Vol 16 No 12, December 2007
1009-1963/2007/16(12)/3879-06
Chinese Physics
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Hawking radiation via tunnelling from general stationary axisymmetric black holes*

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(Received 14 April 2007; revised manuscript received 19 May 2007)

Hawking radiation is viewed as a tunnelling process. In this way the emission rates of massless particles and massive particles tunnelling across the event horizon of general stationary axisymmetric black holes are calculated, separately. The emission spectra of these two different kinds of outgoing particles have the same functional form and both are consistent with an underlying unitary theory.

Keywords: black hole, Hawking radiation, quantum theory
PACC: 9760L, 0420

1. Introduction

In 2000, Parikh and Wilczek presented a method to calculate the emission rate at which particles tunnel across the event horizon.[1−3] They treated Hawking radiation as a tunnelling process. They have found that the barrier is just created by the outgoing particles themselves, and their key insight is to find a coordinate system well-behaved at the event horizon. In this way they calculated the corrected emission spectrum of the particles from the spherically symmetric black holes, such as Schwarzschild black holes and Reissner–Norström black holes. Their results are consistent with an underlying unitary theory. Following this method, a number of static black holes have been studied.[4−17] The same results, that is, Hawking radiation is no longer purely thermal, the unitary theory is satisfied and information is conserved, have been obtained. In 2005 and 2006, in Refs.[18–23] the method was extended, and the emission rates of massive particles and charged particles were calculated, separately. The obtained results are also consistent with the unitary theory and support the information conservation. In this paper, we continue to discuss the emission process and calculate the emission rates of massless particles and massive particles tunnelling across the event horizon of general stationary axisymmetric black holes.

2. Massless particle tunnelling

As is well known, general stationary axisymmetric black hole spacetimes can be described by a general line element

$$\begin{aligned} \mathrm{d}s^2 =& g_{00}\mathrm{d}t_g^2 + g_{11}\mathrm{d}r^2 + g_{22}\mathrm{d}\theta^2 \\ &+ g_{33}\mathrm{d}\varphi^2 + 2g_{03}\mathrm{d}t_k\mathrm{d}\varphi. \end{aligned} \tag{1}$$

This metric contains the Kerr–Newman black hole spacetime and other stationary axisymmetric black hole spacetimes.[34−37] Apart from the Kerr–Newman black hole, the black holes given in Refs.[34–37] all have naked singularities which lie either in a stationary limit surface or in an event horizon surface. Thus, their stationary limit surfaces or event horizons are not topological sphere surfaces. By further study, one has found that because of the existence of naked singularities, the dragging velocities of these black holes either vanish or become negative, but they still have angular momenta. General speaking, there is a coordinate singularity in the metric (1) at the radius of the event horizon. As discussed in Ref.[1], in order to perform a tunnelling computation at the event horizon we should find a coordinate system that is well-behaved at the event horizon. Moreover, since in quantum mechanics, tunnelling across the barrier is an instantaneous process, constant-time slices of the spacetime should satisfy Landau's condition of the coordinate clock synchronization.[38]

We first investigate a dragged coordinate system. Let

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t_g} = -\frac{g_{03}}{g_{33}} = \Omega. \tag{2}$$

In the dragged coordinate system, the line element can

*Project supported by the National Natural Science Foundation of China (Grant Nos 10573005 and 10633010).
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be rewritten as

$$\mathrm{d}s^2 = \hat{g}_{00}\mathrm{d}t_g^2 + g_{11}\mathrm{d}r^2 + g_{22}\mathrm{d}\theta^2, \tag{3}$$

where

$$\hat{g}_{00} = g_{00} - \frac{(g_{03})^2}{g_{33}}. \tag{4}$$

Without loss of generalization, we let[39]

$$\hat{g}_{00} = \frac{r - r_{\mathrm{H}}}{\theta_1}, \quad \text{and} \quad g_{11} = \frac{1}{f \cdot (r - r_{\mathrm{H}})}, \tag{5}$$

where r_{H} is the radial coordinate of the event horizon, θ_1 and f are two functions of coordinates r and θ. From expression (5) we can easily find that the event horizon and the infinite red-shift surface are located at the same place in the dragged coordinate system. We can express the surface gravity κ with the functions θ_1 and f as follows:

The definition of the surface gravity is

$$\kappa = \lim_{r \to r_{\mathrm{H}}} (|b|\sqrt{-\hat{g}_{00}}\,), \tag{6}$$

where b is the proper acceleration of a mass point resting near the event horizon. The expression of b is[39]

$$b = -\frac{1}{2}\frac{1}{\sqrt{g_{11}}}\frac{\hat{g}_{00,1}}{\hat{g}_{00}}. \tag{7}$$

