model for train movement simulations under mixed traffic conditions. A kind of control strategy is employed for trains to reduce energy consumption. In the proposed CA model, the driver controls the train movements by using some updated rules. In order to obtain a good insight into the evolution behaviours of the rail traffic flow, we investigate the space–time diagram of the rail traffic flow and the trajectories of the train movements. The numerical simulation results demonstrate that the proposed CA model can well describe the dynamic behaviours of the train movements. Some complex phenomena of train movements can be reproduced, such as the train delay propagations, etc.

Keywords: train movement, cellular automaton, moving-block
PACC: 0550

1. Introduction

In the past several decades, computer simulation of the train movement has been widely adopted in studying various kinds of rail systems. For example, it can be used to test theoretical control algorithms for improving the performance of a rail system. Train movement is described by the speed and distance profiles while a train travels from one point to another. There are three major approaches to calculating the train movement: i.e. the basic model, the time-based model and the event-based model.[1,2] In the basic model, when the length of track conductor loop is small, a continuous braking profile is a reasonable approximation. The time-based approach makes it easy to design and build simulation models, but it makes great demands on computational time. The event-based model substantially saves computational effort, but it is more complicated to program.

The problem of the train control and energy consumption is one of the problems in the optimal control theory. Many methods of solving this problem have been proposed. Classical numerical methods of optimization have some difficulties in computation:[3] for example, they require a significant amount of computational time and are not feasible to employ an on-board computer. By making some simplifications, the optimization problem can be solved, such as constant constraints on controls.[4,5] However, application of such a solution obtained under some assumptions may be very limited. The optimization problem has also been completely solved. Therein the proposed calculation procedure includes the integration of additional differential equation.[6,7] This procedure is less effective than the calculation under simpler algebraic optimal conditions.

The studies above are based on the analysis made by using mathematical models. The simulation analysis approach provides the solution for the above deficiencies, for it does not have to search for the analytical and mathematical solutions. By taking the simulation analysis approach, it is possible to study and conduct experiments on the complex internal interactions of the rail system. The cellular automata (CA) model is one of the 'micro-' simulation models whose space and time are discrete, and it has been successfully applied to high volume of traffic with shorter execution time.[8−10] It has also be used to simulate the mixed traffic in road traffic.[11] In Ref.[12], we proposed a CA model for train movement simulation, which is a kind of computer-based simulation model. In this paper, an improved CA model is described to simulate the train movements. This is an optimal control model. By using some simple rules, it can capture the complex behaviour of the train movements.

The rest of the paper is organized as follows. In Section 2, we introduce the principle of the moving-

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block train control system. Our previous work is introduced in Section 3; in Section 4, we develop the improved CA model. The numerical and analytical results are presented in Section 5. Finally, conclusion of this approach is also drawn in the paper and presented in Section 6.

2. Moving block signalling system

Automatic train control (ATC) based on moving block (MB) principles is supported by the transmission based signalling (TBS) systems, which can be used to control, protect and regulate the train movements. By employing the MB train control system, more room for headway reduction can be provided. In a moving-block system, trains can establish their own safety distances dynamically. The separation between two successive trains is governed by their safety stopping distance. On a moving-block equipped rail, trains transmit continuously their current speeds, current locations, and braking capabilities to a local computer so that the local computer knows the locations of all the trains in its area at all times. At the same time, the local computer transmits the locations of trains to each train, and gives them their braking curves.

Several types of moving-block schemes have been discussed in Ref.[13], i.e. moving space block (MSB), moving time block (MTB) and pure moving block (PMB). Moving space block (MSB) is the simplest scheme, in which the safety stopping distance between two successive trains is

$$d_{\min} = v_{\max}^2/(2b) + SM, \tag{1}$$

where $v_{\max}$ is the maximum speed, b denotes the deceleration of train and SM is the marginal distance. The marginal distance SM can be written as $SM = T_{\rm r} v_{\max}$, where $T_{\rm r}$ is called the react time. In this scheme, the only information that the following train requires is the position of the preceding train and its own position.

