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Properties of field quantum entropy evolution in the Jaynes–Cummings model with initial squeezed coherent states field*

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The properties of the field quantum entropy evolution in a system of a single-mode squeezed coherent state field interacting with a two-level atom is studied by utilizing the complete quantum theory, and we focus our attention on the discussion of the influences of field squeezing parameter γ, atomic distribution angle θ and coupling strength g between the field and the atom on the properties of the evolution of field quantum entropy. The results obtained from numerical calculation indicate that the amplitude of oscillation of field quantum entropy evolution decreases with the increasing of squeezing parameter γ, and that both atomic distribution angle θ and coupling strength g between the field and the atom can influence the periodicity of field quantum entropy evolution.

Keywords: quantum field entropy, single-mode squeezed coherent states, Jaynes-Cummings model
PACC: 4250, 0365

1. Introduction

The standard Jaynes–Cummings model (JCM)[1] is an exactly solvable model of a single-mode quantized field interacting with a single two-level atom in a lossless cavity. In the years, investigators have paid much attention to the properties of the entropy in the JCM and its generalizations due to the quantum entropy theory of the field interacting with an atom presented by Phoenix and Knight (P-K).[2−4] They have shown that quantum entropy is a very convenient and sensitive operational measure of the purity of the quantum state because it automatically includes all moments of the density operator of the system. The time evolution of the field (atom) quantum entropy reflects the time evolution of the degree of the quantum entanglement between the field and the atom. The higher the quantum entropy is, the greater the quantum entanglement will be.

The squeezed state light field has aroused the interest of both theorists and experimentalists because of its wide applications in the low-noise light communications,[5] weak signal detection technique,[6] quantum-nondestructive measurement technique,[7] and quantum communication field[8] and so on. Some investigators have studied the evolution of the field quantum entropy in the JCM and its generalizations with initial coherent-state field,[9−17] vacuum-state field,[18] and squeezed vacuum-state field.[19,20] Other investigators have examined the interaction between two coupling-atoms and a single-mode squeezed coherent state light field,[21] and the entropic squeezing properties of squeezed coherent states.[22] However, up to now, less attention has been paid to the study of the evolution of the field quantum entropy in the squeezed coherent states of JCM. In this paper, we investigate the properties of the field quantum entropy evolution in the system of a single-mode squeezed coherent state field interacting with a two-level atom by utilizing the complete quantum theory, and we discuss the influence of field squeezing parameter γ, atomic distribution angle θ and coupling strength g between the field and the atom on the properties of the evolution of the field quantum entropy.

The remainder of this paper is organized as follows: in Section 2, we introduce our Hamiltonian model and give an exact expression of reduced density matrix $\rho_{\rm f}(t)$ and eigenvalues $\pi_{\rm f}^{\pm}$ of the field; in Section 3, we calculate the field quantum entropy and entanglement and give the theoretical analysis; finally

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a brief conclusion is given in Section 4.

2. The model and its solution

We consider a single-mode squeezed coherent state field interacting with a two-level atom through one-photon transition process. The Hamiltonian of this system can be written with the dipole interaction in the rotating-wave approximation (RWA) as ($\hbar = 1$)

$$H = \omega a^+ a + \omega_0 S_z + g(aS_+ + a^+ S_-), \tag{1}$$

where a^+ and a are the photon creation and annihilation operators of the single-mode field with frequency ω respectively; S_z and $S_\pm$ are the usual pseudo-spin operators of the two-level atom with a transition frequency ω_0; g is the coupling constant between the two-level atom and the single-mode squeezed coherent field. In what follows, we consider only resonance conditions so that $\omega = \omega_0$. The effective Hamiltonian in the interaction picture is given by

$$H_{\rm I} = g(aS_+ + a^+ S_-). \tag{2}$$

Using the standard technique in Ref.[2], from this Hamiltonian we can obtain the following time-evolution operator:

$$\begin{aligned} U_{\rm I}(t) &= {\rm e}^{-{\rm i}H_{\rm I}t} \\ &= \begin{pmatrix} \cos(gt\sqrt{aa^+}) & -{\rm i}a\dfrac{\sin(gt\sqrt{a^+a})}{\sqrt{a^+a}} \\ -{\rm i}a^+\dfrac{\sin(gt\sqrt{aa^+})}{\sqrt{aa^+}} & \cos(gt\sqrt{a^+a}) \end{pmatrix}, \end{aligned} \tag{3}$$

