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Entanglement properties in a system of a pairwise entangled state*

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Based on the quantum information theory, this paper has investigated the entanglement properties of a system which is composed of the two entangled two-level atoms interacting with the two-mode entangled coherent fields. The influences of the strength of light field and the two parameters of entanglement between the two-mode fields on the field entropy and on the negative eigenvalues of partial transposition of density matrix are discussed by using numerical calculations. The result shows that the entanglement properties in a system of a pairwise entangled states can be controlled by appropriately choosing the two parameters of entanglement between the two-mode entangled coherent fields and the strength of two light fields respectively.

Keywords: field entropy, negative eigenvalues, two entangled atoms, two-mode entangled coherent fields
PACC: 4250, 0365

1. Introduction

The entangled state of two or more particles has not only played a vital role in discussion of nonlocal quantum correlations,[1] but has also formed the basis of quantum communication and information processing, such as quantum computation,[2] quantum cryptography,[3] quantum teleportation,[4] and quantum dense coding.[5] For a bipartite pure state it is straightforward, that there has been a lot of work on bipartite entanglement: for example, with the von Neumann entropy,[6−12] distance between density operators,[13,14] fidelity,[15,16] negativity,[17,18] and concurrence.[19,20] Recently, two two-level atoms in an entangled state have aroused huge research interest. By means of negativity and concurrence, the time evolution of the entanglement of the two atoms and the entanglement between the two-mode fields in the system of two atoms interacting with two-mode fields are investigated.[21] The entanglement time evolution of two entangled two-level atoms that interact resonantly with a single-mode field in the Fock state is studied.[22]

In this paper, we consider the situation of the two entangled two-level atoms interacting with the two-mode entangled coherent fields, the von Neumann entropy (which represents the degree of entanglement between the field and the atom) and the negative eigenvalues of partial transposition of density matrix (which denotes the degree of entanglement between the two atoms) are investigated. The influences of the strength of light field and the two parameters of entanglement between the two-mode fields on field entropy and on the negative eigenvalues of partial transposition of density matrix are discussed by using numerical calculations.

2. The model and its solution

We consider the case that two identical two-level atoms A and B are simultaneously injected into a two-mode light cavity, the dipole interaction between atoms can be neglected. There are same couplings of the two atoms as interacting with two-mode fields, un-

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der these conditions. The Hamiltonian of the system can be written as ($\hbar = 1$)

$$H = \sum_{m=1}^{2} \omega_m a_m^+ a_m + \omega_a \sum_{n=A}^{B} S_n^z + g \sum_{n=A}^{B} (a_1^+ a_2^+ S_n^- + a_1 a_2 S_n^+), \quad (1)$$

where a_m^+ and a_m are the creation and annihilation operators of the mth mode field frequency ω_m respectively, ω_a denotes the atomic transition frequency, S_n^z, $S_n^\pm$ denote the inversion and transition operators of the nth atom respectively, and g describes coupling constant of the field and the atom.

If we consider that at time $t = 0$ the two atoms and the two-mode fields are decoupled, and assume that the two atoms are initially in entangled state:

$$|\varphi_a(0)\rangle = \frac{1}{\sqrt{2}} \left(|e_A, g_B\rangle + |g_A, e_B\rangle\right), \quad (2)$$

where $\left|e_{A(B)}\right\rangle$ and $\left|g_{A(B)}\right\rangle$ denote excited and ground states of two two-level atoms A and B respectively. The two-mode fields are initially in the entangled coherent states:[23]

$$|\varphi_{\rm f}(0)\rangle = \frac{1}{\sqrt{N}} (\varepsilon_+ |\alpha_1\rangle |\alpha_2\rangle + \varepsilon_- |-\alpha_1\rangle |-\alpha_2\rangle), \quad (3)$$

for $|\alpha_m\rangle$ and $|-\alpha_m\rangle$ are coherent states of the mth-mode field. Coherent states $|\alpha_m\rangle$ and $|-\alpha_m\rangle$ are 180° out of phase with respect to each other. $N = \varepsilon_+^2 + \varepsilon_-^2 + 2\varepsilon_+\varepsilon_- {\rm e}^{-2(|\alpha_1|^2+|\alpha_2|^2)}$ is a normalization factor, ε_+ and ε_- represent the parameters of entanglement between the two-mode entangled coherent states and satisfy $\varepsilon_+^2 + \varepsilon_-^2 = 1$.

