Vol 16 No 12, December 2007
1009-1963/2007/16(12)/3692-05

Chinese Physics

© 2007 Chin. Phys. Soc.
and IOP Publishing Ltd

Teleportation of a bipartite entangled coherent state via cavity QED

Zhou Yan-Li(周艳丽), Dai Hong-Yi(戴宏毅), and Yang Li-Jia(杨丽佳)

College of Science, National University of Defense Technology, Changsha 410073, *China*

(Received 15 January 2007; revised manuscript received 16 April 2007)

Based on the atom–cavity–field interaction, this paper proposes a scheme for the teleportation of a bipartite entangled coherent state (ECS) with high fidelity as long as $|\alpha|$ is not too small. In this proposal, only four cavities and a three-level cascade atom are needed. The fidelity of the ECS is calculated and analysed in detail.

Keywords: teleportation, entangled coherence state, atom-cavity field interaction, fidelity
PACC: 4250, 0365

1. Introduction

Since Bennett *et al*[1] proposed the teleportation of an unknown quantum state, much attentions have been paid to quantum teleportation. Inspired by that infusible idea, a large number of schemes have been proposed, some of which have been experimentally realized, e.g., to teleport atomic states,[2,3] photonic states,[4,5] entangled states,[6] and states with continuous variables.[7] In particular, following the theoretical proposal of Ref.[8] continuous-variable teleportation has been realized for coherent states of cavity field.[9−12] Superposition of coherent states not only plays a fundamental role in understanding the transfer of the 'indeterminacy' of a microscopic system to the 'uncertainty' of a macroscopic one, but also can be used as logical qubit encoding for the correction of spontaneous emission errors.[13] So it is very necessary and important to teleport superposition of coherent states. Recently, Wang[14] proposed a scheme for teleportation of both bipartite and multipartite entangled coherent states with successful probability 0.5 based on linear optical devices. Cai *et al*[15] proposed a scheme to teleport a single qubit that is encoded in the space spanned by two equal-amplitude and opposite-phase squeezed vacuum states of a harmonic oscillator with the successful probability of 0.25. Song *et al*[16] presented a proposal for teleporting an entangled coherent state (ECS) through the dispersive atom-cavity-field interaction. In this paper, we present a scheme for the teleportation of a bipartite ECS with high fidelity of nearly 100%. During the teleportation process, only four cavity fields are needed, and only measurements on the cavity field and on the singlet atomic state are used. The transformation required to complete the teleportation is described for four cases at length.

2. Teleportation scheme

We suppose that Alice wants to teleport the following ECS to Bob:

$$|\varphi\rangle_{\mathrm{C1-C2}} = a\,|\alpha\rangle_1\,|\alpha\rangle_2 + b\,|-\alpha\rangle_1\,|-\alpha\rangle_2\,, \qquad (1)$$

where a and b are unknown coefficients which satisfy

$$|a|^2 + |b|^2 + 2\mathrm{e}^{-4|\alpha|^2}\mathrm{Re}(ab^*) = 1. \qquad (2)$$

$|\alpha\rangle_i$ and $|-\alpha\rangle_i$ are two coherent states separated by π phase space, and subscript i refers to mode i. When amplitude $|\alpha|$ is large enough, the states $|\alpha\rangle_i$ and $|-\alpha\rangle_i$ are approximately orthogonal: ${}_i\langle-\alpha|\alpha\rangle_i = \mathrm{e}^{-2|\alpha|^2} \approx 0$.

2.1. Description of the atom–cavity–field interaction

Figure 1 represents the interaction of a single-mode cavity field with a three-level cascade atom A with $|f\rangle$, $|e\rangle$ and $|g\rangle$ being the upper, intermediate and lower states, respectively. We assume that the frequency of $|e\rangle \leftrightarrow |f\rangle$ transition is detuned from the cavity frequency enough so that only virtual transitions occur between these levels. In addition we assume that the transition $|e\rangle \leftrightarrow |g\rangle$ is far from resonance with the cavity field mode and is resonant with