Substituting expressions (5) and (7) into Eq.(6), we obtain

$$\kappa = \Big(\frac{1}{2}\sqrt{\frac{f}{-\theta_1}}\,\Big)_{r_{\mathrm{H}}} = \Big(-\frac{1}{2}\frac{1}{\theta_1\sqrt{-\hat{g}_{00}g_{11}}}\Big)_{r_{\mathrm{H}}}. \tag{8}$$

Now we continue to investigate the dragged coordinate system. In fact, line element (3) represents a three-dimensional hypersurface in a four-dimensional general stationary axisymmetric spacetime. But we can easily obtain the Hawking purely thermal spectrum by using the Damour–Ruffini method in the dragged coordinate system.[19,39] It means that we can also discuss Hawking radiation via tunnelling in this dragged coordinate system. However, this coordinate system is not what we expect, because the metric is still singular at the event horizon and is not of flat Euclidean space in radial. To obtain an coordinate system analogous to Painlevé coordinates,[18,19,40] we first perform a coordinate transformation

$$\mathrm{d}t_g = \mathrm{d}t + F(r,\theta)\mathrm{d}r + G(r,\theta)\mathrm{d}\theta, \tag{9}$$

where $F(r,\theta)$ and $G(r,\theta)$ are two functions to be determined, and they satisfy the integrability condition

$$\frac{\partial F(r,\theta)}{\partial \theta} = \frac{\partial G(r,\theta)}{\partial r}. \tag{10}$$

Then, the new time coordinate can be expressed as

$$t = t_g - \int F(r,\theta)\mathrm{d}r + G(r,\theta)\mathrm{d}\theta. \tag{11}$$

As a corollary, we demand that the metric is flat Euclidean in radial to the constant-time slices. We then obtain the condition

$$g_{11} + \hat{g}_{00}F(r,\theta)^2 = 1. \tag{12}$$

From Eq.(10), we have

$$G(r,\theta) = \int \frac{\partial F(r,\theta)}{\partial \theta}\mathrm{d}r + C(\theta), \tag{13}$$

where $C(\theta)$ is an arbitrary analytic function of θ. There is no need to integrate expression (11). From expressions (9) and (10) we can work out the element

$$\begin{aligned}\mathrm{d}s^2 = {} & \hat{g}_{00}\mathrm{d}t^2 + 2\sqrt{\hat{g}_{00}(1-g_{11})}\mathrm{d}t\mathrm{d}r \\ & + \mathrm{d}r^2 + [\hat{g}_{00}G(r,\theta)^2 + g_{22}]\mathrm{d}\theta^2 \\ & + 2\hat{g}_{00}G(r,\theta)\mathrm{d}t\mathrm{d}\theta \\ & + 2\sqrt{\hat{g}_{00}(1-g_{11})}G(r,\theta)\mathrm{d}r\mathrm{d}\theta. \end{aligned} \tag{14}$$

We refer to expression (14) as the general axisymmetric Painlevé line element. The new coordinate system is well-behaved at the event horizon. Moreover, the metric in the new coordinate system satisfies the condition of coordinate clock synchronization. The verification of this feature is in the following.

According to Landau's theory of the coordinate clock synchronization[38] in a space-time decomposed in (3+1) dimensions, the coordinate time difference of two events, which take place simultaneously in different places, is

$$\Delta T = -\int \frac{g_{0i}}{g_{00}}\mathrm{d}x^i, \qquad (i = 1,2,3). \tag{15}$$

If the simultaneity of coordinate clocks can be transmitted from one place to another and has nothing to do with the integration path, components of the metric should satisfy[41]

$$\frac{\partial}{\partial x^j}\Big(-\frac{g_{0i}}{g_{00}}\Big) = \frac{\partial}{\partial x^i}\Big(-\frac{g_{0j}}{g_{00}}\Big), \quad (i,j = 1,2,3). \tag{16}$$

Substituting the components of metric (14) into Eq.(16) yields

$$\frac{\partial F(r,\theta)}{\partial \theta} = \frac{\partial G(r,\theta)}{\partial r}. \tag{17}$$

Equation (17) is identical with Eq.(10). That is to say, in the general axisymmetric Painlevé coordinate system we can define coordinate clock synchronization though the metric is not diagonal. This feature of the

new coordinates is necessary for us to discuss the tunnelling process.