Then the initial state vector of the system is

$$|\varphi_{\rm s}(0)\rangle = \frac{1}{\sqrt{2N}} \sum_{n_1,n_2=0}^{\infty} F_{n_1,n_2} (|e_A, g_B, n_1, n_2\rangle + |g_A, e_B, n_1, n_2\rangle), \quad (4)$$

where

$$F_{n_1,n_2} = {\rm e}^{-\frac{1}{2}(|\alpha_1|^2+|\alpha_2|^2)} \frac{\varepsilon_+ |\alpha_1|^{n_1} |\alpha_2|^{n_2} + \varepsilon_- |-\alpha_1|^{n_1} |-\alpha_2|^{n_2}}{\sqrt{n_1!}\sqrt{n_2!}}, \quad (5)$$

with $\alpha_m = |\alpha_m| {\rm e}^{{\rm i}\phi_m}$ being a complex parameter, ϕ_m and $|\alpha_m|$ representing the direction angle of the excitation and the strength of light field of mth-mode field, respectively (therein, we suppose $\phi_m = 0$).

For simplicity, we consider only the resonance situation, namely, transition frequency of atomic and effective action resonance frequency of the two-mode fields satisfying $\omega_a = \omega_1 + \omega_2$. Then, along with time evolution, the state vector of the system at any time $t > 0$ is given by

$$\begin{aligned} |\varphi_{\rm s}(t)\rangle = & \sum_{n_1,n_2=1}^{\infty} A_{n_1,n_2}(t) |e_A, e_B, n_1 - 1, n_2 - 1\rangle + \sum_{n_1,n_2=0}^{\infty} B_{n_1,n_2}(t) |e_A, g_B, n_1, n_2\rangle \\ & + \sum_{n_1,n_2=0}^{\infty} C_{n_1,n_2}(t) |g_A, e_B, n_1, n_2\rangle + \sum_{n_1,n_2=0}^{\infty} D_{n_1,n_2}(t) |g_A, g_B, n_1 + 1, n_2 + 1\rangle. \end{aligned} \quad (6)$$

Based on the initial condition, the solution of Schrödinger equation in the interaction representation is given by

$$\begin{aligned} A_{n_1,n_2}(t) &= -{\rm i} \frac{1}{\sqrt{N}} \sqrt{\frac{n_1 n_2}{n_1 n_2 + (n_1+1)(n_2+1)}} \sin\left[\sqrt{2[n_1 n_2 + (n_1+1)(n_2+1)]} gt\right] F_{n_1,n_2}, \\ B_{n_1,n_2}(t) &= C_{n_1,n_2}(t) = \frac{1}{\sqrt{2N}} \cos\left[\sqrt{2[n_1 n_2 + (n_1+1)(n_2+1)]} gt\right] F_{n_1,n_2}, \\ D_{n_1,n_2}(t) &= -{\rm i} \frac{1}{\sqrt{N}} \sqrt{\frac{(n_1+1)(n_2+1)}{n_1 n_2 + (n_1+1)(n_2+1)}} \sin\left[\sqrt{2[n_1 n_2 + (n_1+1)(n_2+1)]} gt\right] F_{n_1,n_2}. \end{aligned} \quad (7)$$

The properties of the quantum entanglement between the atom and the field, and between two atoms will be discussed in the next section by using above results.

3. Entanglement between the atom and the field

We use the von Neumann entropy as a measurement of the degree of entanglement between the atom and the field of the system under consideration. The von Neumann entropy of a system is defined as[24]

