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Intracavity power addition with a close-ended composite four-mirror cavity

Lei Ming(雷　鸣), Gong Ma-Li(巩马理),
Liu Qiang(柳　强), Zhang Hai-Tao(张海涛), and Wang Yun-Xiang(王云祥)

Center for Photonics and Electronics, State Key Laboratory of Tribology, Department of Precision Instruments and Mechanology, Tsinghua University, Beijing 100084, *China*

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We present a novel four-mirror cavity with two active gains to combine power intracavity and also give a detailed theoretical analysis of the combined gain. By using the effective field method, the four-mirror cavity with two gain media can be regarded as a linear resonator with one effective combined gain (ECG), and we procure a theoretical model of the ECG and deduce its exact analytical expression. When the two branch gains are close to each other, the combined gain can be reduced to their product, and the simplified presentation of ECG has been demonstrated. The combined output power which directly reflects the small signal ECG of the four-mirror cavity is studied experimentally, and the results are in good agreement with the theoretical ones.

Keywords: laser, composite four-mirror cavity, power addition, effective combined gain
PACC: 4260D, 4260H

1. Introduction

High power solid-state lasers are required in many applications,[1−4] such as basic scientific research and precise machining. However, a single high-power laser often suffers from poor beam quality, instability, and heat dissipation. The power and beam quality always compete with each other. Beam combining with multiple lasers is an effective approach to obtaining the high power output with good beam quality. In the past, researchers have brought up various cavities to combine laser beams, all of which are mainly based on the Michelson cavity with three end mirrors, such as the modified Michelson cavity,[5] Vernier–Michelson-type cavity.[6−8] However, as to these open-ended resonators, in order to obtain good combination results, the symmetries of the two branches in spacial distribution or polarization must be ensured by using additional components[6,7] or special media,[8] or else more loss will be produced from the open end of the beam splitter (BS). This condition is rather difficult to realize especially in the circumstances of high power, because the output of the three-mirror cavity tends to be more instable due to its open-ended configuration, which becomes the limit of power scaling. In addition, the branch length of cavities of this kind is rather long (at least 43 cm[8] or even longer) in order that more common longitudinal modes can exist within the gain bandwidth of the media (out of several hundreds of longitudinal modes).[6,8]

Recently, we have presented a new close-ended composite-cavity named the Four-Mirror Cavity to combine the beams' intracavity.[9] Unlike other laser configurations with three mirrors, the accession of the fourth mirror makes the alignment easier to realize and improves the output stability greatly. Another advantage of this cavity is that even if the two component laser gains are not fairly identical, no additional loss on the beam splitter is introduced with this close-ended configuration. In addition, the length of cavity can be rather short and there is no need for the additional elements within the cavity, thereby allowing for practical implementation in laser systems. We have reported some experimental investigations of this four-mirror cavity, including power, beam quality and the improved brightness in Ref.[9].

Although the power addition with composite cavity is so attractive for improving brightness, to the best of our knowledge, there is no systemic theoretical study of composite cavity for power combination to date, even in terms of the three-arm cavity. Most of the theories of the complex cavity with multi-arm reported before are mainly from the point of view of frequency selection. DiDomenico[5] first proposed a Michelson-type He–Ne laser with two gain media to

select single frequency output and theoretically analysed some characteristics of this composite cavity. Afterwards, Rigrod successively presented some theoretical analysis of three-arm cavity with three gains,[10] and Vernier–Michelson-type cavity with two gains[6] respectively. But all of these analyses are mostly based on the open-ended cavity with three arms and primarily for the purpose of mode selection. In recent years Binks *et al* and Ko proposed a grazing-incidence cavity with a four-arm configuration,[11−13] which can be regarded geometrically as the four-mirror cavity scheme if the grating is treated as a beam splitter. They have conducted some theoretical analysis of it without mentioning the power output, however, these models still mostly aim at mode selection, and there is only one active gain in the composite cavity. What is more, the whole cavity is based on the grating, which is fairly different from the real four-mirror cavity that uses a beam splitter.

In this paper, we procure a theoretical model to analyse this close-ended four-mirror cavity with two active gains in detail. The combined gain and power output obtained from our model are presented and compared with the experimental values.

2. Theoretical model of the four-mirror cavity with two gain media

A basic configuration for intracavity power addition of two lasers with four-mirror cavity is schematically presented in Fig.1. The cavity is based on a close-ended resonator, which is composed of four end mirrors M_1–M_4 with four arms. The amplification is provided by two amplifying media located in the two of branches, arm 1 and arm 2. Thank to a 50/50 beamsplitter, BS, the lasers produced by the two amplifying media combine together into one beam. They mainly interact in arm 3 and arm 4 and at last pass through the output coupler M_4.