Here we consider the situation that the two-level atom is in a coherent superposition state of the excited state $|e\rangle$ and the ground state $|g\rangle$ at the initial time $t = 0$, namely

$$|\psi_a(0)\rangle = \cos\frac{\theta}{2}|e\rangle + {\rm e}^{-{\rm i}\phi}\sin\frac{\theta}{2}|g\rangle, \tag{4}$$

where $0 \le \theta \le \pi$ with θ denoting the atomic distribution angle and $0 \le \phi \le 2\pi$ with ϕ being the phase angle of the atom dipole, while the field is in the single-mode squeezed coherent state,[23] that is

$$|\psi_{\rm f}(0)\rangle = |\beta\rangle_{\rm g} = \sum_{n=0}^{\infty} F(n)\,|n\rangle, \tag{5}$$

where $F(n)$ is the statistical distribution of the photon numbers of the single-mode squeezed coherent field, and expressed as

$$\begin{aligned} F(n) \;=\; & (n!\mu)^{-1/2}(\nu/2\mu)^{n/2} H_n\left[\beta(2\mu\nu)^{-1/2}\right] \\ & \times \exp\left[-\frac{1}{2}|\beta|^2 + \frac{\nu^*}{2\mu}\beta^2\right], \end{aligned} \tag{6}$$

where γ is the squeezing parameter of the single-mode squeezed coherent field. If both the direction angle of the excitation for the field φ and the squeezing phase angle α of the field are zero, then $\mu = \cosh\gamma$, $\nu = \sinh\gamma$ and $\beta = |\beta| = \bar{n}^{1/2}$, with $\bar{n}$ representing the initial average photon number of the squeezed coherent field.

If we consider that at the initial time $t = 0$ the two-level atom and the single-mode squeezed coherent field are decoupled, then the initial state of the system can be written as

$$\begin{aligned} |\psi(0)\rangle \;&=\; |\psi_a(0)\rangle \otimes |\psi_f(0)\rangle \\ &= \sum_{n=0}^{\infty} F(n)\left[\cos\frac{\theta}{2}|n,e\rangle + {\rm e}^{-{\rm i}\phi}\sin\frac{\theta}{2}|n,g\rangle\right]. \end{aligned} \tag{7}$$

At any time $t > 0$ the state vector of the system is given by

$$|\psi(t)\rangle = U_I(t)\,|\psi(0)\rangle = |C\rangle\,|e\rangle + |D\rangle\,|g\rangle, \tag{8}$$

Where

$$\begin{aligned} |C\rangle \;&=\; \sum_{n=0}^{\infty} F(n)\left[\cos\left(\frac{\theta}{2}\right)\cos(gt\sqrt{n+1})\,|n\rangle - {\rm i}{\rm e}^{-{\rm i}\phi}\sin\left(\frac{\theta}{2}\right)\sin(gt\sqrt{n})\,|n-1\rangle\right], \\ |D\rangle \;&=\; \sum_{n=0}^{\infty} F(n)\left[-{\rm i}\cos\left(\frac{\theta}{2}\right)\sin(gt\sqrt{n+1})\,|n+1\rangle + {\rm e}^{-{\rm i}\phi}\sin\left(\frac{\theta}{2}\right)\cos(gt\sqrt{n})\,|n\rangle\right]. \end{aligned} \tag{9}$$