In the following discussion, we will consider a massless particle tunnelling from the event horizon as an ellipsoid shell. To conserve the symmetry of the general stationary axisymmetric space-time, we think that the outgoing particle should be still an ellipsoid shell during the tunnelling process. That is, the particle does not have motion in the θ-direction ($\mathrm{d}\theta = 0$). Therefore, under these conditions ($\mathrm{d}\theta = \mathrm{d}s^2 = 0$) we obtain the radial null geodesics

$$\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}t} = \pm\sqrt{-\hat{g}_{00}g_{11}} - \sqrt{-\hat{g}_{00}(1-g_{11})}\,, \qquad (18)$$

where the $+(-)$ sign can be identified with outgoing(ingoing) radial motion.

We now focus on a semiclassical treatment of the associated radiation from general stationary axisymmetric black holes. As described above, the event horizon and the infinite red-shift surface are located in the same place. Therefore, when the outgoing wave is traced back towards the horizon, its wavelength as measured by local fiducial observers is ever-increasingly blue-shifted. Near the horizon, the radial wavenumber approaches infinity and the point particle, or Wentzel–Kramers–Brillouin (WKB) approximation is justified. We adopt the picture of a pair of virtual particles spontaneously created just inside the horizon. The positive energy virtual particle can tunnel out – no classical escape route exists – where it materializes as a real particle. The negative energy particle is absorbed by the black hole, resulting in a decrease in mass and angular momentum of the black hole. As mentioned above, we consider the particle as a shell (an ellipsoid shell) of energy ω' and angular momentum j'. If the particle self-gravitation is taken into account, expressions (14) and (18) should be modified. We fix the total mass and the total angular momentum of the space-time and allow the hole mass and hole angular momentum to fluctuate. Then, we should replace M with $M' = M - \omega'$, and J with $J' = J - j'$ in the metric (14) and the geodesic Eq.(18) to describe the motion of the shell.

When a particle tunnels out, the matter-gravity system makes a transition from its initial state to the final state. The action corresponding to this tunnelling process is

$$S = \int_{t_\mathrm{i}}^{t_\mathrm{f}} L\mathrm{d}t, \qquad (19)$$

which is related to the emission rate of the tunnelling particle by

$$\Gamma \sim \exp[-2\mathrm{Im}S]. \qquad (20)$$

However, In the dragged Painlevé coordinate system, the coordinate φ does not appear in the line element (14); that is, φ is an ignorable coordinate in the Lagrangian function L of the matter-gravity system. To eliminate this degree of freedom completely, the action should be rewritten as

$$S = \int_{t_\mathrm{i}}^{t_\mathrm{f}} (L - P_\varphi\dot{\varphi})\mathrm{d}t. \qquad (21)$$

Therefore, the imaginary part of the action is

$$\begin{aligned}\mathrm{Im}S &= \mathrm{Im}\Big\{\int_{r_\mathrm{i}}^{r_\mathrm{f}}\Big[P_r - \frac{P_\varphi\dot{\varphi}}{\dot{r}}\Big]\mathrm{d}r\Big\} \\ &= \mathrm{Im}\Big\{\int_{r_\mathrm{i}}^{r_\mathrm{f}}\Big[\int_{(0,\,0)}^{(P_r,\,P_\varphi)}\mathrm{d}P'_r - \frac{\dot{\varphi}\mathrm{d}P'_\varphi}{\dot{r}}\Big]\mathrm{d}r\Big\}, \qquad (22)\end{aligned}$$

where P_r and P_φ are the canonical momenta conjugate to r and φ, respectively.

To proceed with an explicit calculation, it is useful to apply Hamilton's equations

$$\dot{r} = \frac{\mathrm{d}H}{\mathrm{d}P_r}\Big|_{(r;\varphi,P_\varphi)} = \frac{d(M-\omega')}{\mathrm{d}P_r} = \frac{\mathrm{d}M'}{\mathrm{d}P_r}. \qquad (23)$$

If we treat the black hole as a rotating ellipsoid sphere with an angular speed Ω'_H, considering the self-gravitation we have

$$\Omega'_\mathrm{H} = \dot{\varphi}, \qquad (24)$$

and

$$J' = (M-\omega')a' = P'_\varphi. \qquad (25)$$

Substituting expressions (23), (24) and (25) into expression (22) yields

$$\mathrm{Im}S = \mathrm{Im}\Big\{\int_{r_\mathrm{i}}^{r_\mathrm{f}}\Big[\int_{(M,\,J)}^{(M-\omega,\,J-j)}\frac{1}{\dot{r}}\mathrm{d}M' - \frac{1}{\dot{r}}\Omega'_\mathrm{H}\mathrm{d}J'\Big]\mathrm{d}r\Big\}. \qquad (26)$$