$$S(t) = -\mathrm{Tr}(\rho(t)\ln\rho(t)), \tag{8}$$

where $\rho(t)$ is the density operator for a given quantum system. Suppose we consider $S_{\mathrm{f}}(t)$ and $S_a(t)$ to be the entropies for the field and the atom, respectively, and let $S(t)$ denote the entropy of the composite system. In this case Araki and Lieb[25] showed that these entropies satisfy the 'triangle inequalities' $|S_a(t) - S_{\mathrm{f}}(t)| \leq S(t) \leq S_a(t) + S_{\mathrm{f}}(t)$. On the other hand, Phoenix and Knight[6] succeeded in evaluating the field entropy in a closed form and showed that it is indeed equal to the atomic entropy at all times. In this case the composite entropy is initially zero ($S(0) = 0$), and also remains zero ($S(t) = 0$) for all values of the time when the atom-field system is isolated from its environment. Under those circumstances the former implies that $S_a(t) = S_{\mathrm{f}}(t)$. It explains that a system in which both the atom and the light field start from pure states and that the entropy of the system is always zero $S(t) = 0$. Therefore the entropies of two interacting subsystems are precisely equal $S_a(t) = S_{\mathrm{f}}(t)$. In the following, we only consider the time behaviour of the field entropy $S_{\mathrm{f}}(t)$. In order to derive a calculation formalism of the field entropy, we must obtain the eigenvalues of the reduced field density operator. From Eq.(6), we have the reduced field density operator

$$\begin{aligned}\rho_{\mathrm{f}}(t) &= \mathrm{Tr}_a[\rho(t)] \\ &= |\mu(t)\rangle\langle\mu(t)| + |\nu_1(t)\rangle\langle\nu_1(t)| + |\nu_2(t)\rangle\langle\nu_2(t)| \\ &\quad + |\tau(t)\rangle\langle\tau(t)| \\ &= |\mu(t)\rangle\langle\mu(t)| + |\nu(t)\rangle\langle\nu(t)| + |\tau(t)\rangle\langle\tau(t)|, \end{aligned} \tag{9}$$

where

$$\begin{aligned}|\mu(t)\rangle &= \sum_{n_1,n_2=1}^{\infty} A_{n_1,n_2}(t)\,|n_1-1, n_2-1\rangle, \\ |\nu(t)\rangle &= |\nu_1(t)\rangle + \mathrm{i}\,|\nu_2(t)\rangle \\ &= (1+\mathrm{i})\sum_{n_1,n_2=0}^{\infty} B_{n_1,n_2}(t)\,|n_1, n_2\rangle, \\ |\tau(t)\rangle &= \sum_{n_1,n_2=0}^{\infty} D_{n_1,n_2}(t)\,|n_1+1, n_2+1\rangle. \end{aligned} \tag{10}$$

Then the eigenvalues of the reduced field density operator are given by

$$\begin{aligned}\pi_{\mathrm{f}}^1(t) &= \frac{\theta_1}{3} - \frac{2}{3}(\theta_1^2 - 3\theta_2)^{1/2}\cos\beta, \\ \pi_{\mathrm{f}}^2(t) &= \frac{\theta_1}{3} + \frac{1}{3}(\theta_1^2 - 3\theta_2)^{1/2}(\cos\beta + \sqrt{3}\sin\beta), \\ \pi_{\mathrm{f}}^3(t) &= \frac{\theta_1}{3} + \frac{1}{3}(\theta_1^2 - 3\theta_2)^{1/2}(\cos\beta - \sqrt{3}\sin\beta), \end{aligned} \tag{11}$$

where

$$\begin{aligned}\beta &= \frac{1}{3}\arccos\left(\frac{\xi}{\varsigma}\right), \\ \xi &= 9\theta_1\theta_2 - 2\theta_1^2 - 27\theta_3, \\ \varsigma &= 2(\theta_1^2 - 3\theta_2)^{3/2}, \end{aligned} \tag{12}$$