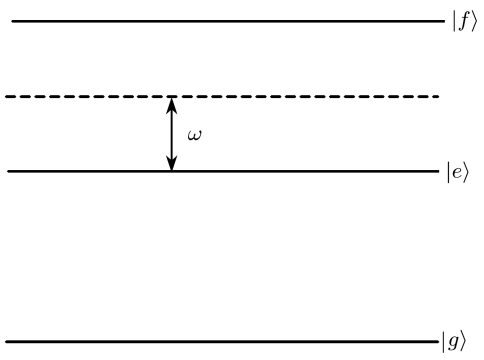


Fig.1. Rydberg-atom energy-level configuration, where ω is the cavity mode frequency.

the Ramsey cavities frequency. The effective Hamiltonian for the dispersive atom-cavity interaction is given by[17]

$$H_{\rm eff}^{\rm i} = \frac{\chi^2}{\Delta} a_{\rm i}^{+} a_{\rm i} (|f\rangle\langle f| - |e\rangle\langle e|), \tag{3}$$

where a_i^+ and a_i are the creation and annihilation operator for cavity mode i respectively, $\Delta = \omega_{\rm f} - \omega_{\rm e} - \omega$ is the detuning where $\omega_{\rm f}$ and $\omega_{\rm e}$ are the frequencies of the upper and intermediate levels respectively, ω is the cavity mode frequency, and χ is the atom–cavity–mode coupling constant. Outside the cavities, the classical fields in the Ramsey zones are resonant with the $|e\rangle \leftrightarrow |g\rangle$ transition, and generate $\pi/2$ pulses having the effects $|e\rangle \leftrightarrow (|e\rangle + |g\rangle)/\sqrt{2}$, $|g\rangle \leftrightarrow (|g\rangle - |e\rangle)/\sqrt{2}$. The atomic states $|g\rangle$, $|e\rangle$ and $|f\rangle$ could be of principal quantum numbers 49, 50 and 51, respectively, with the cavity tuned near the 50–51 circular-to-circular state transition at a frequency of about 50 GHz.[18]

2.2. Generation of the quantum channel

To realize the teleportation of ECS, first Alice and Bob must share a quantum channel. Now we will describe how Alice and Bob generate the quantum channel. We assume that the prepared Rydberg atom A is initially in the state $|e\rangle$, and a subsequent $\pi/2$ pulse in the first Ramsey zone R_1 creates the superposition $(|e\rangle + |g\rangle)/\sqrt{2}$. Meanwhile the cavity modes 3 and 4 are initially prepared in coherent state $|\alpha\rangle_3 |\alpha\rangle_4$. The initial state of the atom A+C3+C4 is

$$|\phi\rangle_{\rm C3-C4-A} = \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle)\,|\alpha\rangle_3\,|\alpha\rangle_4\,. \tag{4}$$

We now inject the atom A into C3 and C4 respectively, and control the interaction time being both fulfilled $(\chi^2/\Delta)\tau = \pi$, then the entangled state of the atom A and cavity modes 3 and 4 is

$$\begin{aligned}
&|\phi\rangle_{\rm C3-C4-A}\,(\tau+\tau)\\
&= e^{-{\rm i}H_{\rm eff}^{3}\tau} e^{-{\rm i}H_{\rm eff}^{4}\tau} |g\rangle + |e\rangle)\,|\alpha\rangle_3\,|\alpha\rangle_4/\sqrt{2}\\
&= \Big(|g\rangle\,|\alpha\rangle_3\,|\alpha\rangle_4 + |{\rm e}\rangle\,\Big|{\rm e}^{{\rm i}(\chi^2/\Delta)\tau}\alpha\Big\rangle_3\,\Big|{\rm e}^{{\rm i}(\chi^2/\Delta)\tau}\alpha\Big\rangle_4\Big)/\sqrt{2}\\
&= (|g\rangle\,|\alpha\rangle_3\,|\alpha\rangle_4 + |e\rangle\,|-\alpha\rangle_3\,|-\alpha\rangle_4)/\sqrt{2}.
\end{aligned} \tag{5}$$

The state shown in Eq.(5) is the quantum channel which Alice and Bob share together. When they are separated, the atom A belongs to Alice and the cavities C3 and C4 are in Bob's side.