Considering the self-gravitation and substituting the modified expression of $\dot{r}$ into expression (26), we obtain

$$\begin{aligned}\mathrm{Im}S = & -\mathrm{Im}\Big\{\int_{r_\mathrm{i}}^{r_\mathrm{f}}\Big[\int_{(M,\,J)}^{(M-\omega,\,J-j)}\frac{\sqrt{-\hat{g}'_{00}g'_{11}}+\sqrt{-\hat{g}'_{00}(1-g'_{11})}}{\hat{g}'_{00}}\mathrm{d}M' \\ & -\frac{\sqrt{-\hat{g}'_{00}g'_{11}}+\sqrt{-\hat{g}'_{00}(1-g'_{11})}}{\hat{g}'_{00}}\Omega'_\mathrm{H}\mathrm{d}J'\Big]\mathrm{d}r\Big\},\end{aligned} \tag{27}$$

where $\hat{g}'_{00}$ and g'_{11} are functions of M' and J'; that is, we should replace M with M', and J with J'. Since $\hat{g}'_{00}$ and g'_{11} can be expressed as expression (5), we have

$$\begin{aligned}\mathrm{Im}S = & -\mathrm{Im}\Big\{\int_{r_\mathrm{i}}^{r_\mathrm{f}}\Big[\int_{(M,\,J)}^{(M-\omega,\,J-j)}\frac{\theta'_1\sqrt{-\hat{g}'_{00}g'_{11}}+\sqrt{-\hat{g}'_{00}(1-g'_{11})}}{r-r'_\mathrm{H}}\mathrm{d}M' \\ & -\frac{\theta'_1\sqrt{-\hat{g}'_{00}g'_{11}}+\sqrt{-\hat{g}'_{00}(1-g'_{11})}}{r-r'_\mathrm{H}}\Omega'_\mathrm{H}\mathrm{d}J'\Big]\mathrm{d}r\Big\}.\end{aligned} \tag{28}$$

where r'_H is the event horizon corresponding to M' and J'. We see that the $r=r'_\mathrm{H}$ is a pole. The integral can be evaluated by deforming the contour around the pole, so as to ensure that the positive energy solution decays in time. Switching the order of integration and performing an integration with respect to r first, we obtain

$$\begin{aligned}\mathrm{Im}S = & \pi\int_{(M,\,J)}^{(M-\omega,\,J-j)}\big(\theta'_1\sqrt{-\hat{g}'_{00}g'_{11}}+\sqrt{-\hat{g}'_{00}(1-g'_{11})}\big)_{r'_\mathrm{H}}\mathrm{d}M' \\ & -\big(\theta'_1\sqrt{-\hat{g}'_{00}g'_{11}}+\sqrt{-\hat{g}'_{00}(1-g'_{11})}\big)_{r'_\mathrm{H}}\Omega'_\mathrm{H}\mathrm{d}J'.\end{aligned} \tag{29}$$

Since

$$(\hat{g}'_{00})_{r'_\mathrm{H}}=0, \tag{30}$$

we have

$$\mathrm{Im}S=-\frac{1}{2}\int_{(M,\,J)}^{(M-\omega,\,J-j)}\frac{1}{T'}\mathrm{d}M'-\frac{\Omega'_\mathrm{H}}{T'}\mathrm{d}J', \tag{31}$$

where

$$T'=\frac{\kappa'}{2\pi}=-\frac{1}{4\pi}\Big(\frac{1}{\theta'_1\sqrt{-\hat{g}'_{00}g'_{11}}}\Big)_{r'_\mathrm{H}}. \tag{32}$$

According to the first law of black hole thermodynamics[42]

$$T'\mathrm{d}S'_\mathrm{BH}=\mathrm{d}M'-\Omega'_\mathrm{H}\mathrm{d}J', \tag{33}$$

namely,

$$\frac{\mathrm{d}M'}{T'}-\frac{\Omega'_\mathrm{H}\mathrm{d}J'}{T'}=\mathrm{d}S'_\mathrm{BH}, \tag{34}$$

we acquire

$$\mathrm{Im}S=-\frac{1}{2}[S_\mathrm{BH}(M-\omega,J-j)-S_\mathrm{BH}(M,J)]. \tag{35}$$

The tunnelling spectrum is therefore

$$\Gamma\sim\exp[-2\mathrm{Im}S]=\mathrm{e}^{\Delta S_\mathrm{BH}}. \tag{36}$$

It is consistent with an underlying unitary theory and takes the same functional form as that for spherically symmetric configurations.