Moving time block (MTB) is another scheme, in which the headway between two trains is always constant and independent of travel speed. In this scheme, the safety stopping distance between two successive trains is

$$d_{\min} = v_{\max} v_{\rm f}/(2b) + SM, \tag{2}$$

where $v_{\rm f}$ is the speed of the following train.

Pure moving-block (PMB) is also a type of moving-block system, in which the requirement for information is the relative position between the successive trains and the speed of the following train. In this scheme, the safety stopping distance between two successive trains is

$$d_{\min} = v_{\rm f}^2/(2b) + SM, \tag{3}$$

where the distance is basically the braking distance required by a train, and is the minimum distance needed to avoid a collision even if the preceding train comes to a sudden halt. For the present paper, only the simplest scheme, MSB, is considered in more detail.

3. Cellular automata model for train movement

In our previous work,[12] we presented a cellular automaton (CA) model for the train control system simulation, which is based on the deterministic NaSch traffic model. Since the optimal model proposed in this paper is based on our previous work,[12] we recall the basic concepts adopted in our previous CA model in this section. The CA model proposed in our previous work is as follows. The rail line consists of a single-lane track, it is divided into L cells numbered by $i = 1, 2, \ldots, L$, and the time is discrete. Each site can be either empty or occupied by a train with an integer speed $v_n = 0, 1, \ldots, v_{\max}$. The updated rules for implementing the nth train are as follows:

Case I: the nth train is behind the $(n+1)$th train.

Step 1. Acceleration:

if $\Delta x_n > d_{\min}$

$$v_n = \min(v_n + a, v_{\max})$$

elseif $\Delta x_n < d_{\min}$

$$v_n = \max(v_n - b, 0)$$

else

$$v_n = v_n$$

end

Step 2. Slowing down:

$$v_n = \min(v_n, gap)$$

Step 3. Movement:

$$x_n = x_n + v_n$$

Case II: the nth train is before a station.

(1) The station is occupied by the $(n+1)$th train, the update rules are the same as those used in the Case I.

(2) The station is empty.

Step 1. Acceleration:

if $\Delta x_n > d_c$

$$v_n = \min(v_n + a, v_{\max})$$

elseif $\Delta x_n < d_c$

$$v_n = \max(v_n - b, 0)$$

else

$$v_n = v_n$$

end

Step 2. Slowing down:

$$v_n = \min(v_n, \Delta x_n)$$

Step 3. Movement:

$$x_n = x_n + v_n$$

where Δx_n is the distance from the nth train to the $(n+1)$th train or the preceding station, a represents the acceleration of the nth train, and *gap* is the empty cell number from the nth train to the $(n+1)$th train or the station in front of the nth train. Δx_n can be calculated by $\Delta x_n = x_{n+1} - x_n$, or $\Delta x_n = x_s - x_n$. Here x_n, x_{n+1} and x_s respectively represent the site of the nth train, the site of the $(n+1)$th train and the site of the station in front of the nth train. *gap* can be calculated by $gap = x_{n+1} - x_n - 1$, or $\Delta x_n = x_s - x_n - 1$. The distance d_c can be calculated by $d_c = v_{\max}^2/(2b)$.

In our method, the boundary condition for CA model is the same as that adopted in Ref.[12]. It is as follows. (1) When the section from site 1 to site L_s is empty, a train with the speed $v_{\max}$ is created. This train immediately moves according to the updated rules. The parameter L_s is called the departure interval, and it is larger than the safety stopping distance. (2) At the site $i = L$, trains simply move out of the system. In order to compare the simulation results with the field measurements, one cellular automaton iteration roughly corresponds to 1 s, and the length of a cell is about 1 m. This means, for example, that $v_{\max} = 10$ cells/update corresponds to $v_{\max} = 36$ km/h.

