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Least mean square error difference minimum criterion for adaptive chaotic noise canceller*

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The least mean square error difference (LMS-ED) minimum criterion for an adaptive chaotic noise canceller is proposed in this paper. Different from traditional least mean square error minimum criterion in which the error is uncorrelated with the input vector, the proposed LMS-ED minimum criterion tries to minimize the correlation between the error difference and input vector difference. The novel adaptive LMS-ED algorithm is then derived to update the weights of adaptive noise canceller. A comparison between cancelling performances of adaptive least mean square (LMS), normalized LMS (NLMS) and proposed LMS-ED algorithms is simulated by using three kinds of chaotic noises. The simulation results clearly show that the proposed algorithm outperforms the LMS and NLMS algorithms in achieving small values of steady-state excess mean square error. Moreover, the computational complexity of the proposed LMS-ED algorithm is the same as that of the standard LMS algorithms.

Keywords: chaotic noise, adaptive noise canceller, error difference
PACC: 0545, 0650D

1. Introduction

An important operation in acoustic noisy environments is the extraction of the acoustic noise from the desired speech,[1−8] where acoustic noise is added to speech. Although the single microphone approach to noise cancelling can be achieved using adaptive filtering, the two-microphone approach using adaptive filtering is a more powerful technique for that purpose.[2] The strength of the adaptive noise cancellers lies in the fact that no prior knowledge of the speech signal or the corrupting noise is required, and a correlation between the noise corrupted speech and the random noise in the secondary input (adaptive filter input), is necessary for adapting least the primary input signal.[3] Due to the fact that a noise process is often modelled as the Gaussian process in many practical applications, the minimum mean square error optimum criterion is generally used to iterate adaptive noise cancellers for cancelling the acoustic noise.[1−8] However, many practical noise processes have chaotic dynamical characteristics in some physical and acoustic speech engineering fields, such as fan noise in duck line.[9] The single sensor approach to chaotic noise suppression has been proposed by using nonlinear inverse filtering;[10−12] however, little attention has been paid to a two-sensor approach to adaptive chaotic noise cancellation.

The least mean square (LMS) algorithm[7] coupled with the desired properties has been successfully used in modelling an unknown system and prediction,[13] in adaptive noise cancelling,[14] in channel equalization,[15] and in many other areas due to the robustness of LMS algorithm in non-stationary environments and its simplicity in computation. Since the LMS algorithm can achieve an asymmetric optimum in Gaussian environments,[8] in which it tries to uncorrelate the error and the input vector, it is not suitable for cancelling chaotic acoustic noise because the chaotic noise is the determinative nonlinear and non-Gaussian noise process.

In this paper, the least mean square error difference (LMS-ED) minimum criterion is proposed to optimize an adaptive chaotic noise canceller. The new adaptive algorithm, referred to as LMS-ED algorithm, is derived from this criterion for cancelling the chaotic acoustic noise, where the error difference tries to be uncorrelated with the input vector difference. Experi-

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mental results show that the proposed adaptive LMS-ED algorithm can outperform traditional LMS-type algorithm for adaptive chaotic noise cancellers.

The rest of this paper is organized as follows. Adaptive LMS noise cancellers are reviewed in Section 2. The proposed LMS-ED minimum criterion and the LMS-ED adaptive algorithm are given in Section 3. In Section 4 given are simulation results. The conclusions are presented in Section 5.

2. Adaptive LMS noise canceller

Figure 1 illustrates a typical adaptive noise canceller, which is composed of two inputs: primary input and reference input.

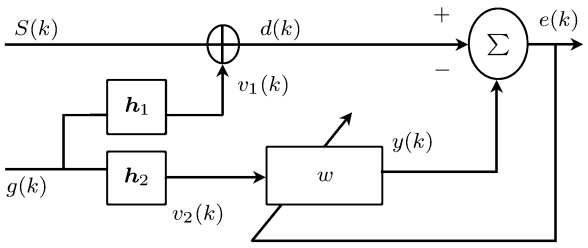


Fig.1. Adaptive noise canceller.