2.3. Teleportation of ECS

Now Alice decides to teleport the ECS described in Eq.(1) to Bob by using the quantum channel generated before. The state of the whole system of the atom and four cavity modes is

$$\begin{aligned}
&|\psi\rangle_{\rm C1-C2-C3-C4-A}\\
&= |\varphi\rangle_{\rm C1-C2} \otimes |\phi\rangle_{\rm C3-C4-A}\,(\tau+\tau)\\
&= \frac{1}{\sqrt{2}}(a\,|\alpha\rangle_1\,|\alpha\rangle_2 + b\,|-\alpha\rangle_1\,|-\alpha\rangle_2)\\
&\quad \otimes(|g\rangle\,|\alpha\rangle_3\,|\alpha\rangle_4 + |e\rangle\,|-\alpha\rangle_3\,|-\alpha\rangle_4).
\end{aligned} \tag{6}$$

Then we inject the atom A into C1 and C2, respectively. Similarly, the interaction time satisfies $(\chi^2/\Delta)\tau = \pi$, then the state shown in Eq.(4) will become

$$\begin{aligned}
&|\psi'\rangle_{\rm C1-C2-C3-C4-A}\\
&= \frac{1}{\sqrt{2}}(a\,|g\rangle\,|\alpha\rangle_1\,|\alpha\rangle_2\,|\alpha\rangle_3\,|\alpha\rangle_4\\
&\quad + a\,|e\rangle\,|-\alpha\rangle_1\,|-\alpha\rangle_2\,|-\alpha\rangle_3\,|-\alpha\rangle_4\\
&\quad + b\,|g\rangle\,|-\alpha\rangle_1\,|-\alpha\rangle_2\,|\alpha\rangle_3\,|\alpha\rangle_4\\
&\quad + b\,|e\rangle\,|\alpha\rangle_1\,|\alpha\rangle_2\,|-\alpha\rangle_3\,|-\alpha\rangle_4).
\end{aligned} \tag{7}$$

After passing the atom A through the second Ramsey zone R_2 we have the state

$$\begin{aligned}
&|\psi''\rangle_{\rm C1-C2-C3-C4-A}\\
&= \frac{1}{\sqrt{2}}[|\alpha\rangle_1\,|\alpha\rangle_2\,|e\rangle\,(a\,|\alpha\rangle_3\,|\alpha\rangle_4 + b\,|-\alpha\rangle_3\,|-\alpha\rangle_4)\\
&\quad + |-\alpha\rangle_1\,|-\alpha\rangle_2\,|e\rangle\,(a\,|-\alpha\rangle_3\,|-\alpha\rangle_4 + b\,|\alpha\rangle_3\,|\alpha\rangle_4)\\
&\quad - |\alpha\rangle_1\,|\alpha\rangle_2\,|g\rangle\,(a\,|\alpha\rangle_3\,|\alpha\rangle_4 - b\,|\alpha\rangle_3\,|\alpha\rangle_4)\\
&\quad + |-\alpha\rangle_1\,|-\alpha\rangle_2\,|g\rangle\,(a\,|-\alpha\rangle_3\,|-\alpha\rangle_4 - b\,|\alpha\rangle_3\,|\alpha\rangle_4)].
\end{aligned} \tag{8}$$

2.4. Transformation performed in the teleportation process

According to Eq.(8), by the results of measurement on the atom A and cavity modes 1 and 2 (how to measure on the cavity field is discussed in Ref.[19]), we know that the modes 3 and 4 in Bob' side will collapse probably on one of the four states described by

$$|\varphi\rangle^{(1)}_{\mathrm{C3-C4}} = a\,|\alpha\rangle_3\,|\alpha\rangle_4 + b\,|-\alpha\rangle_3\,|-\alpha\rangle_4\,, \quad (9)$$

$$|\varphi\rangle^{(2)}_{\mathrm{C3-C4}} = a\,|-\alpha\rangle_3\,|-\alpha\rangle_4 + b\,|\alpha\rangle_3\,|\alpha\rangle_4\,, \quad (10)$$

$$|\varphi\rangle^{(3)}_{\mathrm{C3-C4}} = a\,|\alpha\rangle_3\,|\alpha\rangle_4 - b\,|-\alpha\rangle_3\,|-\alpha\rangle_4\,, \quad (11)$$

$$|\varphi\rangle^{(4)}_{\mathrm{C3-C4}} = a\,|-\alpha\rangle_3\,|-\alpha\rangle_4 - b\,|\alpha\rangle_3\,|\alpha\rangle_4\,. \quad (12)$$

Case 1 If Alice detects the atom A in state $|e\rangle$ and the cavity modes 1 and 2 in the coherent states $|\alpha\rangle_1\,|\alpha\rangle_2$, Bob will get the state shown in Eq.(9) and the teleportation is successful.