4. The improved CA model

It is well known that the CA model is a flexible model, which makes it easy to simulate different types of traffic by modifying the basic rules. In order to make our previous model more realistic, in this paper, we take into account the train traction and train braking in the calculation of the train movements. In general, the motion of a train can be described by the following equation:[6]

$$\mathrm{d}v/\mathrm{d}t = u_{\mathrm{f}} f(v) - u_{\mathrm{b}} b(v) - w(v) - g(x), \qquad (4)$$

where u_{f} is the relative traction force, $f(v)$ is the specific maximum traction force, u_{b} is the relative braking force, $b(v)$ is the specific maximum braking force, $w(v)$ is the specific resistance to motion, and $g(x)$ is the specific external force caused by track grade and curve resistance. If we take into account only the influence of the traction and braking factors, Eq.(4) can be simply written as

$$\mathrm{d}v/\mathrm{d}t = u_{\mathrm{f}} f(v) - u_{\mathrm{b}} b(v)\,. \qquad (5)$$

There exist five results of Eq.(5): (1) full braking, $u_{\mathrm{f}} = 0$, $u_{\mathrm{b}} = 1$; (2) partial braking, $u_{\mathrm{f}} = 0$, u_{b} may vary; (3) inertia motion or coasting, $u_{\mathrm{f}} = 0$, $u_{\mathrm{b}} = 0$; (4) partial power, u_{f} may vary, $u_{\mathrm{b}} = 0$; (5) full power, $u_{\mathrm{f}} = 1$, $u_{\mathrm{b}} = 0$. Equation (5) can be further written as a discrete equation, i.e. $\Delta v(t) = v(t+1) - v(t) = u_{\mathrm{f}} f(v(t)) - u_{\mathrm{b}} b(v(t))$. For simplicity, in our model proposed in the following, only the full braking, inertia motion or coasting and full power are taken into account.

The reduction in energy consumption is regarded as one of the key objectives in metro and rail transit systems, it is also one of the fundamental problems in the optimal control of the train movements. In this field, the main problem is how to control the speed of train and save energy. Numerous empirical data and numerical simulations demonstrate that for the economical travel, three control strategies can be employed:[14] (1) when a train accelerates, its speed is calculated by using the full power; (2) when a train decelerates, its speed is calculated by using the full braking; (3) under some constraint conditions, its speed is calculated by using the inertia motion or coasting. Based on these concepts, we revise the acceleration rule of the nth train in our previous model. First, when $\Delta x_n > d_{\min}$, the speed of the nth train is calculated by using the full power, i.e. $v_n = \min([v_n + f(v_n), v_{\max}])$. Second, when $\Delta x_n < d_{\min}$, the inertia motion or coasting could be employed. It should be pointed out that the nth train represents each train travelling on the rail line.

The choice of the braking method is one of the factors to save energy. The normal service braking sequence of an economical travel becomes coasting first, regenerative braking alone second and the combined braking last.[15] In view of the concept, we revise the updated rules when the cells within the braking distance of the following train are still occupied by the preceding train. In the revised method, when $\Delta x_n < d_{\min}$, if $v_n < V_c$, the nth train can travel at the inertia motion or coasting, and maintain the constant speed, otherwise, as $v_n > V_c$, the nth can travel at the full braking, i.e. $v_n = \max((v_n - b(v_n), 0))$. Here V_c is the maximum allowable speed, it is defined as $V_c = (2a(\Delta x_n - v_{\max} T_r))^{1/2}$. In the simulation, at each time step, we first calculate the maximum allowable speed of the nth train, and then compare it with the current speed of the nth train, finally determine the operation of the nth train at the next time step. According to the above concepts, the acceleration of the nth train outlined in Section 3 can be written as follows:

if $\Delta x_n > d_{\min}$

$$v_n = \min([v_n + f(v_n), v_{\max}])$$

elseif $\Delta x_n < d_{\min}$

if $v_n < V_c$

$$v_n = v_n$$

else

$$v_n = \max((v_n - b(v_n), 0))$$

end

else

$$v_n = v_n$$

end

In the proposed model, the updated rules of the slowing down and movement are the same as those outlined in Section 3. The boundary condition for CA model is also the same as that adopted in Section 3.