with

$$\begin{aligned}\theta_1 &= -(\langle\mu(t)\mid\mu(t)\rangle + \langle\nu(t)\mid\nu(t)\rangle + \langle\tau(t)\mid\tau(t)\rangle), \\ \theta_2 &= \langle\mu(t)\mid\mu(t)\rangle\langle\tau(t)\mid\tau(t)\rangle \\ &\quad + \langle\mu(t)\mid\mu(t)\rangle\langle\nu(t)\mid\nu(t)\rangle \\ &\quad + \langle\nu(t)\mid\nu(t)\rangle\langle\tau(t)\mid\tau(t)\rangle \ - |\langle\mu(t)\mid\nu(t)\rangle|^2 \\ &\quad - |\langle\mu(t)\mid\tau(t)\rangle|^2 - |\langle\nu(t)\mid\tau(t)\rangle|^2, \\ \theta_3 &= \langle\mu(t)\mid\mu(t)\rangle\langle\nu(t)\mid\nu(t)\rangle\langle\tau(t)\mid\tau(t)\rangle \\ &\quad + \langle\mu(t)\mid\nu(t)\rangle\langle\tau(t)\mid\mu(t)\rangle\langle\nu(t)\mid\tau(t)\rangle \\ &\quad - \langle\mu(t)\mid\mu(t)\rangle|\langle\nu(t)\mid\tau(t)\rangle|^2 \\ &\quad - \langle\nu(t)\mid\nu(t)\rangle|\langle\tau(t)\mid\mu(t)\rangle|^2 \\ &\quad - \langle\tau(t)\mid\tau(t)\rangle|\langle\mu(t)\mid\nu(t)\rangle|^2. \end{aligned}$$

The field entropy $S_{\mathrm{f}}(t)$ may be expressed in terms of the eigenvalues $\pi_{\mathrm{f}}^i(t)$ ($i = 1,\ 2,\ 3$) for the reduced field density operator as

$$\begin{aligned}S_{\mathrm{f}}(t) &= -\{\pi_{\mathrm{f}}^1(t)\ln[\pi_{\mathrm{f}}^1(t)] + \pi_{\mathrm{f}}^2(t)\ln[\pi_{\mathrm{f}}^2(t)] \\ &\quad + \pi_{\mathrm{f}}^3(t)\ln[\pi_{\mathrm{f}}^3(t)]\}. \end{aligned} \tag{13}$$

If $S_{\mathrm{f}}(t)$ takes its minimum value 0, the field and the atom are in the disentangled state. If $S_{\mathrm{f}}(t)$ takes its maximum value 1, the field and the atom are in the maximum entangled state.

Figures 1, 2 and 3 present the numerical results of the time evolution of the field entropy $S_{\mathrm{f}}(t)$ for four parameters ε_+, ε_-, $|\alpha_1|$ and $|\alpha_2|$, respectively. It is shown that the field entropy is strongly dependent on ε_+ and ε_- of the parameters of entanglement degree for two-mode entangled coherent states, and the

strength of the two-mode light fields $|\alpha_1|$ and $|\alpha_2|$. In Figs.1–3, the ordinates represent $S_{\rm f}(t)$ (therein, (a) $S_{\rm f}+8$, (b) $S_{\rm f}+6$, (c) $S_{\rm f}+4$, (d) $S_{\rm f}+2$, (e) $S_{\rm f}$), the x-coordinates represent the scaled time gt.

Figure 1 shows the influence of the strength of the light field on time evolution of the field entropy $S_{\rm f}(t)$ in a system of the two-mode coherent state field interacting with the two atoms, namely, $\varepsilon_+ = 1$, $\varepsilon_- = 0$. The result tells us that for different $|\alpha_m|$, the time evolution of the field entropy $S_{\rm f}(t)$ is quite complicated. When $|\alpha_m|$ is small, the field entropy $S_{\rm f}(t)$ is the change of orderliness, the atom and the field are in alternation in the entangled state and disentangled state. With increasing $|\alpha_m|$, the field entropy $S_{\rm f}(t)$ is the change of disorderliness. When $|\alpha_m|$ is large enough, the curve of time evolution returns to orderliness, but with longer cycles of shake in comparison with those when $|\alpha_m|$ is small.

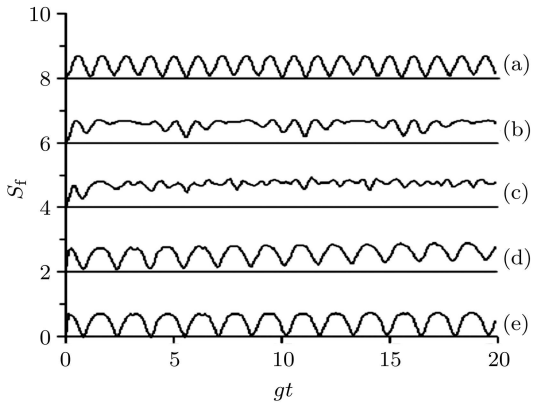


Fig.1. Time evolution of the entanglement between field and atom for $\varepsilon_+ = 1$, $\varepsilon_- = 0$. (a) $|\alpha_1| = |\alpha_2| = 0.1$, (b) $|\alpha_1| = |\alpha_2| = 0.5$, (c) $|\alpha_1| = |\alpha_2| = 1$, (d) $|\alpha_1| = |\alpha_2| = 5$, (e) $|\alpha_1| = |\alpha_2| = 10$.