The primary input $d(k)$ consists of the original signal, $S(k)$, corrupted by an additive noise $v_1(k)$. The reference signal $v_2(k)$, the input to the adaptive filter, is correlated with $v_1(k)$, but uncorrelated with $S(k)$. The effectiveness of the adaptive noise cancellers depends on how closely $v_1(k)$ and $v_2(k)$ are correlated. The noise source is represented by g, and the transmission paths from the noise source to the primary and reference inputs are represented by the low pass filters, $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$, respectively. The filter weights $\boldsymbol{w}$ are adapted by means of an LMS-based algorithm to minimize the power in the output signal. This minimization is achieved by processing $v_2(k)$ via the adaptive filter to provide an estimate of $v_1(k)$, $y(k) = \hat{v}_1(k)$, and then subtracting it from $d(k)$ to obtain $e(k)$. Thus,

$$e(k) = S(k) + v_1(k) - y(k). \tag{1}$$

The performance function J_{LMS} is defined as

$$J_{\mathrm{LMS}} = \frac{1}{2}e^2(n) = \frac{1}{2}\left(d(n) - \boldsymbol{W}^{\mathrm{T}}(n)\boldsymbol{X}(n)\right)^2, \tag{2}$$

where n is the time index in the discrete-time domain, $e(n)$ is the filter output error, $\boldsymbol{X} \in \boldsymbol{R}^N$ is the filter input data vector, d is the desired scalar output signal of the filter and T denotes transpose.

A stochastic gradient algorithm is obtained as

$$\boldsymbol{W}(n+1) = \boldsymbol{W}(n) - \mu\frac{\partial J_{\mathrm{LMS}}}{\partial \boldsymbol{W}}, \tag{3}$$

where $\boldsymbol{W} \in \boldsymbol{R}^N$ is the weight vector of an N^{th}-order transversal filter, and μ is a constant step size or adaptation gain. Therefore the LMS algorithm from the above performance function (Eq.(1)) is

$$\boldsymbol{W}(n+1) = \boldsymbol{W}(n) + \mu_1 e(n)\boldsymbol{X}(n). \tag{4}$$

This LMS algorithm is a stochastic gradient optimization technique which attempts to minimize the performance function so that the filter weight vector converges to the optimal value $\boldsymbol{W}_{\mathrm{o}}$.

Many two-microphone adaptive noise cancellers have been proposed in the literature[1−5] using the modified LMS algorithms that alter the step-size of the update equation to improve the tracking ability and speed of convergence as well. One of the well-known algorithms in this category is the normalized LMS algorithm given by

$$\boldsymbol{W}(n+1) = \boldsymbol{W}(n) + \mu_2\frac{1}{\delta + ||\boldsymbol{X}||^2}e(n)\boldsymbol{X}(n). \tag{5}$$

where adaptation constants $\delta > 0$, and $0 < \mu_2 < 2$.

Since the LMS algorithm in Eq.(4) coupled with its desired properties, makes it and its variants an important part of adaptive techniques, its main drawback is the tradeoff relationship of the convergence rate and the residual misadjustment errors after convergence[2,4] as compared with other complex algorithms such as the recursive least square (RLS) algorithm and the recursive lattice filter algorithm. A common major drawback of adaptive noise cancellers using these LMS-based algorithms is the large value of excess mean-square error (EMSE) that results in signal distortion and reverberation in the noise-cancelled speech.

In the present work, we propose an LMS-ED adaptive chaotic noise canceller using an LMS-ED minimum criterion rather than the LMS minimum criterion. The proposed algorithm provides significant improvements in decreasing EMSE and consequently minimizing signal reverberation and distortion.

3. The proposed LMS-ED algorithms

Due to uncorrelation between the error and the input vector in the LMS algorithm, the weights updating equations (4) and (5) result in a highly steady state misadjustment in cancelling chaotic acoustic noise as illustrated later. To reduce steady state misadjustment in chaotic noise cancellation, here we propose an LMS-ED minimum criterion instead of the least mean square error minimum criterion for updating the weights of adaptive chaotic noise cancellers.