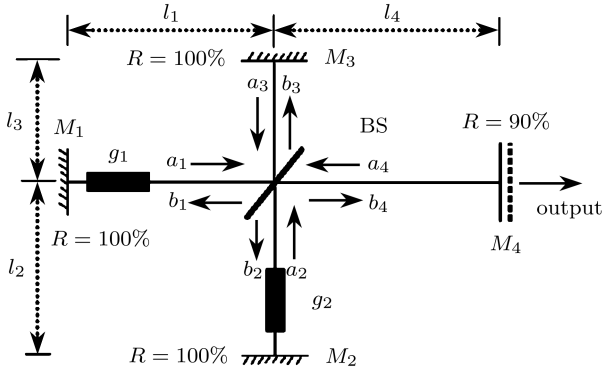


Fig.1. Schematic diagram of close-ended resonator of four-mirror cavity including inward coming fields and outward going fields. BS: beam splitter; M_1–M_3: perfectly reflecting end mirrors; M_4: output coupler; a_1–a_4: inward coming waves; b_1–b_4: outward going waves; g_1, g_2: the gain media

We assume that used in our experiment are uniform plane waves, a lossless beam splitter, perfectly reflecting end mirrors (M_1–M_3) at the laser wavelength (1064 nm), and M_4 that is the output coupler with partly reflectance of 1064 nm. Shown in Fig.1 is the four-mirror cavity laser with four sets of waves coupled by beam splitter BS. From the point of view of the beam splitter, each set of waves is made up of an outward going wave (b_i) and an inward coming wave (a_i). The waves b_i and a_i, are related by the transmission properties of each arm when they propagate from the beam splitter to a mirror and back to the beam splitter. The changes in the amplitude and the phase of a wave on a round trip in the jth arm are related by the function f_i

$$f_i = r_i \cdot \exp(\mathrm{i} \cdot 2kl_i), \tag{1}$$

where $k = 2\pi \cdot \gamma / c$, with γ being the frequency in units of radian and c the speed of light. So

$$a_i = b_i \cdot f_i = b_i \cdot r_i \cdot \exp(\mathrm{i} \cdot 2kl_i). \tag{2}$$

In this paper, because M_1–M_3 are highly reflective at the laser wavelength (1064 nm), we have $r_1 = r_2 = r_3 = 1$.

A lossless beam splitter is optically equivalent to a perfect directional coupler. We define its complex amplitude reflection coefficient r and its complex amplitude transmission coefficient t respectively as

$$r = |r| \cdot \exp(\mathrm{i} \cdot \alpha) \quad \text{and} \quad t = |t| \cdot \exp(\mathrm{i} \cdot \beta), \tag{3}$$

where

$$r \cdot r^* + t \cdot t^* = R + T = 1. \tag{4}$$

When the beam splitter's reference planes are chosen symmetrically, it can be shown[10] that

$$\beta = \alpha \pm \pi/2. \tag{5}$$

Therefore

$$r^2 = R \cdot \exp(\mathrm{i} \cdot 2\alpha), \tag{6}$$

$$t^2 = T \cdot \exp(\mathrm{i} \cdot 2\beta) = -T \cdot \exp(\mathrm{i} \cdot 2\alpha). \tag{7}$$

For the case of a 3-dB beam splitter, which is of interest here, we have,[5] without loss of generality,

$$r = 1/\sqrt{2} \cdot \exp(\mathrm{i} \cdot \pi/4),$$
$$t = 1/\sqrt{2} \cdot \exp(-\mathrm{i} \cdot \pi/4). \quad (8)$$

As there are two gain media and four arms in the four-mirror cavity configuration, in order to simplify the analysis, the four-mirror cavity can be regarded as a simple linear resonator geometry. Besides the output coupler M_4, the beam splitter and the mirrors M_1–M_3 can be considered to be one end mirror M_5 and an effective combined gain (ECG) g_{eff} respectively as shown in Fig.2. In a simple two-mirror laser cavity with a single gain medium, the net internal amplitude gain can be equivalent to the ratio of the emerging field amplitude to the amplitude of the field passing through the gain medium from the output side to the incident side. In a similar way, the effective combined amplitude gain can also be described by

$$g_{\mathrm{eff}} = \frac{b_4}{a_4}, \quad (9)$$

where a_4 is the incidence field to the BS and b_4 is the emerging field from the BS.

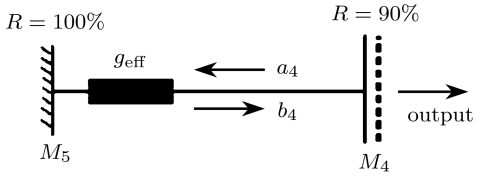


Fig.2. Schematic of the effective two-mirror laser cavity with the combined gain g_{eff}.

Using Smith's notation,[14] we designate the round-trip amplitude gain in arm l_i by g_i in the four-mirror cavity, and neglect the dispersion associated with laser gain. As shown in Fig.1, the waves are interrelated on the beam splitter. Thus, at the beam splitter we have

$$b_1 = r \cdot a_3 + t \cdot a_4 = r \cdot f_3 \cdot b_3 + t \cdot a_4, \quad (10)$$
$$b_2 = t \cdot a_3 + r \cdot a_4 = t \cdot f_3 \cdot b_3 + r \cdot a_4, \quad (11)$$
$$b_3 = r \cdot a_1 + t \cdot a_2 = r \cdot g_1 \cdot f_1 \cdot b_1 + t \cdot g_2 \cdot f_2 \cdot b_2, \quad (12)$$
$$b_4 = t \cdot a_1 + r \cdot a_2 = t \cdot g_1 \cdot f_1 \cdot b_1 + r \cdot g_2 \cdot f_2 \cdot b_2, \quad (13)$$

where r is the complex amplitude reflection coefficient and t is the complex amplitude transmission coefficient of the beam splitter.