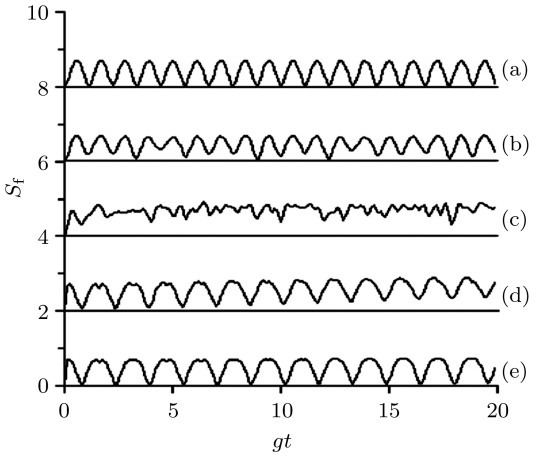


Fig.2. Time evolution of the entanglement between field and atom for $\varepsilon_+ = \varepsilon_- = 1/\sqrt{2}$. (a) $|\alpha_1| = |\alpha_2| = 0.1$, (b) $|\alpha_1| = |\alpha_2| = 0.5$, (c) $|\alpha_1| = |\alpha_2| = 1$, (d) $|\alpha_1| = |\alpha_2| = 5$, (e) $|\alpha_1| = |\alpha_2| = 10$.

Figure 2 shows the influence of the strength of the light field on the time evolution of the field entropy $S_{\rm f}(t)$ in a system of the maximally two-mode entangled coherent state field interacting with the two atoms, namely $\varepsilon_+ = \varepsilon_- = 1/\sqrt{2}$. We can see that when $|\alpha_m|$ is small, or $|\alpha_m|$ is large enough, the amplitude of vibration of the field entropy $S_{\rm f}(t)$ resembles Fig.1.

Figure 3 shows the influence of the strength of the light field on the time evolution of the field entropy $S_{\rm f}(t)$ in a system of the nonmaximally two-mode entangled coherent state field interacting with the two atoms. When $|\alpha_m|$ is small, the amplitude of vibration of the field entropy $S_{\rm f}(t)$ does not present periodicity. When $|\alpha_m|$ is large enough, it is shown that the amplitude of vibration of the field entropy $S_{\rm f}(t)$ resembles Fig.1 and Fig.2.

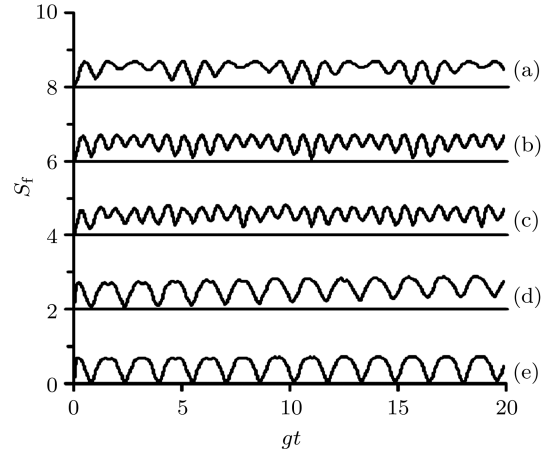


Fig.3. Time evolution of the entanglement between field and atom for $\varepsilon_+ = 1/2$, $\varepsilon_- = -\sqrt{3}/2$. (a) $|\alpha_1| = |\alpha_2| = 0.1$, (b) $|\alpha_1| = |\alpha_2| = 0.5$, (c) $|\alpha_1| = |\alpha_2| = 1$, (d) $|\alpha_1| = |\alpha_2| = 5$, (e) $|\alpha_1| = |\alpha_2| = 10$.