Therefore, at any time $t > 0$ the density matrix for the system is given by

$$\rho(t) = |\psi(t)\rangle\langle\psi(t)| = \begin{pmatrix} |C\rangle\langle C| & |C\rangle\langle D| \\ |D\rangle\langle C| & |D\rangle\langle D| \end{pmatrix}, \quad (10)$$

and the reduced density matrix of the single-mode squeezed coherent field is given by

$$\rho_{\rm f}(t) = {\rm tr}_a[\rho(t)] = |C\rangle\langle C| + |D\rangle\langle D|. \quad (11)$$

Using the standard technique in Ref.[2], it can be shown that the eigenvalues of the reduced density operator $\hat{\rho}_{\rm f}(t)$ are given by

$$\begin{aligned} \pi_f^{\pm} &= \langle C \mid C\rangle \pm \exp(\mp\delta)\,|\langle C \mid D\rangle| \\ &= \langle D \mid D\rangle \pm \exp(\pm\delta)\,|\langle D \mid C\rangle| \end{aligned} \quad (12)$$

where

$$\delta = \sinh^{-1}\left[\frac{1}{2\,|\langle C \mid D\rangle|}(\langle C \mid C\rangle - \langle D \mid D\rangle)\right], \quad (13)$$

$$\begin{aligned} \langle C|C\rangle =& \sum_{n=0}^{\infty}\left\{ |F(n)|^2\left[\cos^2\left(\frac{\theta}{2}\right)\cos^2(gt\sqrt{n+1}) + \sin^2\left(\frac{\theta}{2}\right)\sin^2(gt\sqrt{n})\right]\right. \\ &\left. -\frac{1}{4}{\rm i}{\rm e}^{-{\rm i}\phi}F^*(n-1)F(n)\sin\theta\sin(2gt\sqrt{n}) + \frac{1}{4}{\rm i}{\rm e}^{{\rm i}\phi}F^*(n+1)F(n)\sin\theta\sin(2gt\sqrt{n+1})\right\}, \end{aligned} \quad (14)$$

$$\begin{aligned} \langle D|D\rangle =& \sum_{n=0}^{\infty}\left\{ |F(n)|^2\left[\cos^2\left(\frac{\theta}{2}\right)\sin^2(gt\sqrt{n+1}) + \sin^2\left(\frac{\theta}{2}\right)\cos^2(gt\sqrt{n})\right]\right. \\ &\left. -\frac{1}{4}{\rm i}{\rm e}^{{\rm i}\phi}F^*(n+1)F(n)\sin\theta\sin(2gt\sqrt{n+1}) + \frac{1}{4}{\rm i}{\rm e}^{-{\rm i}\phi}F^*(n-1)F(n)\sin\theta\sin(2gt\sqrt{n})\right\}, \end{aligned} \quad (15)$$

$$\begin{aligned} \langle C|D\rangle =& \sum_{n=0}^{\infty}\left\{ {\rm i}\sin(gt\sqrt{n+1})F^*(n+1)F(n)\left[\sin^2\left(\frac{\theta}{2}\right)\cos(gt\sqrt{n}) - \cos^2\left(\frac{\theta}{2}\right)\cos(gt\sqrt{n+2})\right]\right. \\ &+\frac{1}{2}{\rm e}^{-{\rm i}\phi}\,|F(n)|^2\sin\theta\cos(gt\sqrt{n+1})\cos(gt\sqrt{n}) \\ &\left.+\frac{1}{2}{\rm e}^{{\rm i}\phi}F^*(n+2)F(n)\sin\theta\sin(gt\sqrt{n+2})\sin(gt\sqrt{n+1})\right\}. \end{aligned} \quad (16)$$

3. The time evolution of the field quantum entropy and quantum entanglement between the single-mode squeezed coherent field and the two-level atom

Following the work by P-K in Ref.[2], we can express the reduced quantum entropy $S_{\rm f}(t)$ of the single-mode squeezed coherent field in terms of the eigenvalue $\pi_{\rm f}^{\pm}$ of the reduced density matrix $\rho_{\rm f}(t)$, given by expression (12), that is

$$S_{\rm f}(t) = -[\pi_{\rm f}^{+}\ln(\pi_{\rm f}^{+}) + \pi_{\rm f}^{-}\ln(\pi_{\rm f}^{-})]. \quad (17)$$

