Vol 16 No 2, February 2007
1009-1963/2007/16(02)/0287-05

Chinese Physics

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A realistic model for complex networks with local interaction, self-organization and order*

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(Received 10 May 2006; revised manuscript received 28 August 2006)

In this paper, a new mechanism for the emergence of scale-free distribution is proposed. It is more realistic than the existing mechanism. Based on our mechanism, a model responsible for the scale-free distribution with an exponent in a range of 3-to-5 is given. Moreover, this model could also reproduce the exponential distribution that is discovered in some real networks. Finally, the analytical result of the model is given and the simulation shows the validity of our result.

Keywords: local interaction, self-organization, order, complex network
PACC: 0250, 0565

1. Introduction

In view of the Newtonian paradigm developed in the last century, every phenomenon can be reduced to an aggregation of atoms and particles whose characteristics can be observed; thus we obtain the whole world through the gathering of these small parts.[1] However, more and more evidence shows that the interactions between these parts are as important as the parts themselves.[1,2] This has led to a revolution in most disciplines of science.[3−16] Before the study of these interactions, the topology of the interactions among small parts should be given. However, in most systems, the actual topology is difficult to obtain. This is partly because of the large number of nodes and edges and our incapacity of recognizing real systems. Therefore, in order to study such systems, some models are proposed that attempt to give the possible topology of the real systems. Among these models, some are classical, such as random graph model,[17] small world model,[18] Barabaśi–Albert (BA) model,[19−21] in short BA model, etc.[22−24]

The BA model has its own advantages such as reproducing the scale-free distribution observed in most of the networks,[24] evolving which is consistent with the real world, etc.

The BA model is described as follows:[19]

- starting with m_0 nodes, and adding a node with m edges connected to the nodes existing in the network at each timestep.
- selecting the endpoint of edges, with a probability being proportional to the degree of that node.

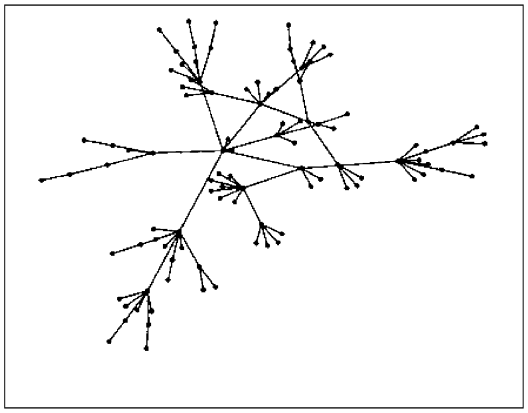


Fig.1. An example of scale-free network, with the number of nodes being 100 while $m = 1$.

However, the BA model also encounters some problems as follows:

*Project supported by the National Natural Science Foundation of China (Grant Nos 60374037 and 60574036), the Program for New Century Excellent Talents of High Education of China(Grant No NCET 2005-290), The Special Research Fund for the Doctoral Program of High Education of China (Grant No 20050055013).

- The scaling exponent is fixed and equals 3. However, among most of the real networks, they are either scale-free networks with an exponent in a range of 2-to-4 or networks with an exponential distribution.

- The mechanism generating scale-free distribution is the preferential linking to the introduction of new nodes. However, in this model, the manipulation of the links between existing nodes is lacking. This is critical. For example, when a baby is born in the world, the only persons he or she knows at the moment may be his or her farther and mother. However, as the child grows up, he or she will know his or her classmates, colleagues *et al*. This means that the linking between existing nodes should not be neglected and our real networks do not work in that manner as described in BA model. Thus the emergence of power law distribution does not follow the mechanism proposed by BA model.

The rest of this paper is organized as follows. In Section 2, a realistic model based on local interaction is given. Then the exact solution of degree distribution for this model and some simulations are given for illustration and verification in Section 3. Moreover, a particular case of this model generating the exponential distribution is studied in Section 4. Finally, some conclusions are drawn and some further work is proposed in Section 5.