5. Numerical simulations

We use the improved CA model proposed in this paper to simulate the mixed rail traffic under the moving-block condition. The moving-block scheme adopted in this work is moving space block (MSB). For convenience, we consider only two types of the maximum speeds of trains that are taken as $v^1_{\max} = 15$ and $v^2_{\max} = 10$. The total distance L and the evolution time T are respectively set to be $L = 1000$ and $T = 1000$. One station is designated in the middle of the system (i.e. the site $i = 500$). During the numerical simulations, the react time T_r, the acceleration a and deceleration b are respectively taken as $T_r = 1$, $a = 1$ and $b = 1$. The parameters $f(v)$ and $b(v)$ are taken as $f(v) = 1$ and $b(v) = 1$.

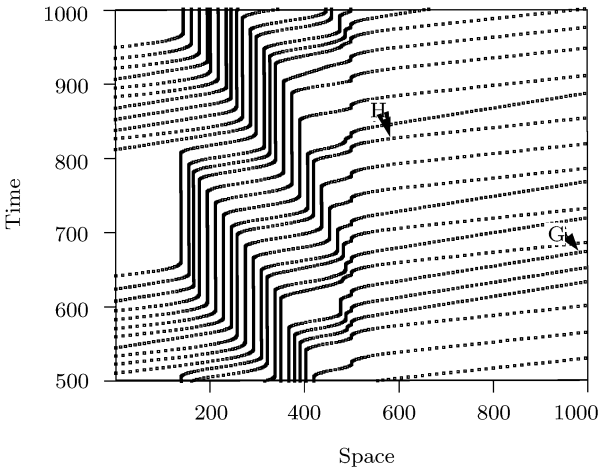


Fig.1. Space–time diagram of rail traffic flow.

Under the moving-block condition, the characteristic behaviour of the train movement is similar to that observed in road traffic. Figure 1 shows the space–time diagram of rail traffic flow for $T_d = 4$ and $L_s = 150$. Here the condition $L_s = 150$ means that in the simulation, when the distance that the last train has just travelled from the departure site is equal to or larger than 150, a new train will set off from the departure site. In Fig.1, we plot 1000 sites in 500 consecutive time steps. The horizontal direction indicates the direction in which trains move ahead, and the vertical direction indicates time. From Fig.1, it can be clearly seen that before the station, the go-and-stop waves occur. The reason is that trains before the station are delayed within the station dwell time T_d, and the train delays propagate backward. In addition, after the station, we can find that as the simulation proceeds in time, some trajectories of trains converge, such as around point G, while some other trajectories of trains diverge, such as around point H. In the former case, it is possible that the fast trains can be caught up with by their preceding slow trains, and the safety stopping distance must be maintained among them.

In order to investigate the characteristic behaviour of rail traffic flow, we study the train queue speed at which the train delay propagates backward, and compare it with the car queue speed of road traffic. For a vehicle queue, the vehicle queue speed v_q

can be defined as the average number of vehicles joining this queue per time step. In theory, the speed v_q is largely related to the gaps among vehicles. The larger the gap between vehicles is, the smaller the vehicle queue speed will be. From Fig.1, the train queue speed can be roughly estimated to be about $v_q = 0.05$. We can also estimate the car queue speed of road traffic. If the car density is large enough, the car queue speed will be larger than the train queue speed. The reason is that in rail traffic, trains must be kept in the safety stopping distance, so that the gaps among trains in rail traffic are larger than the gaps among cars in road traffic.