If the minimum value of $S_{\rm f}(t)$ is taken to be zero, the single-mode squeezed coherent field and the two-level atom are disentangled; if the maximum value of $S_{\rm f}(t)$ is taken to be one, the single-mode squeezed coherent field and the two-level atom are in maximal quantum entangled state; if the value of $S_{\rm f}(t)$ is between zero and one, the single-mode squeezed coherent field and the two-level atom are in usual quantum entangled states. It does not appear to be possible to express the sum in expression (17) in the closed form, but for not too large $\bar{n}$, the direct numerical temporal evolution of the entropy can take place based on the analytical solution presented by expression (17). In what follows, we mainly consider the influence of squeezing parameter γ, atomic distribution angle θ and coupling constant g between the two-level atom and the single-mode squeezed coherent field on the time evolution behaviours of the field quantum entropy and the quan-

tum entanglement between the single-mode squeezed coherent cavity field and the two-level atom.

3.1. The influence of squeezing parameter γ on the evolution of field quantum entropy

Figure 1 presents the numerical results of the time evolution of the field quantum entropy for different values of squeezing parameter γ with $\bar{n} = 10$, $\theta = \pi/3$, $g = 0.2$ and $\phi = \pi/4$. It is shown that the reduced field quantum entropy is strongly dependent on squeezing parameter γ. From the figures one can see that the amplitude of vibration of the quantum field entropy decreases with the increasing of the squeezing parameter γ and the maximum value of the field quantum entropy increases with the increasing of squeezing parameter when the squeezing of the field is weaker, which shows that the quantum entanglement between the field and the atom becomes greater in that case, (see Figs.1(a)–(c)), while when the squeezing of the field is stronger, the maximum value of the field quantum entropy decreases with the increasing of squeezing parameter, which indicates that the quantum entanglement between the field and the atom turns worse and worse (see Figs.1(d)–(f)), the field quantum entropy reaches its minimum value zero and almost becomes a straight line (see Fig.1(f)) when the squeezing parameter is big enough, that is to say, the field and the atom are nearly disentangled at those times.