4. Entanglement between the two entangled atoms

For two subsystems in the mixed state, we may use the negative eigenvalues of partial transposition of density matrix as a measurement of the degree of entanglement between the two subsystems. Consider a density matrix $\rho(t)$ and its partial transposition $\rho^{\rm T}(t)$ for a systems of two spin-1/2. The measure of entanglement $E_{AB}(t)$ is then defined by[26]

$$E_{AB}(t) = -2\sum_i \lambda_i^-, \tag{14}$$

where λ_i^- is a negative eigenvalue of $\rho^{\mathrm{T}}(t)$. Then $E_{AB}=0$ denote that two atoms are separated. Then $E_{AB}=1$ denote that two atoms are in the maximum entangled state. Then $0<E_{AB}<1$ denote that two atoms are in entanglement state. In the following, let us only consider the time behaviour of the degree of entanglement between the two atoms $E_{AB}(t)$. In order to derive a calculation formalism of the degree of entanglement between the two atoms, we must obtain the eigenvalues of the reduced atom density operator. From Eq.(6), we have the reduced atom density operator

$$\rho_{\mathrm{a}}(t)=\mathrm{Tr}_{\mathrm{f}}[\rho(t)]=\begin{pmatrix}\rho_{11} & \rho_{12} & \rho_{13} & \rho_{14}\\ -\rho_{12} & \rho_{22} & \rho_{22} & \rho_{24}\\ -\rho_{12} & \rho_{22} & \rho_{22} & \rho_{24}\\ \rho_{41} & \rho_{42} & \rho_{42} & \rho_{44}\end{pmatrix}, \quad (15)$$

where

$$\begin{aligned}
\rho_{11} &= \sum_{n_1,n_2=1}^{\infty}|A_{n_1,n_2}(t)|^2,\\
\rho_{12} &= \rho_{13}=\sum_{n_1,n_2=0}^{\infty}A_{n_1+1,n_2+1}(t)B^*_{n_1,n_2}(t)\ ,\\
\rho_{14} &= \sum_{n_1,n_2=0}^{\infty}A_{n_1+2,n_2+2}(t)D^*_{n_1,n_2}(t)\ ,\ \rho_{21}=\rho_{12}^*\ ,\\
\rho_{22} &= \rho_{23}=\sum_{n_1,n_2=0}^{\infty}|B_{n_1,n_2}(t)|^2,\\
\rho_{24} &= \sum_{n_1,n_2=0}^{\infty}B_{n_1+1,n_2+1}(t)D^*_{n_1,n_2}(t)\ ,\ \rho_{31}=\rho_{21},\\
\rho_{32} &= \rho_{33}=\rho_{22},\ \rho_{34}=\rho_{24}\ ,\\
\rho_{41} &= \rho_{14}^*,\rho_{42}=\rho_{24}^*,\rho_{43}=\rho_{42},\\
\rho_{44} &= \sum_{n_1,n_2=0}^{\infty}|D_{n_1,n_2}(t)|^2. \qquad (16)
\end{aligned}$$

Based on the definition of partial transposition of density matrix, we can write $\rho_a^{\mathrm{T}}(t)$ as

$$\begin{aligned}
\rho_a^{\mathrm{T}}(t) &= \sum_{\langle i|}\rho_a(t)\,|j\rangle\otimes|j\rangle\,\langle i|\\
&= \begin{pmatrix}\rho_{11} & \rho_{12} & -\rho_{12} & \rho_{22}\\ -\rho_{12} & \rho_{22} & \rho_{14} & -\rho_{24}\\ \rho_{12} & \rho_{14} & \rho_{22} & \rho_{24}\\ \rho_{22} & \rho_{24} & -\rho_{24} & \rho_{44}\end{pmatrix}. \qquad (17)
\end{aligned}$$