The proposed LMS-ED minimum at time k is defined as

$$J_k(\boldsymbol{W}) = \frac{1}{2}(e(k) - e(k-1))^2 \Rightarrow \min . \tag{6}$$

From this new performance function, we can derive a stochastic gradient algorithm named LMS-ED algorithm as follows:

$$\begin{aligned} \boldsymbol{W}(n+1) = & \boldsymbol{W}(n) - \mu_3 \frac{\partial J_n(\boldsymbol{W})}{\partial \boldsymbol{W}} \\ = & \boldsymbol{W}(n) - \mu_3 (e(n) - e(n-1)) \\ & \times \left(\frac{\partial e(n)}{\partial \boldsymbol{W}} - \frac{\partial e(n-)}{\partial \boldsymbol{W}} \right) \\ = & \boldsymbol{W}(n) + \mu_3 (e(n) - e(n-1)) \\ & \times (\boldsymbol{X}(n) - \boldsymbol{X}(n-1)). \end{aligned} \tag{7}$$

Comparing with Eq.(4), Eq.(7) shows that the proposed LMS-ED algorithm tries to minimize the correlation between the error difference and input vector difference, which results in steady state misadjustment reduction in chaotic noise cancellation, and the computational complexity of the proposed LMS-ED algorithm is the same as that of LMS algorithms.

To continue our analysis, we consider the Taylor series expansion truncated to the linear term of gradient around the optimal weight vector $\boldsymbol{W}^*$ as follows:

$$\nabla J(\boldsymbol{W}) = \nabla J(\boldsymbol{W}_*) + \frac{\partial \nabla J(\boldsymbol{W}_*)}{\partial \boldsymbol{W}} (\boldsymbol{W} - \boldsymbol{W}_*). \tag{8}$$

Notice that truncating the gradient at the linear term corresponds to approximating the cost function around the optimal point by a quadratic curve. The Hessian matrix of this quadratic performance surface is $\boldsymbol{R}/2$, where $\boldsymbol{R}$ is given as

$$\begin{aligned} \boldsymbol{R} = & \frac{\partial \nabla J(\boldsymbol{W}_*)}{\partial \boldsymbol{W}} = \frac{\partial^2 J(\boldsymbol{W}_*)}{\partial \boldsymbol{W}} \\ = & (\boldsymbol{X}_k - \boldsymbol{X}_i)(\boldsymbol{X}_k - \boldsymbol{X}_i)^{\mathrm{T}}. \end{aligned} \tag{9}$$

By defining a new weight vector space $\bar{\boldsymbol{W}} = \boldsymbol{W} = \boldsymbol{W}_*$ whose origin is translated to the optimal solution w^*, we can rewrite the linearized dynamics of the weight equations in the vicinity of the solution in terms of the step size and the Hessian matrix as given in Eq.(10). These are coupled equations for the translated weights

$$\bar{\boldsymbol{W}}(n+1) = [\boldsymbol{I} + \eta \boldsymbol{R}] \bar{\boldsymbol{W}}(n). \tag{10}$$

In order to obtain decoupled equations, we rotate the vector space by defining $\boldsymbol{V} = \boldsymbol{Q}^{\mathrm{T}} \bar{\boldsymbol{W}}$ as the orthonormal matrix consisting of the eigenvectors of $\boldsymbol{R}$. Thus, the uncoupled dynamics for the translated and rotated weights are

$$\boldsymbol{V}(n+1) = [\boldsymbol{I} + \eta \boldsymbol{\Lambda}] \boldsymbol{V}(n), \tag{11}$$

where Λ is the diagonal eigenvalue matrix with entries ordered in correspondence with the ordering in Q. From this set of equations, we can isolate the dynamics of the weight vector along each mode of the matrix $\boldsymbol{R}$. Specifically, for the ith mode, the dynamic equation will depend on the ith eigenvalue of $\boldsymbol{R}$ as

$$v_t(n+1) = |1 + \eta \lambda_i| v_i(n), \quad i = 1, \ \ldots, \ M. \tag{12}$$

Note that $\boldsymbol{R}$ is Hessian matrix of the performance surface evaluated at a maximum point, its eigenvalues are negative. For a stable dynamics, all of the coefficients in Eqs.(12) must be inside the unit circle, that is $|1 + \eta \lambda_i| < 1$, which results in the following bound for the step size for stability:

$$0 < \eta < \frac{1}{\max_i |\lambda_i|}. \tag{13}$$

This condition is very similar (as it should be) to what we obtain for the MSE criterion,[2,4] except for the fact that we consider the eigenvalues of the Hessian matrix of error difference instead of those of the autocorrelation matrix of the input vector difference. At this point, it also becomes possible to talk about time constants of the modes in the neighbourhood of the optimum point. We can determine an approximate time constant for each individual mode whose dynamic equations are governed by the inequality (13). Specifically, for the kth mode, we write

$$(1 + \eta \lambda_k) = \mathrm{e}^{-1/\tau_k}, \tag{14}$$

from which the time constant is evaluated as

$$\tau_k = \frac{1}{\ln(1+\eta\lambda_k)} \approx \frac{-1}{\eta\lambda_k} = \frac{1}{\eta|\lambda_k|} \qquad (15)$$

The time constants allow us to make a comparison among the convergence times of different modes. In order to evaluate the overall convergence speed, one must consider the slowest mode, which corresponds to the largest time constant, i.e. the one that corresponds to the largest (smallest absolute value) eigenvalue.

4. Simulation results

The simulations of the proposed adaptive chaotic noise cancellers are carried out using a sinusoidal signal with a digital frequency $f = 0.01$, and real speech. In all simulations, the impulse responses of the two IIR low pass filters used in the simulations are $\boldsymbol{h}_1 = [1 - 0.3 - 0.1]$ and $\boldsymbol{h}_2 = [1 - 0.2]$, respectively. The used chaotic noises are generated by
Kent map: $x_{k+1} = 1 - 2 \times |x_k - 0.1|$, $x_k \in (-1, 1)$;
Ulam map: $x_{k+1} = 1 - 2x_k^2$, $x_k \in (-1, 1)$;
Cubic map: $x_{k+1} = 4x_k - 3x_k^3$.