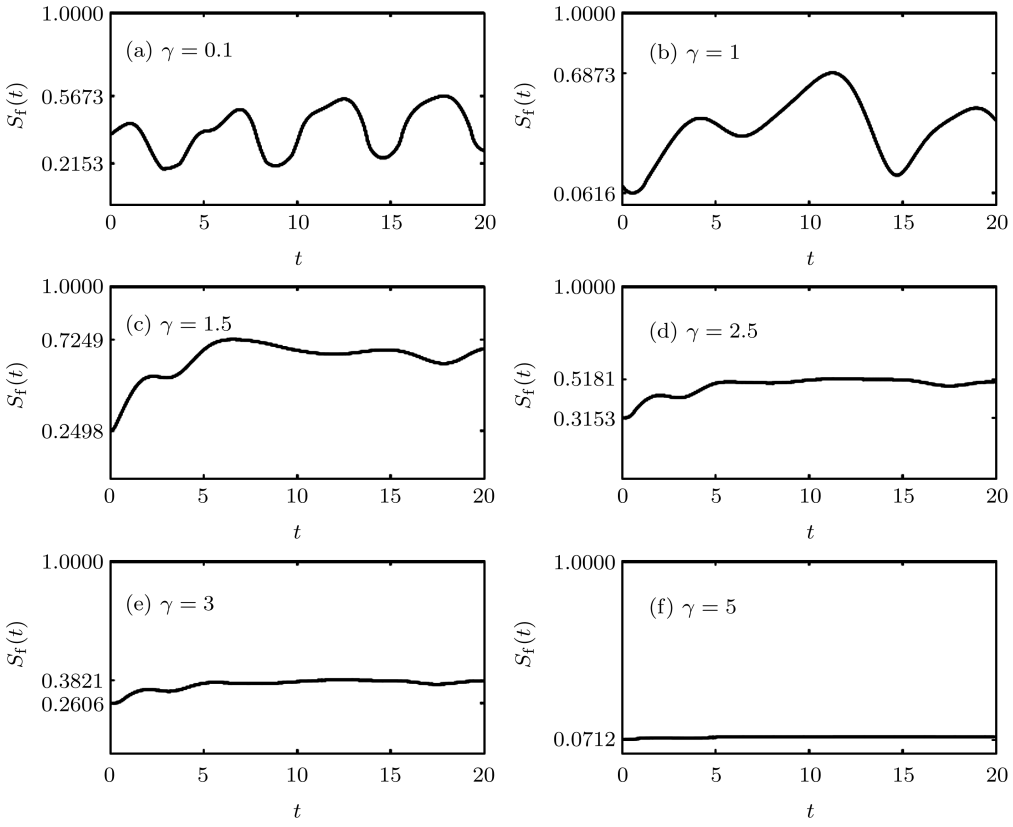


Fig.1. Time evolution of the field quantum entropy $S_{\rm f}(t)$ for $\bar{n} = 10$, $\theta = \pi/3$, $g = 0.2$ and $\phi = \pi/4$.

3.2. The influence of atomic distribution angle θ on the evolution of field quantum entropy

Figure 2 shows the numerical results of the time evolution of the field quantum entropy for different values of atomic distribution angle θ with $\bar{n} = 10$, $g = 0.2$, $\phi = \pi/4$, and $\gamma = 1.5$. Based on the plots, one can find that atomic distribution angle θ almost has no influence on the maximum and minimum values of the field quantum entropy evolution, but it affects the period of the evolution of the field quantum entropy and influences correspondingly the periodicity of the quantum entanglement between the field and the atom.