Solving Eqs.(10)–(13) and rearranging the result, we obtain

$$g_{\mathrm{eff}} = \frac{b_4}{a_4} = \frac{t^2 \cdot f_1 g_1 + r^2 \cdot f_2 g_2 - f_1 g_1 \cdot f_2 g_2 \cdot f_3 (r^2 - t^2)^2}{1 - f_3 \cdot (r^2 \cdot f_1 g_1 + t^2 \cdot f_2 g_2)}. \quad (14)$$

Introduce the values of r and t in expression (8) into expression (14), then the following expression will be obtained:

$$g_{\mathrm{eff}} = \frac{-f_1 g_1 + f_2 g_2 - 2\mathrm{i} \cdot f_1 g_1 \cdot f_2 g_2 \cdot f_3}{-2\mathrm{i} - f_3 \cdot f_1 g_1 + f_3 \cdot f_2 g_2}, \quad (15)$$

with f_i shown in expression (1), and the complex combined effective gain will be given by

$$g_{\mathrm{eff}} = \frac{-g_1 \mathrm{e}^{\mathrm{i}2kl_1} + g_2 \mathrm{e}^{\mathrm{i}2kl_2} - 2\mathrm{i} \cdot g_1 g_2 \mathrm{e}^{\mathrm{i}2k(l_1+l_2+l_3)}}{-2\mathrm{i} - g_1 \mathrm{e}^{\mathrm{i}2k(l_1+l_3)} + g_2 \mathrm{e}^{\mathrm{i}2k(l_2+l_3)}}. \quad (16)$$

Once the system is fixed, all the parameters in expression (16) are already known except k, and $k = 2\pi \cdot \gamma/c$, so the combined effective gain can be expressed by a function of frequency γ.

Here we give the analytic expression of the complex effective gain for the four-mirror cavity as a function of g_1 and g_2. Therefore, if the branch gains (BGs) are threshold gains $g_{1\mathrm{th}}$ and $g_{2\mathrm{th}}$, and by introducing them into expression (16), we can accordingly obtain the threshold effective combined gain (ECG) g_{effth}. On the other hand, if the BGs are small-signal gains g_1^0 and g_2^0, we will obtain the small-signal ECG g_{eff}^0.

2.1. Mode resonance condition (MRC) and threshold ECG of the cavity for mode selecting

Like the modes in a linear resonator with two mirrors, the modes of the four-arm resonator occur when the round-trip factor increases the initial field without introducing a phase change. Any phase change that was introduced on a single pass would eventually lead to destructive interference after many round trips. Therefore, the modes occur when the gain is both positive and real.[14] Whether these potential modes actually are oscillatory is determined by the low loss they experience.[11,12] We define these three qualifications as the mode resonance condition (MRC) and it is necessary for the modes to fully satisfy all the requirements of the MRC. Therefore, we can conclude

that the actual combined gain must be a real one, and the magnitude of the gain at the chosen principal frequencies is just the actual gain really existing.

From among the three conditions of MRC, firstly we analyse the loss (L) of the four-mirror cavity in this sub section by using the 'self-consistent wave' method. One can use the gain function to calculate the modulation of the cavity loss. When the gain obtained from expression (16) is equal to the threshold gain (i.e. the saturation gain), based on the self-consistency condition the cavity loss can be expressed as[14]

$$L = 1 - \frac{1}{g_{\rm th} \cdot g_{\rm th}^*}. \tag{17}$$

For a laser resonator in which all losses are negligible except for transmission through the output coupler, the output is composed of modes at the frequencies determined by expression (16), which will be modulated by expression (17).

The four-arm cavity in Fig.1 can be regarded as a simple linear resonator consisting of a mirror, M_1, the gain g_1 and a complex output coupler composed of the beam splitter, the gain g_2 and mirrors M_2–M_4, as Fig.3(a) shows. Using the same method we analyse the combined gain $g_{\rm eff}$, where the reflectivity of the complex output coupler, $r_{\rm eff1}$, can be given by

$$\begin{aligned} r_{\rm eff1} &= \frac{b_1}{a_1} = \frac{-f_4 + f_3 - 2{\rm i} \cdot f_2 g_2 f_3 f_4}{-2{\rm i} - f_2 g_2 (f_4 - f_3)} \\ &= \frac{-r_4 {\rm e}^{{\rm i}2kl_4} + {\rm e}^{{\rm i}2kl_3} - 2{\rm i} \cdot g_2 \cdot r_4 \cdot {\rm e}^{{\rm i}2k(l_2+l_3+l_4)}}{-2{\rm i} - g_2 r_4 {\rm e}^{{\rm i}2k(l_2+l_4)} + g_2 {\rm e}^{{\rm i}2k(l_2+l_3)}}, \end{aligned} \tag{18}$$

where b_1 and a_1 are, respectively, the outward going wave and the inward coming wave to the beam splitter, as shown in Fig.1.