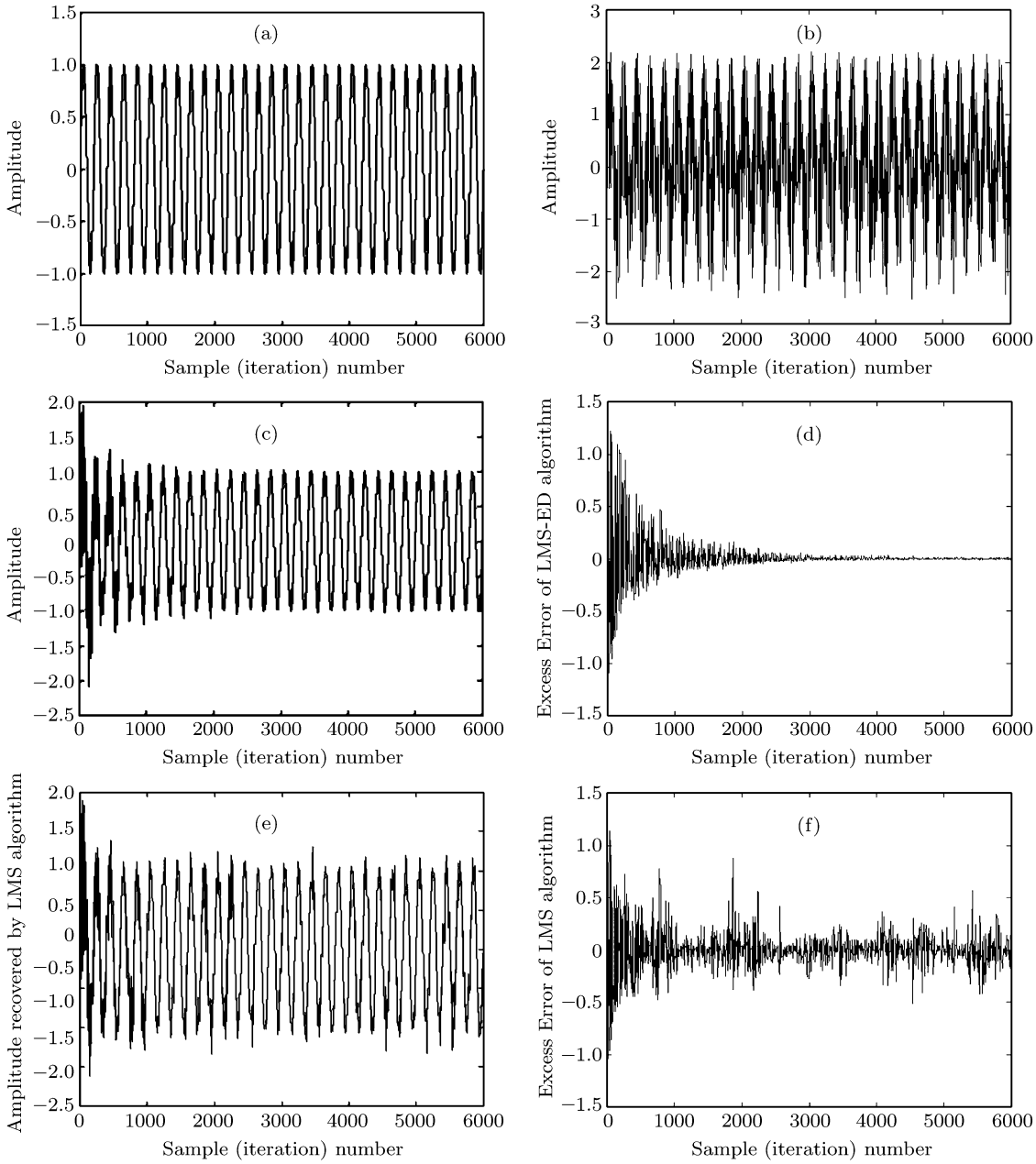


Fig.2. (a) Original clean sinusoidal signal S, (b) chaotic noise-corrupted sinusoidal signal, (c) chaotic noise-cancelling sinusoidal signal e using proposed LMS-ED algorithm; (d) excess error (S-e) of the proposed LMS-ED algorithm; (e) chaotic noise-cancelling sinusoidal signal e by LMS algorithm; (f) excess error (S-e) of the LMS algorithm.

In order to evaluate the performance of the adaptive chaotic acoustic noise canceller, the steady-state EMSE is used as the evaluation of performance, which is estimated by averaging square error after the algorithm has reached the steady-state. The EMSE is defined as

$$\text{EMSE} = \frac{1}{K-P}\sum_{k=P}^{K}|e(k)-S(k)|^2, \qquad (16)$$

where, $e(k)-S(k)$ is the excess (residual) error. K is the total number of samples of the signal, and P is the number of samples after which the algorithm reaches the steady-state.

Now we use adaptive chaotic noise cancellers to extract tone signal from chaotic acoustic environments. In this simulation, the following values of parameters are used: $K = 6000$ and $P = 4001$ for computing the steady state MSE; $\mu_1 = 0.01$ in Eq.(4) and $\mu_3 = 0.01$ in Eq.(7) for a comparison among the steady-state EMSE performances and convergences of the LMS and LMS-ED algorithms for adaptive chaotic noise cancellers. Figure 2 demonstrates the results using the proposed LMS-ED and LMS algorithms for cancelling Ulam chaotic noise with an initial value of 0.31. Figures 2(a), (b), (c), (d), (e) and (f) show original clean signal and chaotic noise-corrupted signal, the noise-cancelled signal and the residual error by using the LMS-ED algorithm, the noise-cancelled signal and the residual error by LMS algorithm, respectively. Figure 2 shows that the proposed LMS-ED algorithm outperforms the LMS algorithms in chaotic acoustic noise environments. Although the initial speed of convergence of LMS-ED algorithm is a little bit slower, its steady-state residual error of the LMS-ED algorithm is smaller than that of the LMS algorithm. We can increase the step size to speed the initial convergence of the LMS-ED algorithms.