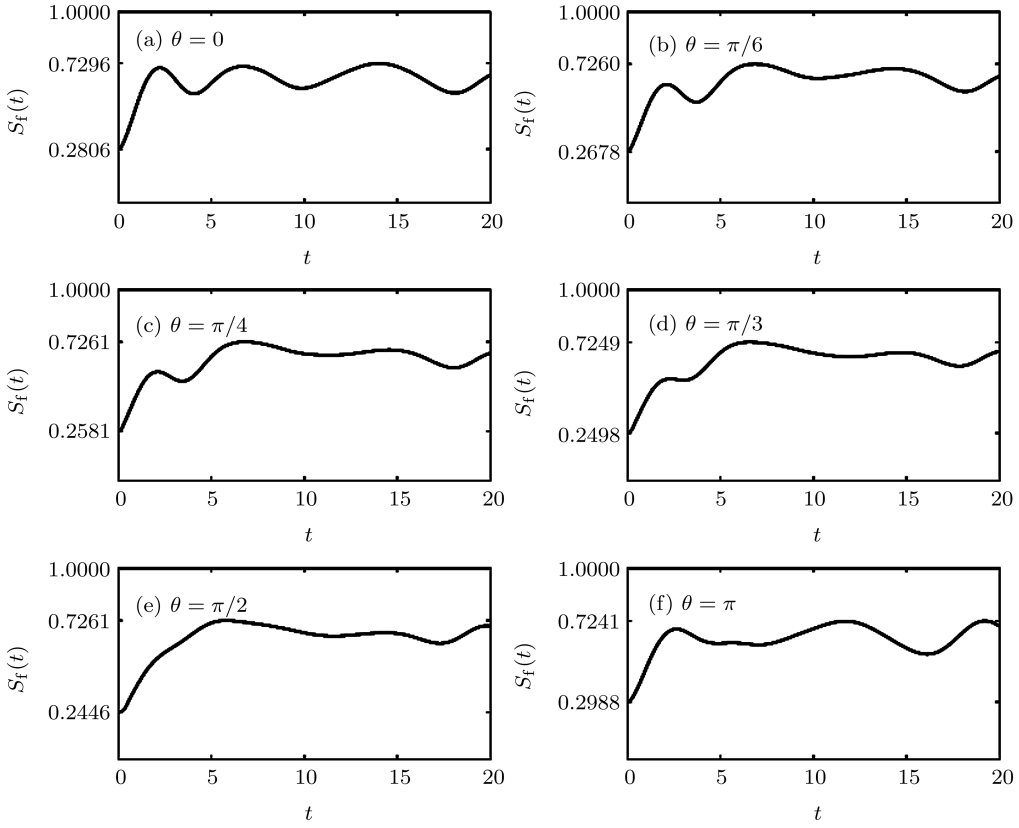


Fig.2. Time evolution of the field quantum entropy $S_{\rm f}(t)$ for $\bar{n} = 10$, $g = 0.2$, $\phi = \pi/4$ and $\gamma = 1.5$.

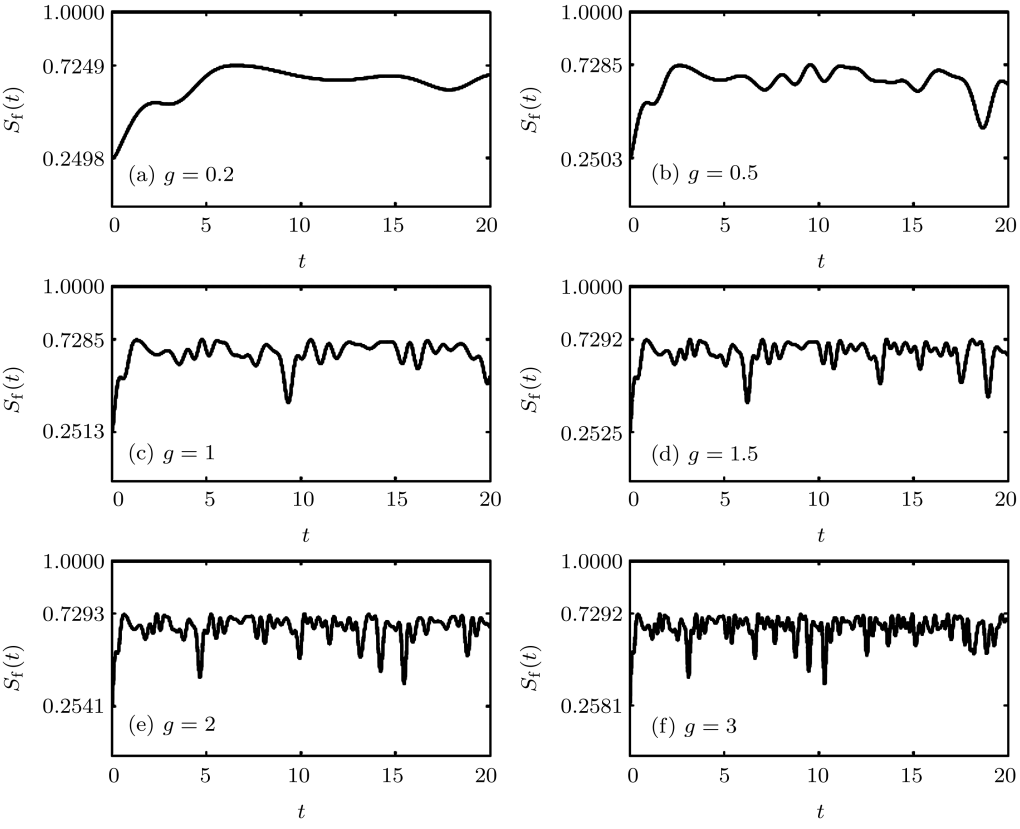


Fig.3. Time evolution of the field quantum entropy $S_{\rm f}(t)$ for $\bar{n} = 10$, $\theta = \pi/3$, $\phi = \pi/4$ and $\gamma = 1.5$.