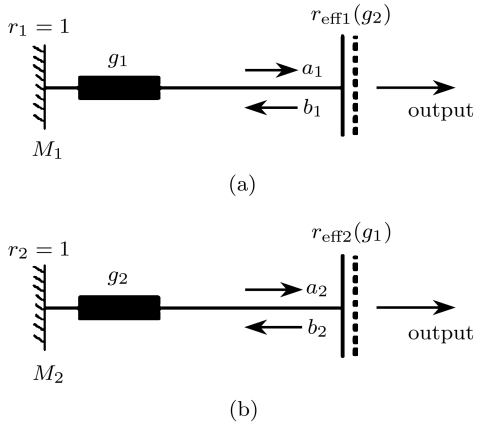


Fig.3. Schematic of the effective two-mirror laser cavity with the effective reflectivity $r_{\rm eff1}$ (a) and schematic of the effective two-mirror laser cavity with the effective reflectivity $r_{\rm eff2}$ (b).

When the laser system is in a stationary state, the wave after travelling one round trip is equivalent to the initial one, hence we can obtain that

$$a_1 \cdot r_{\rm eff1} \cdot g_{\rm 1th} \cdot f_1 = a_1. \tag{19}$$

Here $g_{\rm 1th}$ is the saturation gain or the threshold gain. Therefore

$$g_{\rm 1th} = \frac{1}{r_{\rm eff1} \cdot f_1} = \frac{-2{\rm i} - g_{\rm 2th} r_4 {\rm e}^{{\rm i}2k(l_2+l_4)} + g_{\rm 2th} {\rm e}^{{\rm i}2k(l_2+l_3)}}{-r_4 {\rm e}^{{\rm i}2k(l_1+l_4)} + {\rm e}^{{\rm i}2k(l_1+l_3)} - 2{\rm i} \cdot g_{\rm 2th} \cdot r_4 \cdot {\rm e}^{{\rm i}2k(l_1+l_2+l_3+l_4)}} = \frac{D_1 + {\rm i}E_1}{F_1}. \tag{20}$$

As discussed above, the mode exists only when the imaginary part of the gain is zero, so the numerator of the imaginary part E_1 is equal to zero. And we obtain

$$\begin{aligned} E_1 &= [2r_4 \cos(l_1 + l_4) - 2r_4^2 \cos(l_1 + l_3)] \cdot g_{\rm 2th}^2 + \left[\begin{array}{l} (-r_4 \sin(l_4 + l_2 - l_1 - l_3) - 4r_4 \sin(l_1 + l_2 + l_3 + l_4) \\ +(1 + r_4^2) \sin(l_2 - l_1) - r_4 \sin(l_3 + l_2 - l_1 - l_4) \end{array} \right] \cdot g_{\rm 2th} \\ &\quad \times [-2 \cos(l_1 + l_3) + 2r_4 \cos(l_1 + l_4)] \\ &= 0. \end{aligned} \tag{21}$$

Solve this equation we can find the root, i.e. the threshold branch gain $g_{\rm 2th}$.

In the same way, we regard the four-arm cavity as a simple linear resonator consisting of mirror M_2, the gain g_2 and a complex output coupler composed of the beam splitter, the gain g_1 and mirrors M_1, M_3 and M_4 as shown in Fig.3(b). Using the method of calculating $g_{\rm 1th}$, mentioned above, we can obtain the expression of $g_{\rm 2th}$ as follows:

$$g_{\rm 2th} = \frac{1}{r_{\rm eff2} \cdot f_2} = \frac{-2{\rm i} + g_{\rm 1th} r_4 {\rm e}^{{\rm i}2k(l_1+l_4)} - g_{1th} {\rm e}^{{\rm i}2k(l_1+l_3)}}{r_4 {\rm e}^{{\rm i}2k(l_2+l_4)} - {\rm e}^{{\rm i}2k(l_2+l_3)} - 2{\rm i} \cdot g_{\rm 1th} \cdot r_4 \cdot {\rm e}^{{\rm i}2k(l_1+l_2+l_3+l_4)}} = \frac{D_2 + {\rm i}E_2}{F_2}. \tag{22}$$

Take the imaginary part of the numerator above and let it be zero, then we will have

$$E_2 = \left[-2r_4\cos(l_2+l_4)+2r_4^2\cos(l_2+l_3)\right]\cdot g_{1th}^2 + \begin{bmatrix}(-r_4\sin(l_3+l_1-l_4-l_2)-4r_4\sin(l_1+l_2+l_3+l_4)\\ +(1+r_4^2)\sin(l_1-l_2)-r_4\sin(l_1+l_4-l_2-l_3)\end{bmatrix}\cdot g_{1\mathrm{th}} \times\left[2\cos(l_2+l_3)-2r_4\cos(l_4+l_2)\right] = 0. \quad (23)$$

Solving this equation, we can obtain the root, i.e. the other threshold branch gain of the four-mirror cavity $g_{1\mathrm{th}}$.

Introduce $g_{1\mathrm{th}}$ and $g_{2\mathrm{th}}$ into the combined gain expression (16) and then into the cavity loss expression (17), then we will be finally able to plot the profile of the cavity loss against the frequency γ as shown in Fig.5, and we will use this loss modulation profile to select the potential mode which will actually oscillate.