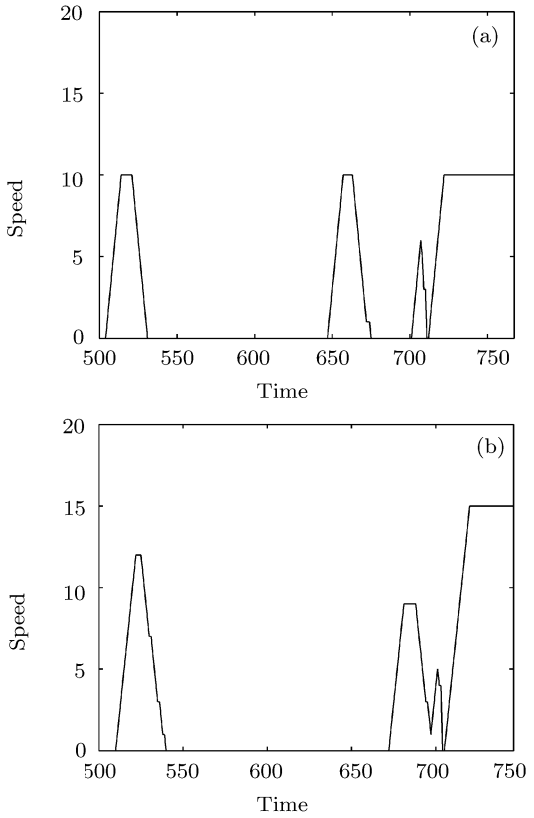


Fig.2. The speeds and times of trains for $T_d = 4$ and $L_s = 150$. (a) is for $v_{max} = 10$, and (b) for $v_{max} = 15$.

As mentioned above, we simulate the mixed rail traffic under the moving space block condition. In simulations, the maximum speed of trains is chosen randomly. Figure 2 shows how the speeds vary with time for $T_d = 4$, which is obtained by different measurements. Here the trains are tracked at the time $t = 500$. In Fig.2, Fig.2(a) shows the simulation results representing the train speed of $v_{max} = 10$, and Fig.2(b) shows the simulation results representing the train speed of $v_{max} = 15$. From Fig.2, we can clearly observe the complex behaviour of train movements. In fact, the motion of a train is governed by its maximum speed v_{max}, acceleration and deceleration rates, etc. In order to maintain a safety distance between two successive trains on the same rail line, the following train needs to adjust its speed continuously according to the position and speed of its leading train. These simulation results indicate that the proposed model can capture the dynamical behaviour of train movement under the mixed traffic condition.

When the traction force $f(v)$ and the braking force $b(v)$ are set to be $f(v) = 1$ and $b(v) = 1$, the acceleration rate is 1 at full power, and the deceleration rate is also 1 at full braking. The trajectory of one train is shown in Fig.3. It displays the distances and speeds of the tracked train for $v_{max} = 15$. In Fig.3, this train is tracked at the time $t = 500$. From Fig.3, we can see that during the tracked train travelling toward the station, this train adjusts its speed continuously: sometimes it accelerates at full power, and sometimes it decelerates at full braking. In addition, we find in Fig.3 that the tracked train travels at the inertia motion or coasting several times. In these cases, the tracked train maintains the constant speed. The simulation result demonstrates that the proposed CA model can reproduce the characteristic behaviour of train movement under the economical travel condition, and the speed controlling simulation is close to actual control by the driver.

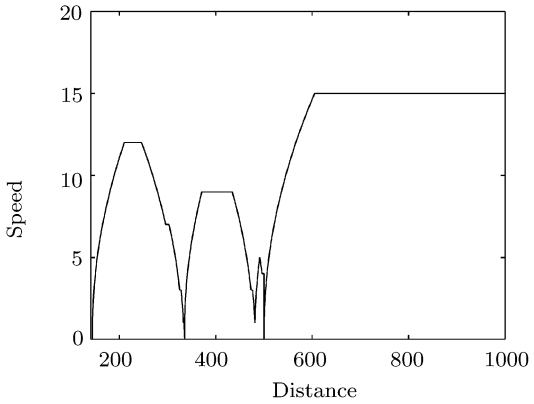


Fig.3. The speeds and distances of one train for $T_d = 4$ and $L_s = 150$.