2. Model based on local interaction

The complexity is ubiquitous in nature, man made systems and human societies.[1] However, in most of the complex systems, there exist indeed some orders, for example, the emergence of so many large corporations in Silicon Valley, similar to the scale-free distribution in many real networks, the flock of the migrating geese, etc. Now the question has arisen. How do these orders come about? It is generally believed that the orders are generated through local interactions, which means that the nodes in the system only interact with their neighbours and finally the order of the whole networks comes into being.[1]

In the following, we present a realistic model based on local interactions. The model is described as follows. Starting with n_0 nodes, at each time:

- randomly selecting a node i and link α edges to the nodes in the network. When selecting the other endpoint of the edges, the probability of the node j selected is proportional to the number of links to the neighbours of node i, hence the duplicating linking and self linking are eliminated.

- adding a node with β edges linked to the existing nodes randomly.

In this section, the exact solution of the degree distribution of this model is given. The method used here is the mean field method.[24] First, consider the change of the degree d_i of node i. At time t, the change of the d_i is due to the following three terms:

- The probability of the node i selected as the node to link to the existing nodes.

- The probability of the node i connected to the node selected.

- The probability of the node i connected to the new node.

In respect of the second term, the number of links from node i to the node selected is proportional to the degree d_i. Thus the change of d_i can be described as

$$\frac{\partial d_i}{\partial t} = \frac{\alpha}{t-1} + \alpha\frac{d_i}{2(\alpha+\beta)t} + \frac{\beta}{t-1}\,. \tag{1}$$

When $\alpha \neq 0$ and $t \to \infty$, Eq.(1) could reduce to

$$\frac{\partial d_i}{\partial t} = \alpha\frac{d_i}{2(\alpha+\beta)t}\,, \tag{2}$$

which is a differential equation with the initial condition

$$d_i(t_i) = m\,, \tag{3}$$

where t_i is the adding time of node i. Solving Eq.(2) yields

$$d_i = \beta\Big(\frac{t}{t_i}\Big)^{\alpha/(2\alpha+\beta)}. \tag{4}$$

Thus the distribution function of the degree of node i is

$$P(d_i \leq k) = P\Big(t_i \geq \frac{t}{(k/\beta)^{(2\alpha+\beta)/\alpha}}\Big). \tag{5}$$

The adding time t_i is equally distributed in t, i.e.

$$P(t_i) = \frac{1}{t}\,. \tag{6}$$

Thus,

$$P(d_i \leq k) = 1 - \frac{1}{(k/\beta)^{(2\alpha+\beta)/\alpha}}\,, \tag{7}$$

and the degree distribution is

$$p(d) = \frac{\partial p(d_i \leq k)}{\partial k}$$

$$= \frac{2\alpha+\beta}{\alpha}\Big(\frac{k}{\beta}\Big)^{-[(2\alpha+\beta)/\alpha]-1}(1/\beta)\,. \qquad (8)$$

Thus the degree distribution of this model is obtained. It is evident that the model obeys the power law distribution with the exponent $-[(2\alpha+\beta)/\alpha]-1$. This exponent depends on the values of α and β. Therefore, we can adjust the values of α and β to obtain the desired exponent.

3. Simulation and results

In the following, we give the simulations of the degree distribution of this model.

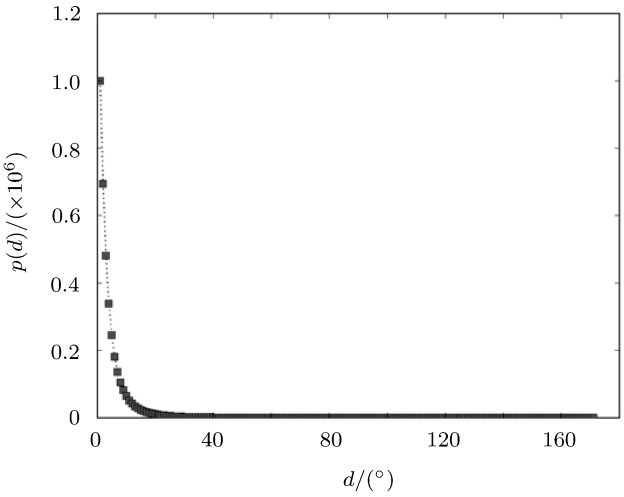


Fig.2. The degree distribution of the present model when $\alpha = 1$ and $\beta = 1$.