Case 2 If Alice detects the atom A in state $|e\rangle$ and the cavity modes 1 and 2 in the coherent state $|-\alpha\rangle_1\,|-\alpha\rangle_2$, Bob will get the state shown in Eq.(10). Then he just sends an auxiliary three-level cascade atom B which is the same as A, prepared in the excited state $|e_{\mathrm{B}}\rangle$, to interact with C3 and C4 respectively, by selecting the interaction time such that $(\chi^2/\Delta)\tau = \pi$ the cavity modes 3 and 4 evolve exactly to the original ECS.

Case 3 For the outcome $|g\rangle\,|\alpha\rangle_1\,|\alpha\rangle_2$, Bob can still obtain the original ECS from the projected states in C3 and C4. However, this demands considerable effort, working against the fidelity of the teleportation process. Because in order to leave the modes 3 and 4 in the original state ECS, Bob needs to perform a transformation as follows:

$$\begin{aligned} & U(a\,|\alpha\rangle_3\,|\alpha\rangle_4 - b\,|-\alpha\rangle_3\,|-\alpha\rangle_4) \\ \rightarrow\ & (a\,|\alpha\rangle_3\,|\alpha\rangle_4 + b\,|-\alpha\rangle_3\,|-\alpha\rangle_4), \end{aligned} \quad (13)$$

which is in general not a unitary transformation except the limit case $|\alpha| \rightarrow \infty$,[20] and $|U| = 1/\sqrt{2(|a|^2+|b|^2)-1}$, which is the normalization constant. In those cases, we achieve the nonunitary transformation by the method discussed as follows. First, send a two-level atom C prepared in its lower state $|g_{\mathrm{C}}\rangle$ through the cavity C3, and the atom is resonant with the cavity, with $|g_{\mathrm{C}}\rangle$ and $|e_{\mathrm{C}}\rangle$ being the lower and upper levels respectively. The state of the system, atom C + modes 3 and 4, at the initial time $t = 0$, is $U\,|g_{\mathrm{C}}\rangle\,(a\,|\alpha\rangle_3\,|\alpha\rangle_4 - b\,|-\alpha\rangle_3\,|-\alpha\rangle_4)$.[21] Under the Jaynes–Cummings dynamics, at a later time $t > 0$, the fidelity of the cavity modes' state with respect to the desired state $(a\,|\alpha\rangle_3\,|\alpha\rangle_4 + b\,|-\alpha\rangle_3\,|-\alpha\rangle_4)$ can be expressed as

$$\begin{aligned} F(a,b,\alpha,t) &= (a_3^*\,\langle\alpha|_4\,\langle\alpha| + b_3^*\,\langle-\alpha|_4\,\langle-\alpha|)\rho(a\,|\alpha\rangle_3\,|\alpha\rangle_4 + b\,|-\alpha\rangle_3\,|-\alpha\rangle_4) \\ &= |U|^2\,\mathrm{e}^{-2|\alpha|^2}\left\{\left|\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!}[|a|^2-|b|^2+2(-1)^n\mathrm{i}\mathrm{Im}(ab^*)\mathrm{e}^{-2|\alpha|^2}]\cos(\sqrt{n}\chi t)\right|^2\right. \\ &\quad \left.+\left|\sum_{n=0}^{\infty}\frac{|\alpha|^{2n+1}}{\sqrt{n!(n+1)!}}\left[|a|^2+|b|^2+2(-1)^n\mathrm{Re}(ab^*)\mathrm{e}^{-2|\alpha|^2}\right]\sin(\sqrt{n+1}\chi t)\right|^2\right\}, \end{aligned} \quad (14)$$

where ρ is the reduced density operator of the field at time t.

Figures 2(a) and 2(b) show the fidelity F of Eq.(14) as a function of time, along with excitation probability P_{e} at time t that the atom is found in its upper state $|e_{\mathrm{C}}\rangle$, for the case $\alpha = 4$ and for $a = b = 1/\sqrt{2(1+\mathrm{e}^{-64})}$, and $a = 2b = 2/\sqrt{5+4\mathrm{e}^{-64}}$. It is seen that F and P_{e} have the same extreme point, and both get the largest value when $t \cong \pi/2\,|\alpha|\,\chi$, as long as $|\alpha|$ not too small ($|\alpha| > 3$).