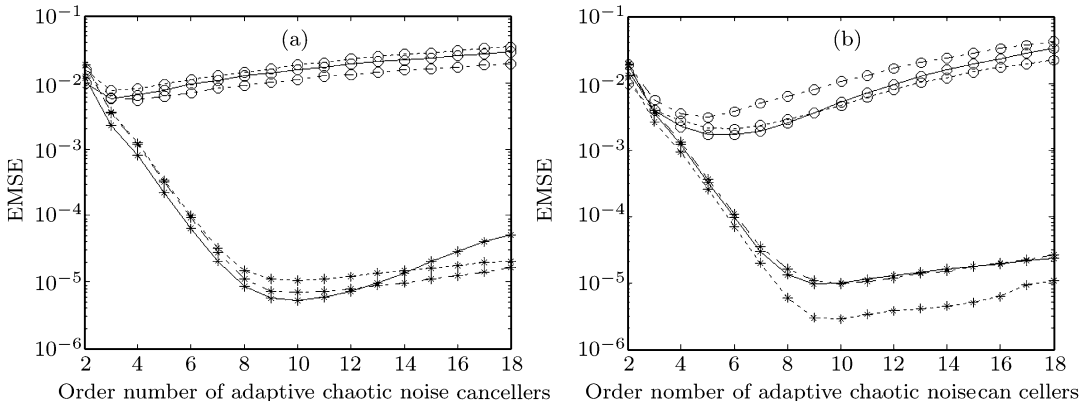


Fig.3. Comparison among the EMSEs of different adaptive algorithms for three kinds of chaotic noise. (a) the EMSE performances of LMS and LMS-ED algorithms (LMS: 'o', LMS-ED: '*', solid lines: Kent map, dotted lines: Ulam map, dashed lines: Cubic map), (b) EMSE Performances of NLMS and LMS-ED algorithms (NLMS: 'o', LMS-ED: '*', solid lines: Kent map, dotted lines: Ulam map, dashed lines: Cubic map).

To evaluate the MSE performances of adaptive LMS, NLMS and LMS-ED algorithms for cancelling chaotic noise, three kinds of chaotic noise are generated by using the Ulam map, Kent map and Cubic map with the same initial value of 0.31, respectively. In this simulation, the following values of parameters are used: $K = 6000$ and $P = 4001$ for computing the steady-state MSE; $\mu_1 = 0.01$ in Eq.(4), $\mu_2 = 0.01$ in Eq.(5) and $\mu_3 = 0.06$ in Eq.(7) as a compromise between fast rate of convergence and good tracking capability. The EMSE curves are plotted in Fig.3. As expected, the proposed LMS-ED algorithm outperforms the LMS and NLMS algorithms for cancelling chaotic noises; it can effectively and efficiently cancel three kinds of chaotic noises with the order of adaptive noise cancellers being equal to 10, while LMS and NLMS algorithms fail to cancel these chaotic noises.

To investigate the efficiency of the proposed LMS-ED algorithm in the chaotic noise-corrupted speech application, the following values are used: $K = 32000$ and $P = 10001$ for computing the steady state MSE; $\mu_1 = 0.01$, $\mu_3 = 0.01$ for a comparison among the steady-state EMSE performances and convergences of the LMS and LMS-ED algorithms for adaptive chaotic noise cancellers with the order 10, and the chaotic noise is generated by Ulam map with an initial values of 0.31. Figure 4 shows the results extracted by LMS and LMS-ED algorithms. The residual error curves

in Figs.4(d) and (f) illustrate that the proposed LMS-ED algorithm is superior to the LMS algorithm for cancelling chaotic acoustic noise. It is also confirmed by listening test that demonstrates that the recovered speech by the proposed LMS-ED algorithm is of a high quality and very close to the original speech.