3. Massive particle tunnelling

In this section we investigate the tunnelling of massive particles. Since the worldline of a massive quanta is not light-like, it does not follow radial-light-like geodesics (18) when it tunnels across the horizon. According to the wave-particle duality, the outgoing massive particle can be taken as a de Broglie wave. In a similar manner to that in Refs.[20–23], we can easily obtain the massive particle equation of motion, namely

$$\begin{aligned}\dot{r}=v_p=&-\frac{1}{2}\frac{\hat{g}_{00}}{\hat{g}_{01}} \\ =&\frac{1}{2}(r-r_\mathrm{H})\sqrt{\frac{f}{(-\theta_1)[1-f\cdot(r-r_\mathrm{H})]}}\end{aligned} \tag{37}$$

Note that to calculate the emission rate correctly, we should take into account the self-gravitation of the tunnelling particle with energy ω' and angular momentum j'. That is to say, we should replace M with $M'=M-\omega'$, and J with $J'=J-j'$ in the metric (14) and the equation of motion (37) to describe the motion of the shell. Similarly, we adopt the dragged painlevé-coordinate system (14). In this coordinate system, the action should be rewritten as

$$S=\int_{t_\mathrm{i}}^{t_\mathrm{f}}(L-P_\varphi\dot{\varphi})\mathrm{d}t. \tag{38}$$

Therefore, the imaginary part of the action is

$$\mathrm{Im}S=\mathrm{Im}\Big\{\int_{r_\mathrm{i}}^{r_\mathrm{f}}\Big[P_r-\frac{P_\varphi\dot{\varphi}}{\dot{r}}\Big]\mathrm{d}r\Big\}=\mathrm{Im}\Big\{\int_{r_\mathrm{i}}^{r_\mathrm{f}}\Big[\int_{(0,\,0)}^{(P_r,\,P_\varphi)}\mathrm{d}P'_r-\frac{\dot{\varphi}\mathrm{d}P'_\varphi}{\dot{r}}\Big]\mathrm{d}r\Big\}. \tag{39}$$

Similarly treating the imaginary part of action for massless particle gives

$$\mathrm{Im}S=\mathrm{Im}\Big\{\int_{r_\mathrm{i}}^{r_\mathrm{f}}\Big[\int_{(M,\,J)}^{(M-\omega,\,J-j)}\frac{1}{\dot{r}}\mathrm{d}M'-\frac{1}{\dot{r}}\Omega'_\mathrm{H}\mathrm{d}J'\Big]\mathrm{d}r\Big\}. \tag{40}$$

Substituting expression (37) into expression (40) yields

$$\mathrm{Im}S=\mathrm{Im}\Big\{\int_{r_\mathrm{i}}^{r_\mathrm{f}}\int_{(M,\,J)}^{(M-\omega,\,J-j)}\frac{2}{(r-r'_\mathrm{H})}\sqrt{\frac{(-\theta'_1)[1-f'\cdot(r-r'_\mathrm{H})]}{f'}}\mathrm{d}M'-\Omega'_\mathrm{H}\mathrm{d}J')\mathrm{d}r\Big\}. \tag{41}$$

Switching the order of integration and performing an integration with respect to r first, we obtain

$$\mathrm{Im}S=-2\pi\int_{(M,\,J)}^{(M-\omega,\,J-j)}\sqrt{\frac{-\theta'_1}{f'}}(\mathrm{d}M'-\Omega'_\mathrm{H}\mathrm{d}J'). \tag{42}$$

In fact, from expression (8) we have

$$\mathrm{Im}S=-\frac{1}{2}\int_{(M,\,J)}^{(M-\omega,\,J-j)}\frac{1}{T'}\mathrm{d}M'-\frac{\Omega'_\mathrm{H}}{T'}\mathrm{d}J'. \tag{43}$$

Using the first law of black hole thermodynamics, we obtain

$$\mathrm{Im}S=-\frac{1}{2}[S_\mathrm{BH}(M-\omega,J-j)-S_\mathrm{BH}(M,J)]. \tag{44}$$

Therefore, the emission spectrum of the massive particles is

$$\Gamma\sim\exp[-2\mathrm{Im}S]=\mathrm{e}^{\Delta S_\mathrm{BH}}. \tag{45}$$

It is also consistent with the underlying unitary theory and takes the same functional form as that of massless particles.

4. Conclusion

We have investigated the tunnelling processes of the massless particles and massive particles from general stationary axisymmetric black holes. The emission spectra of these two different kinds of outgoing particles have the same functional form and both are consistent with an underlying unitary theory. That is to say, the information is conserved during the evaporation of these stationary axisymmetric black holes, no matter which particles they emit.

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