Figure 3 represents $F(t = \pi/2\,|\alpha|\,\chi)$ as a function of $|\alpha|$ and a/b at $t = \pi/2\,|\alpha|\,\chi$. Apparently, as $|\alpha|$ becomes larger, the fidelity F at $t = \pi/2\,|\alpha|\,\chi$ becomes close to unity, meanwhile it varies slowly with respect to a and b for $|\alpha| > 1$.[21] Especially, from Fig.3, we know that when $|\alpha| = 7$ the fidelity can catch up to 0.98. So if the atom C is measured in the state $|e_{\mathrm{C}}\rangle$ when the interaction time is $t = \pi/2\,|\alpha|\,\chi$, Bob can get the initial state ECS with high fidelity of nearly 100%.

Case 4 For the outcome $|g\rangle\,|-\alpha\rangle_1\,|-\alpha\rangle_2$, Bob can first apply the transformation described in case 2, and then allow the cavity C3 to interact with a single atom as described in case 3.

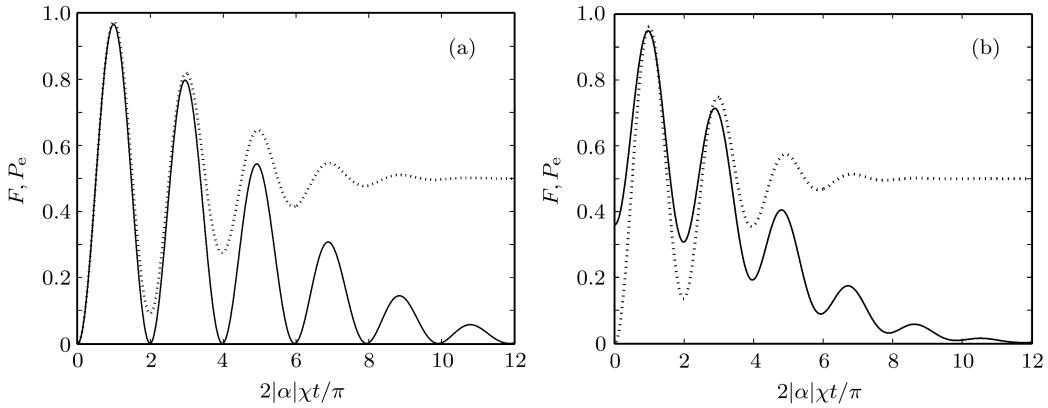


Fig.2. The fidelity F (solid line) of Eq.(14) and excitation probability P_e (dashed line) as a function of time for the case of $|\alpha| = 4$ and (a) $a = b = 1/\sqrt{2(1+e^{-64})}$, (b) $a = 2b = 2/\sqrt{5+4e^{-64}}$.

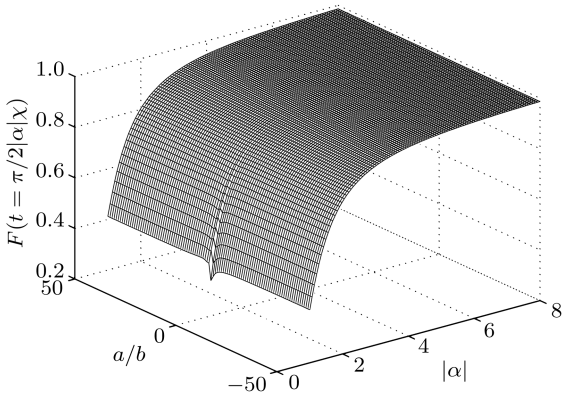


Fig.3. The fidelity $F(t = \pi/2\,|\alpha|\,\chi)$ as a function of $|\alpha|$ and a/b.

In a word, for case 3 and case 4, the transformation that needs Bob to carry out can be realized by allowing the cavity mode 3 to interact with a single two-level atom in its lower state and waiting for a time of $\pi/2\,|\alpha|\,\chi$ to measure the state of the atom, decoupling the atom from the cavity modes 3 and 4, and confirming that the atom has indeed excited. If $|\alpha|$ is not too small, the fidelity becomes to unity (for example, $F > 0.91$ when $|\alpha| > 3$), and for reasonably large $|\alpha|$, the fidelity F can be regarded as practically equal to 1.