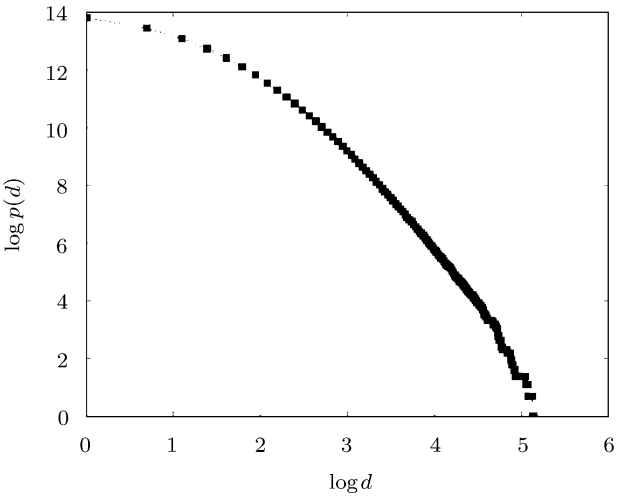


Fig.3. The log–log transformation of the present model when $\alpha = 1$ and $\beta = 1$.

Figures 2 and 3 show the degree distribution and the corresponding logarithmic transform of the present model when $\alpha = 1$ and $\beta = 1$. The number of networks is 1000000. From Figs.2 and 3 we can see that the degree distribution of the present model follows the power law distribution. The scaling exponent is shown in Table 1 . The method of calculating such a scaling exponent is the least square method. The simulation shows that the degree distribution of the present model is in good agreement with our analytical result. However, the figures in Table 1 are a bit smaller than ours. This is partly due to the simplification made when transforming from Eq.(1) to Eq.(2).

Table 1. The scaling exponents when $\alpha = 1$ and $\beta = 1$.

n	real data	analytical result
1	−4.342	−5
2	−4.417	−5
3	−4.489	−5
4	−4.478	−5
5	−4.545	−5
6	−4.457	−5
7	−4.430	−5
8	−4.501	−5
9	−4.478	−5
10	−4.556	−5
average	−4.469	−5

Thus, we may predict that when $\alpha \gg \beta$, the simulation result will be much better than in the case when $\alpha = 1$ and $\beta = 1$, which is confirmed here. The following gives the simulation when $\alpha = 20$ and $\beta = 1$. Figures 4 and 5 show the degree distribution and the corresponding log transform with $\alpha = 20$ and $\beta = 1$. The number of nodes in the networks is 100000.

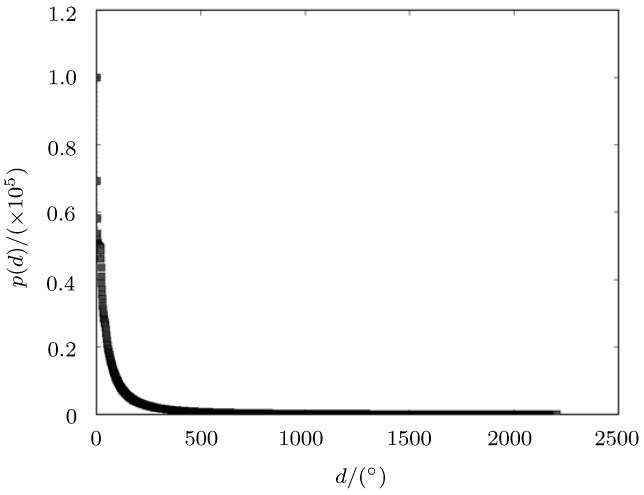


Fig.4. The degree distribution of the present model when $\alpha = 20$ and $\beta = 1$.

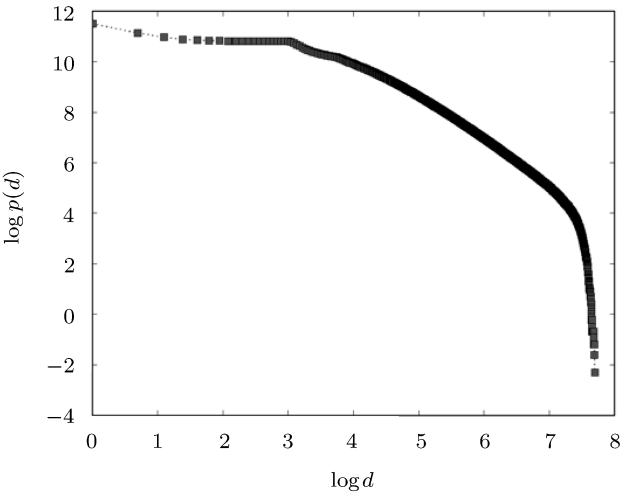


Fig.5. The log–log transformation of the present model when $\alpha = 20$ and $\beta = 1$.