Consequently, according to the eigenvalue equation of the density operator $\rho_a^{\mathrm{T}}(t)$, the eigenvalues of the partial transposition of density matrix of the atom are given by

$$\begin{aligned}
\lambda_a^1(t) &= \rho_{22}+\rho_{14},\\
\lambda_a^2(t) &= \frac{\eta_1}{3}-\frac{2}{3}(\eta_1^2-3\eta_2)^{1/2}\cos\beta,\\
\lambda_a^3(t) &= \frac{\eta_1}{3}+\frac{1}{3}(\eta_1^2-3\eta_2)^{1/2}(\cos\beta+\sqrt{3}\sin\beta),\\
\lambda_a^4(t) &= \frac{\eta_1}{3}+\frac{1}{3}(\eta_1^2-3\eta_2)^{1/2}(\cos\beta-\sqrt{3}\sin\beta),
\end{aligned} \quad (18)$$

where

$$\begin{aligned}
\beta &= \frac{1}{3}\arccos\left(\frac{\xi}{\varsigma}\right),\\
\xi &= 9\eta_1\eta_2-2\eta_1^2-27\eta_3,\\
\varsigma &= 2(\eta_1^2-3\eta_2)^{3/2}, \qquad (19)
\end{aligned}$$

with

$$\begin{aligned}
\eta_1 &= -(\rho_{11}+\rho_{22}+\rho_{44}-\rho_{14}),\\
\eta_2 &= \rho_{11}\rho_{22}+\rho_{11}\rho_{44}+2\rho_{12}^2+2\rho_{24}^2+\rho_{22}\rho_{44}\\
&\quad -\rho_{22}^2-\rho_{11}\rho_{14}-\rho_{14}\rho_{44},\\
\eta_3 &= -\rho_{11}\rho_{22}\rho_{44}-2\rho_{12}^2\rho_{44}-2\rho_{11}\rho_{24}^2+\rho_{14}\rho_{22}^2\\
&\quad +4\rho_{12}\rho_{22}\rho_{24}+\rho_{22}^3+\rho_{11}\rho_{14}\rho_{44}.
\end{aligned}$$

The degree of entanglement between the two atoms $E_{AB}(t)$ may be expressed in terms of the negative eigenvalues $\lambda_a^i(t)$ ($i=1,\ 2,\ 3{,}4$) for the partial transposition of density matrix.

Figures 4, 5 and 6 present the numerical results of the entanglement time evolution of the two atoms for four parameters ε_+, ε_-, $|\alpha_1|$ and $|\alpha_2|$, respectively. In Figs.4–6, the ordinates represent the degree of entanglement between the two atoms $E_{AB}(t)$ (therein, (a) $E_{AB}+4$, (b) $E_{AB}+3$, (c) $E_{AB}+2$, (d) $E_{AB}+1$, (e) E_{AB}), the x-coordinates represent the scaled time gt.

Figures 4 and 5 show the influence of the strength of the light field on the time evolution of the $E_{AB}(t)$ in a system of the two-mode coherent state field and the maximally two-mode entangled coherent state field interacting with the two atoms, respectively. The result tells us that for different $|\alpha_m|$, the time evolution of the degree of entanglement between the two atoms $E_{AB}(t)$ is too complicated. When $|\alpha_m|$ is small, the degree of entanglement between the two atoms $E_{AB}(t)$ is the change of orderliness, the two atoms are alternately in the entangled state and disentangled state. With increasing $|\alpha_m|$, the degree of entanglement between the two atoms $E_{AB}(t)$ is the change of disorderliness. When $|\alpha_m|$ is large enough, we can see that the degree of entanglement between the two atoms $E_{AB}(t)$ is weak and basically in disentanglement status.

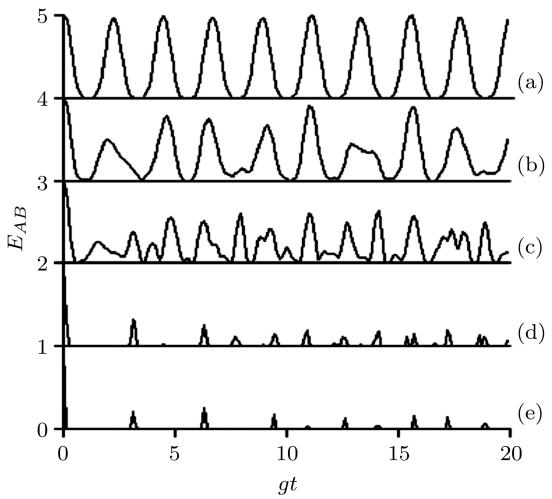


Fig.4. Time evolution of the entanglement between two atoms for $\varepsilon_+ = 1$, $\varepsilon_- = 0$. (a) $|\alpha_1| = |\alpha_2| = 0.1$, (b) $|\alpha_1| = |\alpha_2| = 0.5$, (c) $|\alpha_1| = |\alpha_2| = 1$, (d) $|\alpha_1| = |\alpha_2| = 5$, (e) $|\alpha_1| = |\alpha_2| = 10$.