For the train service planners and signalling design engineers, graphs about the relations among the speed, distance and time are usually used as a basic planning tool. These graphs give the average kinetic performance of trains. The dynamic response of one train before the station is shown in Fig.4. This figure presents the local space–time diagram, which displays the positions and speeds of the trains within the go-and-stop waves. Here numbers represent the speeds

of trains, and dots correspond to empty sites. From Fig.4, it is obvious that during the train travelling, in order to avoid the collision between two successive trains, it must adjust its speed continuously. Sometimes the tracked train accelerates or decelerates at full power or full braking; sometimes it maintains a constant speed at the inertia motion or coasting. As mentioned above, in this case, the tracked train can reduce its energy consumption.

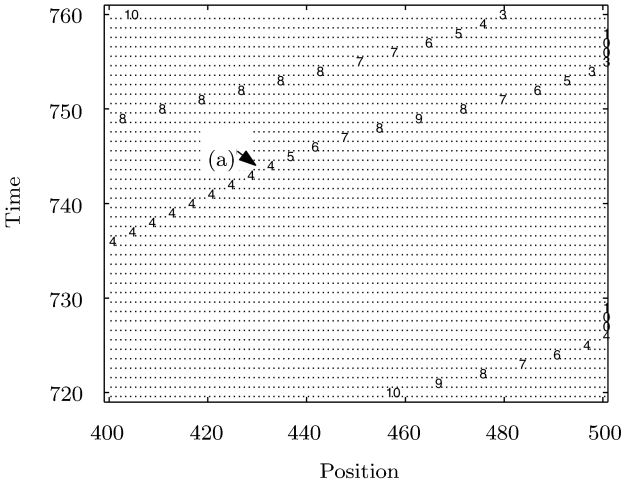


Fig.4. The positions and speeds of trains for $T_{\rm d} = 4$ and $L_{\rm s} = 200$.

Comparing the simulation result obtained in this paper with that in our previous work, there are two main distinctions. First, in this paper, we consider the mixed traffic, where trains have different maximum speeds. This is different from the case in our previous work. Since trains have different maximum speeds, fast trains could be caught up with by their preceding slow trains. The behaviour of train movement makes the space–time evolution of the rail traffic flow more complicated. This character can be seen in Fig.1 and Fig.2, such as point G in Fig.1. Second, the main improvement in this paper is the establishment of an optimal control model. During trains travelling, in order to reduce the energy consumption, trains need to maintain the constant speeds. This can be seen in Figs.2–4. However, this characteristic behaviour of train movement has not been taken into consideration in our previous work.

6. Conclusions

In conclusions, an improved CA model based on our previous work has been used to simulate the mixed rail traffic under the MSB condition. In order to reduce the energy consumption of trains, a kind of optimal strategy is employed for controlling trains. The simulation results demonstrate that the proposed CA model can well capture the characteristic behaviour of train movement. It should be pointed out that the proposed CA model is a basic model, which can be generalized to more complicated traffic circumstances by modifying the basic rules.

In real rail traffic, train movement is subject to not only the speed restriction, but also the constraints of the track geometry and specific external force, etc. In our model, these factors have not been taken into account. Stations are modelled in a rather simple way, on the assumption that each train can use every platform track, and all trains have the same dwell time at each station. In addition, the partial power and partial braking are omitted, which are usually adopted in the real train control. These factors could lead some details of train movement to be omitted in calculation. This means that the optimal control of train operation must be considered in detail. So we think that it is worth to study further.

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