Table 2. The scaling exponents when $\alpha = 20$ and $\beta = 1$.

n	real data	analytical result
1	−2.961	−3.1
2	−3.102	−3.1
3	−3.105	−3.1
4	−2.953	−3.1
5	−3.056	−3.1
6	−2.969	−3.1
7	−2.984	−3.1
8	−3.082	−3.1
9	−2.965	−3.1
10	−2.924	−3.1
average	−3.010	−3.1

From Table 2 we can see that the simulation result is in very good agreement with our result and it is much better than the result when $\alpha = 1$ and $\beta = 1$. Thus our prediction is verified. Moreover, if the number of step is larger, the simulation result can be better.

4. An extreme case

In the previous section, we have considered the case when $\alpha \neq 0$. Next, we will give the analytical result when $\alpha = 0$. The change of d_i can be described as

$$\frac{\partial d_i}{\partial t} = \frac{\beta}{t-1} \tag{9}$$

with the initial condition

$$d_i(t_i) = \beta\,. \tag{10}$$

Combining Eqs.(9) and (10), we obtain

$$d_i = \beta \log^{t/t_i}\,. \tag{11}$$

Then we calculate the cumulative degree distribution of d_i,

$$P((d_i) \leq k) = 1 - \mathrm{e}^{-k/\beta}\,, \tag{12}$$

and

$$p(k) = \frac{\partial P((d_i) \leq k)}{\partial k} = \frac{1}{\beta}\mathrm{e}^{-k/\beta}\,, \tag{13}$$

The formula (13) shows that the degree distribution of the present model with $\alpha = 0$ and $\beta = 1$ follows the exponential distribution. This is consistent with our prediction. Figure 6 shows the degree distribution and the corresponding logarithmic transform when $\alpha = 0$ and $\beta = 1$. The largest degree is only 22° compared with the largest degree 2250° in the case where $\alpha = 20$ and $\beta = 1$. The largest degree combining with the logarithmic transform in Fig.6 shows that the degree distribution of the present model follows the exponential distribution when $\alpha = 0$ and $\beta = 1$.

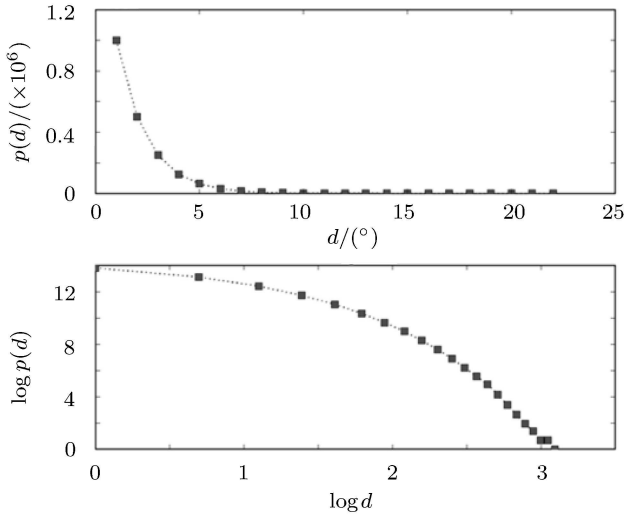


Fig.6. The degree distribution when $\alpha = 0$ and $\beta = 1$.

5. Conclusion

In this paper, we have proposed a new mechanism for generating power law distribution. It is more realistic than the preferential linking proposed by Barabási and Albert. Using this mechanism, a model is shown to generate the power law distribution with an exponent in a range of 3-to-5. Moreover, the present model could also generate the exponential degree distribution that is discovered in some networks.

The model proposed here would help us find the rules guiding one in forming real networks and also in modelling the real complex systems. In the future, we will give further analysis of the present model, such as the clustering coefficient, diameter, etc.

Acknowledgments

The authors would like to thank Réka Albert for useful discussion and are grateful to the anonymous referees for their valuable suggestions and comments, which have made this paper improved.

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