3.3. The influence of coupling constant g between the two-level atom and the single-mode squeezed coherent field on the evolution of field quantum entropy

Figure 3 gives the numerical results of the time evolution of the field quantum entropy for different values of the coupling constant g between the field and the atom with $\bar{n} = 10$, $\theta = \pi/3$, $\phi = \pi/4$ and $\gamma = 1.5$. Comparing the plots in Figs.3(a)–(f), one can see that the periodicity of the field quantum entropy evolution is greatly influenced by the coupling strength between the two-level atom and the single-mode squeezed coherent field. The period of the field quantum entropy evolution decreases with the increasing of coupling constant g, and the period of the quantum entanglement between the field and the atom becomes smaller and smaller accordingly.

4. Conclusions

We have investigated the time evolution of the field quantum entropy for the single-mode squeezed coherent field interacting with a single two-level atom via the one-photon transition process and obtained the analytical expression of the field quantum entropy. Besides, we have presented the numerical results for $\bar{n} = 10$ and different values of squeezing parameter γ, atomic distribution θ and coupling constant g between the field and the atom and examined the influence of these parameters on the time evolution of the field quantum entropy. The results show that the amplitude of oscillation of the field quantum entropy evolution decreases with the increasing of squeezing parameter γ, and both atomic distribution angle θ and coupling strength g between the field and the atom can influence the periodicity of the field quantum entropy evolution.

References

[1] Jaynes E T and Cummings F W 1963 *Proc. IEEE* **51** 89
[2] Phoenix S J D and Knight P L 1988 *Ann. Phys.* (NY) **186** 381
[3] Phoenix S J D and Knight P L 1991 *Phys. Rev.* A **44** 6023
[4] Phoenix S J D and Knight P L 1990 *J. Opt. Soc. Am.* B **7** 116
[5] Peng K C 2001 *Physics* **30** 300
[6] Walls D F 1983 *Nature* **36** 141
[7] Zhang X L, Guo G C, Peng K C and Xie C D 1994 *Prog. Phys.* **14** 173 (in Chinese)
[8] Li Y M, Zhang K S and Xie C D 2002 *Acta Sin. Quantum Opt.* **8** 71 (in Chinese)
[9] Fang M F and Zhou P 1993 *Acta Opt. Sin.* **13** 799 (in Chinese)
[10] Fang M F and Liu H E 1994 *Acta Opt. Sin.* **14** 475 (in Chinese)
[11] Fang M F 1994 *Acta Phys. Sin.* **43** 1776 (in Chinese)
[12] Zhou Q C and Zhu S N 2005 *Acta Phys. Sin.* **54** 1184 (in Chinese)
[13] Huang C J, He H Y, Zhou M, Fang J Y and Huang Z H 2006 *Acta Phys. Sin.* **55** 1764 (in Chinese)
[14] Yang X, Tong Z Y and Xiang S H 2007 *Acta Phys. Sin.* **56** 2148 (in Chinese)
[15] Liu T K, Wang J S, Feng J and Zhan M S 2005 *Chin. Phys.* **14** 536
[16] Gao Y F, Feng J and Wang J S 2005 *Chin. Phys.* **14** 980
[17] Zhou Q C and Zhu S N 2005 *Chin. Phys.* **14** 1147
[18] Huang S X 1996 *Acta Sci. Nat. Univ. Norm. Hunan* **19** 30 (in Chinese)
[19] Huang Y X and Guo G C 1996 *Acta Sin. Quantum Opt.* **2** 97 (in Chinese)
[20] Wang C Z and Fang M F 2002 *Acta Phys. Sin.* **51** 1989 (in Chinese)
[21] Liu S M 2004 *Acta Photonica Sinica* **33** 113 (in Chinese)
[22] Li Y P and Wang B Q 2003 *J. Dezhou University* **19** 28 (in Chinese)
[23] Peng J S and Li G X 1996 *Introduction to Modern Quantum Optics* 179