2.2. Value of the small- signal ECG of four-mirror cavity

(1) $g_1^0 = g_2^0$

Inasmuch as the ECG is a function of frequency γ, according to expression (16), one can readily find the modes by plotting the real and imaginary parts of the gain against frequency. When the two BGs in arm 1 and arm 2 are equal, we plot the effective gain for the four-mirror cavity in Fig.4 with the experimental four-mirror cavity parameters listed in Table 1. From Fig.4, we can obtain the potential modes which most likely oscillate. We can find a rule from Fig.4 that when the real part of the ECG attains to a local maximum value, the imaginary part of the effective gain happens to be zero. Therefore, we only need to find the potential mode with a low loss, i.e. the principal mode, and the magnitude of the real part of the combined gain at this mode is just the effective gain g_1 that we expect to obtain.

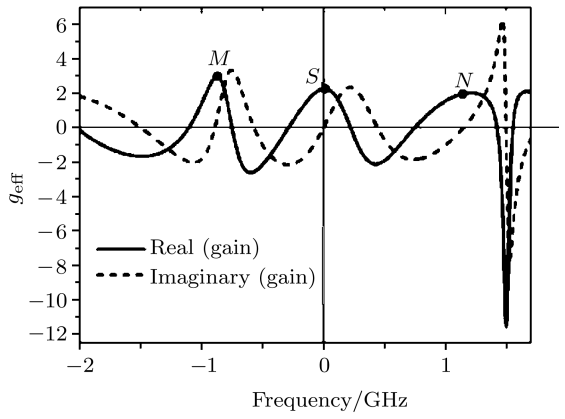


Fig.4. Real and imaginary components of ECG for the four-mirror cavity with identical BGs: $g_1 = 1.5$ and g_2=1.5 calculated with the values listed in Table 1. The positions of the potential modes M, S and N are marked with solid circles.

Table 1. Cavity parameters for the four-mirror cavity.

Cavity Parameter	Value
Length of arm 1, l_1 /mm	40
Length of arm 2, l_2 /mm	60
Length of arm 3, l_3 /mm	50
Length of arm 4, l_4 /mm	80
Reflectance of M_4, R_4	0.90

As the mode profile varies periodically with frequency, we draw the graph against relative frequency from −2 to +1.7 GHz in Fig.4. The positions of the possible modes M, S and N are given by the points where the imaginary component of the gain is zero and the real component is positive. Which of these potential modes will actually oscillate depends on the relatively low loss that each of them experiences. Apply the method depicted in subsection 2.2 to calculating the loss and draw the profile of it in Fig.5, then it will be evident that there are large differences in loss among different modes. The loss of mode M is negative because the associated gain at this mode in expression (17) is less than 1, thus this mode will not oscillate. In addition, mode N happens to suffer high loss and hence is suppressed too. Only mode S at 0 GHz, with a low loss near to zero which agrees well with all the qualifications for MRC simultaneously, will oscillate in the end. Hence insert this principal frequency $\gamma = 0$ GHz into expression (16), then the analytical expression of the effective gain with equal

individual gains will be obtained as follows:

$$\begin{aligned} g_{\text{eff}} &= \frac{-g_1 e^{i\cdot 0} + g_2 e^{i\cdot 0} - 2i \cdot g_1 g_2 e^{i\cdot 0}}{-2i - g_1 e^{i\cdot 0} + g_2 e^{i\cdot 0}} \\ &= \frac{-g_1 + g_2 - 2i \cdot g_1 g_2}{-2i - g_1 + g_2} \\ &= g_1 g_2 = g_1^2 = g_2^2. \end{aligned} \tag{24}$$

We can see that when the gains in the two branch arms are equivalent, the ECG equals the square of the individual gain. For example, the position of point S in Fig.4 clearly indicates that the value of the ECG is 2.25 when the two BGs both are 1.5 at 0 GHz.

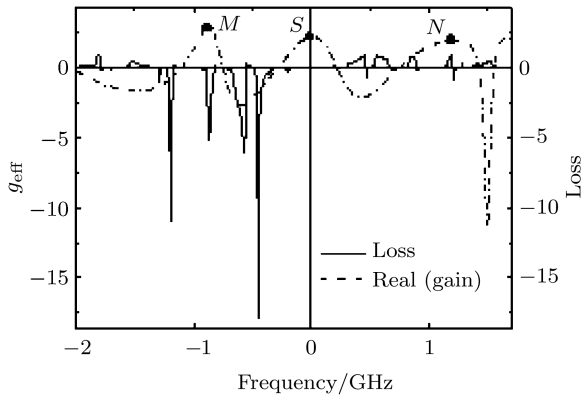


Fig.5. Loss modulation for the mode selecting of the four-mirror cavity with equal BG calculated with the values listed in Table 1. Solid line represents the fractional loss and dashed line refers to the real part of the combined gain. The positions of the potential modes are marked with solid circles.