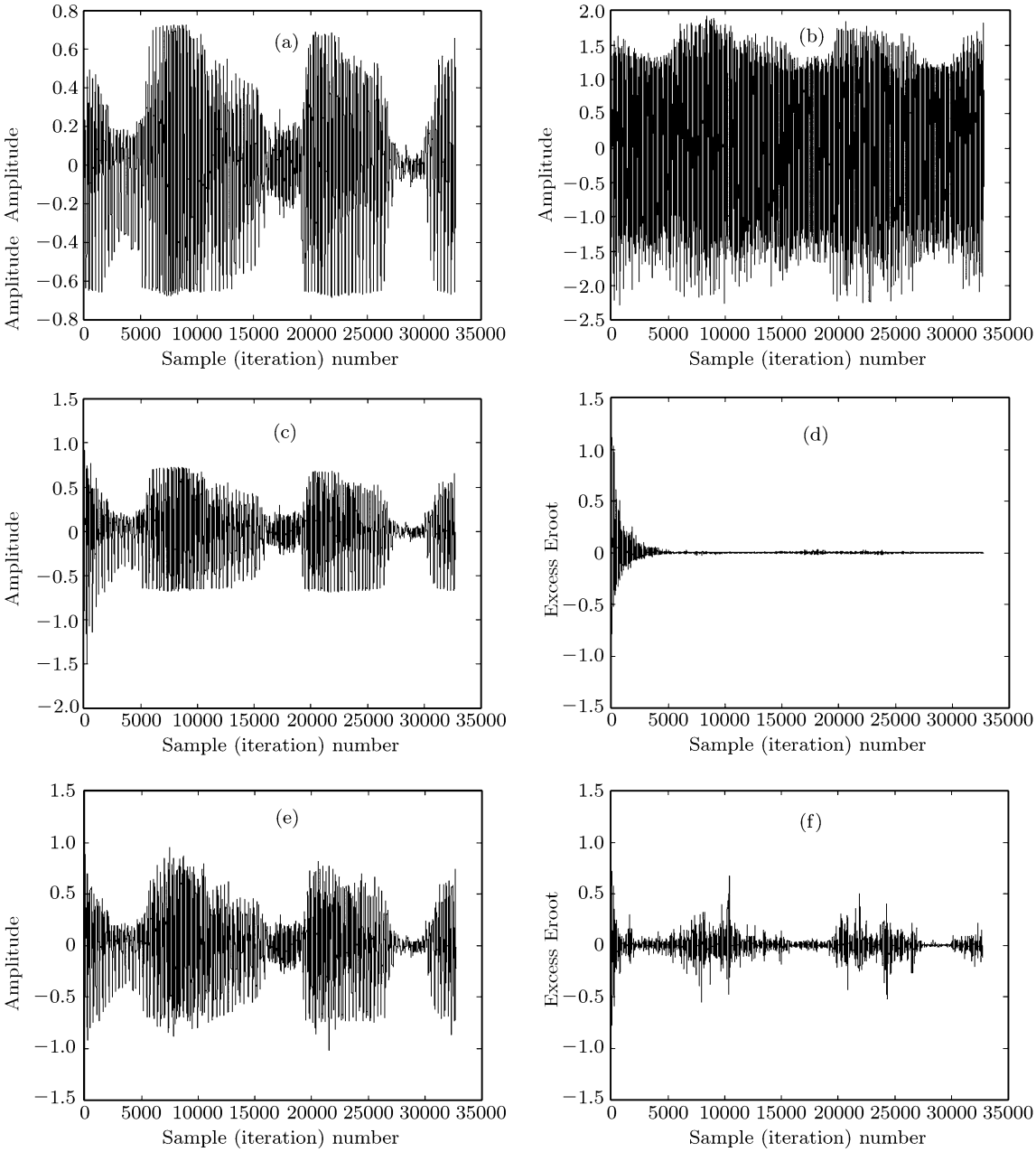


Fig.4. (a) Original clean speech S, (b) Chaotic noise-corrupted speech, (c) Chaotic noise-cancelling speech e using the proposed LMS-ED algorithm; (d) Excess error (S-e) of the proposed LMS-ED algorithm; (e) Chaotic noise-cancelling speech e by LMS algorithm; (f) Excess error (S-e) of the LMS algorithm.

5. Conclusions

This paper presents an LMS-ED minimum criterion, and a novel adaptive LMS-ED algorithm is then derived to update the weights of adaptive noise canceller. Different from uncorrelation between error and the input vector in LMS algorithms, the proposed LMS-ED algorithm tries to uncorrelate the error difference and input vector difference. A comparison among cancelling performances of adaptive LMS, NLMS and proposed LMS-ED algorithms is simulated using three kinds of chaotic noises. As a result, re-

verberation and misadjustment in the proposed algorithm are much less than those in other algorithms, while the computational complexity of the proposed LMS-ED algorithm remains the same as that of the standard LMS algorithms.

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