3. Discussion

In conclusion, we have presented a scheme for teleporting the bipartite ECS, and quantum channel is generated by using interaction of a three-level atom A with two dispersive cavities C3 and C4, respectively. Our scheme has the following advantages. First, it uses fewer cavities than the previous scheme proposed in Ref.[16], where five cavities and an auxiliary atom are used. Next we give the transformation required in the teleportation process. At last, we calculate the fidelity of the scheme and get a high fidelity. By using Rydberg atoms of long radiative lifetime and high-Q superconducting microwave cavities, the negative effects due to decoherence can be disregarded during the required interaction times.[18] In fact, according to Ref.[22], the radiative time of the Rydberg atoms with the principal quantum number 49, 50 and 51 is about 3×10^{-2} s. It is reported that the cavity lifetime is about $T_{cav} \sim 10^{-12}Q$ corresponding to frequency $\nu \sim 50$ GHz.[16] We choose $Q > 10^{11}$ so that $T_{cav} > 0.1$ s, for cavities with $Q \sim 10^{12}$ have been reported in Ref.[23]. The coupling constant of the atom to the cavity is about $\chi/2\pi = 25$ kHz.[24] If we take $\Delta = 10\chi$, the interaction time between the atom and the four cavity-fields is of the order of $4\pi\Delta/\chi^2 \cong 8 \times 10^{-4}$ s. Additional time needed for the classical field pulse can be negligible at this stage. Thus the interaction time needed to complement the total procedure is much shorter than the radiative time of the Rydberg atom and the cavity lifetime with an enough high-Q. Hence, our scheme would be realized experimentally with present and future cavity QED technology.

References

[1] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895

[2] Davidovich L, Zagury N, Brune M, Raimond J M and Haroche S 1994 *Phys. Rev.* A **50** R895

[3] Zheng S B 2005 *Chin. Phys.* **14** 1825

[4] Bouwmeester B, Pan J W, Mattle K, Eibl M, Weinfurter H and Zeilinger A 1997 *Nature* **390** 575

[5] Li Y, Zhang T C, Zhang J X and Xie C D 2003 *Chin. Phys.* **12** 861

[6] Ikram M, Zhu S Y and Zubairy M S 2000 *Phys. Rev.* A **62** 022307

[7] Marian P and Marian A 2006 *Phys. Rev.* A **74** 042306

[8] Braunstein S L and Kimble H J 1998 *Phys. Rev. Lett.* **80** 869

[9] Zheng S B and Guo G C 1997 *Phys. Lett.* A **236** 180

[10] de Almeida N G, Napolitano R and Moussa M H Y 2000 *Phys. Rev.* A **62** 010101(R)

[11] Dai H Y, Chen P X, Liang L M and Li C Z 2004 *Acta Phys. Sin.* **53** 441 (in Chinese)

[12] Liu C L and Zheng Y Z 2006 *Acta Phys. Sin.* **55** 4631 (in Chinese)

[13] An N B 2003 *Phys. Rev.* A **68** 022321

[14] Wang X G 2001 *Phys. Rev.* A **64** 022302

[15] Cai X H and Kuang L M 2002 *Phys. Lett.* A **300** 103

[16] Song K H and Zhang W J 2001 *Phys. Lett.* A **290** 214

[17] Gerry C C 1996 *Phys. Rev.* A **53** 3818

[18] Gerry C C 1996 *Phys. Rev.* A **54** R2529

[19] Enk S J and Hirota O 2001 *Phys. Rev.* A **64** 022313

[20] Gerry C C 1996 *Phys. Rev.* A **53** 2857

[21] Cheong Y K, Kim H and Lee H W 2004 *Phys. Rev.* A **70** 032327

[22] Brune M, Hagley E, Dreyer J, Maitre X, Maali A, Wunderlich C, Raimond J M and Haroche S 1996 *Phys. Rev. Lett.* **77** 4887

[23] Rempe G, Schmidt Kaler F and Walther H 1990 *Phys. Rev. Lett.* **64** 2783

[24] Osnaghi S, Bertet P, Auffeves A, Maioli P, Brune M, Raimond J M and Haroche S 2001 *Phys. Rev. Lett.* **87** 037902