(2) $g_1^0 \neq g_2^0$

When the two BGs are not identical, the oscillating mode structure of the ECG is a little different. The potential modes M', S' and N' are shown in Fig.6

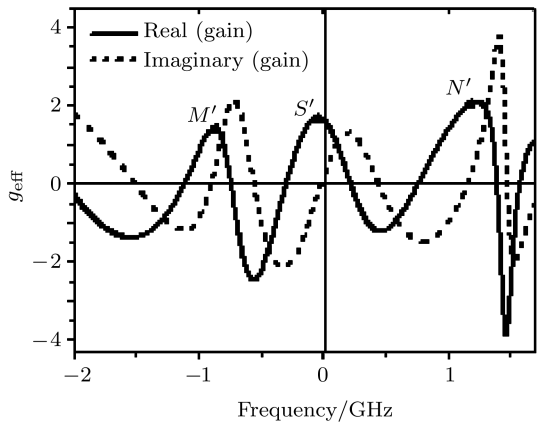


Fig.6. Real and imaginary components of ECG for the four-mirror cavity with different BGs: $g_1 = 1.5$ and g_2=1.1 calculated with the other values listed in Table 1. The positions of the potential modes M', S' and N' are marked with solid circles.

and the loss modulation for mode selecting are indicated in Fig.7. Therefore we can see that the principal mode (S') does not oscillate at 0 GHz any more. There is a little deviation from 0GHz. Therefore, we will use the mathematical approach to calculate the gain at the principal mode with different BGs.

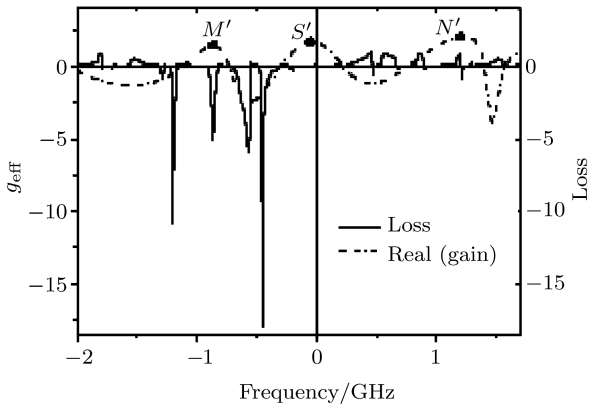


Fig.7. Loss modulation for the mode selecting of the four-mirror cavity with two different BGs calculated with the values listed in Table 1. Solid line represents the fractional loss and dashed line indicates the real part of the combined gain. The positions of the potential modes are marked with solid circles.

Firstly, we differentiate the real part of the effective gain g_{eff} in expression (16) with respect to frequency γ. Secondly, we construct differential equation $[\text{Re}(g_{\text{eff}})]' = \left[\frac{B}{A}\right]' = 0$, which yields one of the roots around 0 GHz (γ_0) in terms of some pairs of g_1 and g_2. Thirdly, insert γ_0 into expression (16), then the real values of ECG g_{cal} will be deduced correspondingly by using the model with respect to the principal mode γ_0. We define the error as follows:

$$E_{\text{error}} = g_{\text{cal}}^0 - g_1^0 \times g_2^0. \tag{25}$$

Assuming that g_1 changes from 1 to 1.5 and g_2 changes from 1 to 1.5, the g_1- and g_2-dependent error is shown in Fig.8.

From the analysis above and the curves shown in Fig.8, we can draw the conclusions as follows: when the gain imbalance of the two elementary lasers attaints to 24% (for example, $g_1 = 1.4$ and $g_2 = 1.1$), the error brought by the approximation is only 0.015, which is so small that the error can be neglected. Therefore, the combined gain of the four-mirror cavity can be simplified into the product of the two BGs, i.e.

$$g_{\text{eff}}^0 \approx g_1^0 \times g_2^0. \tag{26}$$

When the two BGs are equal, $g_{\text{eff}}^0 = g_1^0 \times g_2^0$.

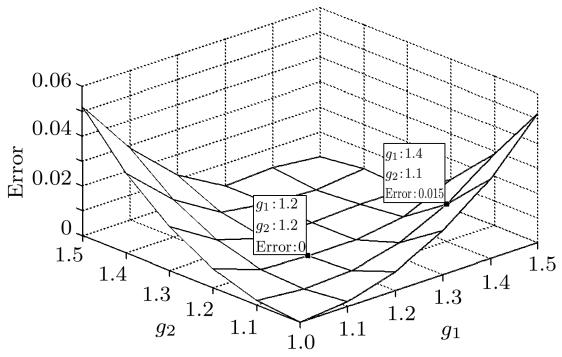


Fig.8. Error brought by the difference between $g_1 \times g_2$ and the calculated gain by the model. There is no error in the case of $g_1 = g_2$.

In summary, we have constructed the theoretical model of the combined gain for the four-mirror cavity with two gain media. The exact analytic expressions of the ECG are demonstrated here for the first time to our knowledge. Based on this, we give the simplified representation of the combined gain. In the next section, we will verify this theoretical model with the experiment results.

3. Theoretical model and experimental results of the power output

3.1. Theoretical analysis of the power output

As is known, the signal gain is closely related to the power output. When the laser system is fixed, the intensity of the power output directly reflects the scale of the small-signal gain. Therefore, here we use the power output to test the small-signal ECG.

One can obtain the power output expression related to the small-signal gain as follows:[11]

$$P_{\text{out}} = A \cdot I_{\text{s}} \cdot \left(\frac{1-R}{1+R}\right)\left(\frac{2g_{\text{e}}^0 l'}{-\ln R + \delta} - 1\right), \quad (27)$$

where I_{s} is a material parameter representing the saturation intensity; A and l' are the cross-section and the length of the laser rod, respectively; R is the reflectivity of the output coupler as defined above; δ is the loss factor of the resonator. These quantities are usually known when the laser system is fixed, except the unsaturated gain coefficient (i.e. small-signal gain coefficient) g_{e}^0. We relate g_{e}^0 to system parameters, and describe the methods of deducing the combined gain and the power output of laser.