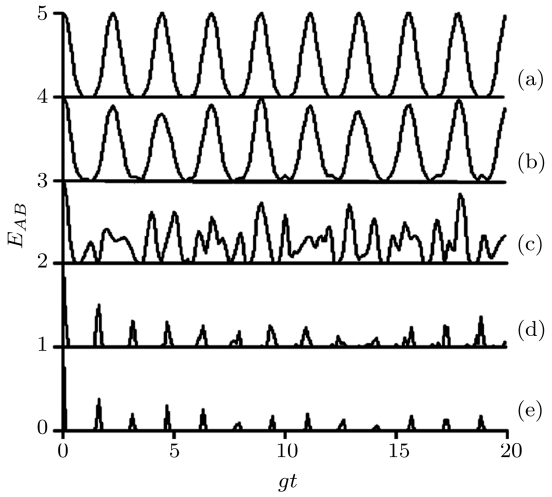


Fig.5. Time evolution of the entanglement between two atoms for$\varepsilon_+ = \varepsilon_- = 1/\sqrt{2}$. (a) $|\alpha_1| = |\alpha_2| = 0.1$, (b) $|\alpha_1| = |\alpha_2| = 0.5$, (c) $|\alpha_1| = |\alpha_2| = 1$, (d) $|\alpha_1| = |\alpha_2| = 5$, (e) $|\alpha_1| = |\alpha_2| = 10$.

Figure 6 shows the influence of the strength of the light field on the time evolution of the $E_{AB}(t)$ in a system of the nonmaximally two-mode entangled coherent state field interacting with the two atoms. When $|\alpha_m|$ is small, the degree of entanglement between the two atoms $E_{AB}(t)$ is the change of disorderliness, but the two atoms are alternately in the entangled state and disentangled state. When $|\alpha_m|$ is large enough, we can see that the degree of entanglement between the two atoms $E_{AB}(t)$ resembles Fig.4 and Fig.5.

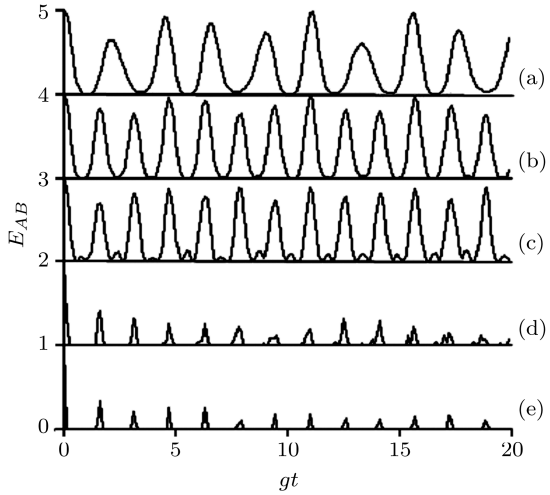


Fig.6. Time evolution of the entanglement between two atoms for $\varepsilon_+ = 1/2$, $\varepsilon_- = -\sqrt{3}/2$. (a) $|\alpha_1| = |\alpha_2| = 0.1$, (b) $|\alpha_1| = |\alpha_2| = 0.5$, (c) $|\alpha_1| = |\alpha_2| = 1$, (d) $|\alpha_1| = |\alpha_2| = 5$, (e) $|\alpha_1| = |\alpha_2| = 10$.

5. Conclusions

In conclusion, we investigate the entanglement properties in a system of two-mode entangled coherent field interacting with the two entangled two-level atoms. The quantum entanglement between the field and the atom by using the field entropy, and the quantum entanglement between two atoms by using the negative eigenvalues of partial transposition of density matrix are discussed, respectively. The result shows that the field entropy and negative eigenvalues of partial transposition of density matrix are strongly dependent on the two parameters of entanglement between two-mode entangled coherent field and the strength of the light field. Our results will be helpful to the understanding of the multipartite entanglement and the control of quantum entanglement in quantum computation and quantum information.

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