As discussed in Section 2, we have defined g_{i} as the round-trip amplitude gain in arm l_{i}. It is important to note that here the small-signal gain coefficient g_{e}^0 is the exponential factor and it is referred to as small signal, unsaturated, single pass gain. So there is a relationship between g_{i} and g_{ei}^0 , i.e. $g_{\text{i}} = \text{e}^{2\cdot g_{\text{ei}}^0 \cdot l'}$. For a given pump power P_{pump} in the laser rod, the small-signal exponential gain coefficient g_{e}^0 can be expressed as

$$\begin{aligned} g_{\text{e}}^0 &= \sigma_{21} \cdot \Delta n^0 = \sigma_{21} \cdot \tau_2 \cdot \frac{\eta \cdot P_{\text{pump}}}{h\gamma_l \cdot V} \\ &= \frac{\eta \cdot P_{\text{pump}}}{I_{\text{s}} \cdot V}, \end{aligned} \quad (28)$$

where I_{s} is the saturation intensity and expressed as $I_{\text{s}} = h\gamma_l/(\sigma_{21} \cdot \tau_2)$ with γ_l being the output laser frequency; Δn^0 means the population inversion density; η is the overall efficiency including all the factors involved in the power transfer mechanism of a laser; V is the volume expressed as $V = A \cdot l' = \pi\omega_0^2 \cdot l'$ with ω_0 being the radius of the laser beam and its value being set to be corresponding to the beam analyser (Spiricon M^2-200) representation in the experiment. Table 2 shows the parameters of Nd:YAG crystal and laser system used in calculation.

When the two branches (arm1 and arm2 shown in Fig.1.) are pumped by some given pumping power, from expression (28) we can obtain the exponential gain coefficients g_{e1}^0 and g_{e2}^0 respectively, and then the round-trip amplitude gains g_1 and g_2 from $g_1 = \text{e}^{2\cdot g_{\text{e1}}^0 \cdot l_1'}$ and $g_2 = \text{e}^{2\cdot g_{\text{e2}}^0 \cdot l_2'}$. As two similar laser diodes (LDs) and laser rods are used in the experiment, the two BGs are close to each other. From the analysis in Section 2 we know that the ECG can be expressed as $g_{\text{eff}} = g_1 \times g_2$ in the four-mirror cavity. So the combined gain g_{eff} can be deduced from the known g_{i}. As shown in the power-output expression (27), the small-signal gain g_{ei}^0 is the exponential factor, hence

$$\begin{aligned} g_{\text{e_eff}}^0 &= \ln(g_{\text{eff}}) = \ln(g_1 \cdot g_2) \\ &= \ln(\text{e}^{2\cdot g_{\text{e1}}^0 \cdot l_1'} \cdot \text{e}^{2\cdot g_{\text{e2}}^0 \cdot l_2'}) \\ &= 2(g_{\text{e1}}^0 \cdot l_1' + g_{\text{e2}}^0 \cdot l_2'). \end{aligned} \quad (29)$$

Table 2. Parameters of Nd:YAG crystal and laser system.

Laser wavelength	λ/ nm	1064
Stimulated emission cross section	$\sigma_{21}/10^{-23}\mathrm{m}^2$	2.8
Planck's constant	$h/10^{-34}$J· S	6.62
Upper laser level lifetime	τ_2/ms	0.23
Saturation intensity	$I_s/(\mathrm{W}\ /\ \mathrm{m}^2)$	2.9×10^7
Beam radius	ω_0/mm	0.2
Resonator loss factor	δ	0.05
Efficiency of four-mirror cavity 1	$\eta_1/\%$	36
Efficiency of four-mirror cavity 2	$\eta_2/\%$	30
Rod radius	r/mm	3.5
Rod length	l'/mm	5.2
Reflectivity of output coupler	R	0.9

Introduce $g^0_{\mathrm{e_eff}}$ into expression (27), we can finally obtain the theoretical combined power output of the four-mirror cavity for two given individual pump powers P_{pump} in the two laser rods as shown in Fig.9.

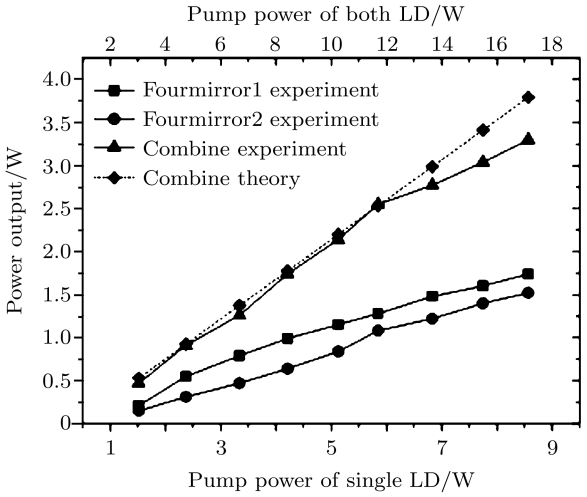


Fig.9. Dependence of the experimental and theoretical power outputs on the pump power launched on each diode laser. The solid circles and the solid squares represent the experimental power output of the four-mirror cavity when only one LD (LD_1 or LD_2) is pumping; The solid triangles and the solid diamonds refer to the experimental and theoretical power outputs of the four-mirror cavity when LD_1 and LD_2 are pumping simultaneously with the pump power at the top.

3.2. Experimental results of the power output

In this section, we use the experiment that we have conducted[9] to verify the theoretical model of the small-signal ECG and the power output. The experimental setup for intracavity power addition of two lasers with four-mirror cavity is schematically presented in Fig.10. Two flat input mirrors M_1 and M_2 are highly reflective at the laser wavelength (1064 nm) and anti-reflection coated at the pump wavelength (808 nm); M_3 is a flat 100% reflector and M_4 is the output mirror with 90% transmission at 1064 nm. Two individual Nd:YAG 7 mm×7 mm×5.2 mm crystals with 1.1-at.% doping concentration each have an end pumped by two similar fibre-coupled CW diode lasers.

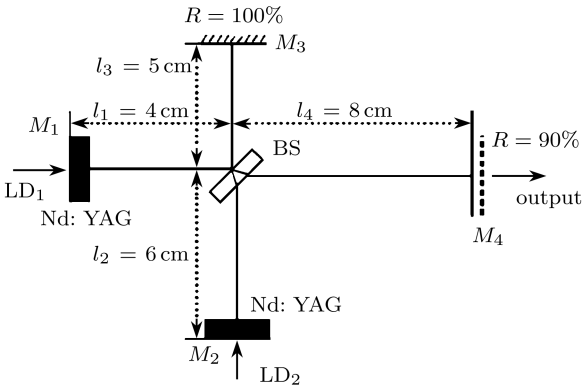


Fig.10. Schematic of the experimental setup of four-mirror cavity. BS: beam splitter; LD: laser diode; M_1–M_4: mirrors.

The powermeter EPM2000 is used to measure the power outputs of the two individual lasers and the combined laser. We show the experimental results, i.e. the power output of four-mirror cavity when only one LD (LD_1 or LD_2) is pumping and that of the combined power when both LD_1 and LD_2 are pumping simultaneously in Fig.10.

A single beam output exceeding 3.3 W is obtained when $LD_1$and LD_2 work simultaneously in the four-

mirror cavity. The combined power outputs calculated by using the method presented in Section 3.1 are also shown in Fig.10 for a comparison with the experimental combined power outputs. We can see that the experimental results of combined power output agree well with the theoretical ones, which convincingly confirms that the theoretical model built in this paper, which gives the analytical expression of the ECG and then the power output of the four-mirror cavity laser, is reasonable and correct.

4. Conclusions

We have demonstrated a new four-mirror cavity configuration which is compact and easy to combine power intracavity and have conducted detailed theoretical study on this cavity. Regarding the four-mirror cavity with two BGs as a simple linear resonator with only one ECG, the exact analytical expression of ECG is presented in this paper, which is reported for the first time, we believe. As ECG is a function of the two BGs, when BG is a threshold gain, the threshold ECG and the loss can be obtained so that the real resonant mode is correspondingly selected out. When BG is the small-signal gain, the small-signal ECG can be obtained to calculate the scale of the resonant mode and then the power output. It is shown that the ECG can be simplified into the product of the two elementary gains BGs when these two BGs are close to each other. Especially when they two are equal to each other, the product can express the combined gain exactly. At last the small-signal ECG is tested experimentally by the combined power outputs, and the experimental results show good agreement with the theoretical ones, which effectively verify the theoretical model established in this paper.

References

[1] Fu Y J, Jian W, Zheng K *et al* 2005 *Chin. Phys.* **14** 2338
[2] Zhang Y P, Zheng Y, Zhang H Y *et al* 2006 *Chin. Phys.* **15** 2018
[3] Liu Q, Gong M L, Chen L *et al* 2005 *Acta Phys. Sin.* **54** 721 (in Chinese)
[4] Lei M, Gong M L, Qiang L *et al* 2007 *Proc. CLEO*
[5] DiDomenico M J 1966 *Appl. Phys. Lett.* **8** 20
[6] Sabourdy D, Kermene V, Berthelemot A D *et al* 2002 *Appl. Phys. B: Las. Opt.* **75** 503
[7] Ishaaya A A, Davidson N, Shimshi L *et al* 2004 *Appl. Phys. Lett.* **85** 2187
[8] Zhang Y P, Zheng Y, Zhang H Y *et al* 2006 *Chin. Phys.* **15** 2018
[9] Lei M and Gong M L 2006 *Las. Phys. Lett.* **4** 16
[10] Rigrod W W 1970 *IEEE J. Quant. Elect.* **6** 9
[11] Binks D J, Ko D K, Gloster L A *et al* 1998 *J. Mod. Opt.* **45** 1249
[12] Binks D J, Ko D K, Gloster L A *et al* 1998 *J. Opt. Soc. Am.* B **15** 2395
[13] Koechner W 1999 *Solid-State Laser Engineering* (New York: Springer)
[14] Smith P W 1965 *IEEE J. Quant. Elect.* **1** 343