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Preparation and control of entangled states in the two-mode coherent fields interacting with a moving atom via two-photon process*

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We investigate the preparation and the control of entangled states in a system with the two-mode coherent fields interacting with a moving two-level atom via the two-photon transition. We discuss entanglement properties between the two-mode coherent fields and a moving two-level atom by using the quantum reduced entropy, and those between the two-mode coherent fields by using the quantum relative entropy. In addition, we examine the influences of the atomic motion and field-mode structure parameter p on the quantum entanglement of the system. Our results show that the period and the duration of the prepared maximal atom-field entangled states and the frequency of maximal two-mode field entangled states can be controlled, and that a sustained entangled state of the two-mode field, which is independent of atomic motion and the evolution time, can be obtained, by choosing appropriately the parameters of atomic motion, field-mode structure, initial state and interaction time of the system.

Keywords: two-mode coherent fields, entangled states, reduced entropy, relative entropy, atomic motion and field-mode structure

PACC: 4250

1. Introduction

Quantum entanglement(QE) is one of the most striking features of quantum mechanics. The manipulation of entangled states that have nonclassical and nonlocal properties leads to a better understanding of basic quantum phenomena. On the other hand, QE is an essential ingredient and entangled state that constitutes a valuable resource in quantum information processing,[1] such as quantum cryptography,[2] computer,[3] and teleportation.[4] Therefore, it is of great importance to quantify the entanglement to assess the efficacy of quantum information processing and to prepare and control quantum entangled states in well-controlled experimental conditions. In the latter case, the entanglement results from the interaction of a two-level atom with a cavity mode field. Interrupting the coherent interaction at a chosen time, one can generate atom-cavity entanglement,[5] exchange the atom and cavity states,[6] or realize a two-qubit quantum logics gate[7] for a basic unit of quantum information processing systems. These functions can be used to realize a quantum non-demolition measurement of a single photon[8] or to generate a three-particle entangled state.[9] Up to now, these experiments have involved a single cavity mode field interacting with one, two, or three atoms crossing it successively. Recently, unconditional quantum teleportation of an unknown coherent state has been realized experimentally by exploiting a two-mode squeezed vacuum state as an entanglement resource,[10] but the effect of environment on the two-mode fields has not been considered in the scheme. In another scheme,[11] the authors presented the realization of a more complex entanglement in cavity quantum electrodynamics, by involving in the experiment two independent cavity modes via emitting a single photon. These multi-mode entanglement opens interesting perspectives for quantum information processing and nonlocality tests.[12−15] In real experimental situations, due to its coupling to the environment, the entangled state of the two-mode fields inevitably loses its pu-

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rity, and it becomes mixed. This phenomenon, i.e. decoherence, is the most dangerous obstacle for all entanglement manipulations.

Even so, all these theoretical results are obtained only under the condition in which the atomic motion is neglected and the field-mode structure is not taken into account. Recently, the atomic space motion has been taken into consideration in order to achieve cooled atoms and the super-cooled atoms with the development of laser cooling and trapped atom technology.[16] Schlicher[17] has examined the influences of the atomic motion and the field-mode structure on the atomic dynamics. Joshi and Lawande[18] have analysed the influences of the atomic motion and the field-mode structure on the squeezing of the field. References [19, 20] have studied the effects of the atomic motion and the field-mode structure on entropy squeezing in JCM. But these outcomes show that the quantum effect due to the atomic motion cannot be underrated in experiment.

In this paper, we investigate the entanglement properties of the two-mode coherent fields interacting with a moving two-level atom via the two-photon transition by regarding the atom as the environment. We study the QE between the two-mode coherent fields and the two-level atom by using the quantum reduced entropy and that between two-mode coherent fields by using the quantum relative entropy.[21] We examine the influences of the atomic motion and field-mode structure parameter p on the entanglement properties of the system. We find that three types of entangled states, i.e. a maximal-entangled state between the two-mode coherent fields and the atom, a maximal-entangled state between the two-mode coherent fields, and a sustained entangled state between the two-mode coherent fields, can be prepared, that the period, the duration and the frequency of the maximal entangled states can be controlled, and that the atomic motion does not hamper that a king of sustained entangled state is produced via choosing appropriately the parameters of the system. Our study provides a good way for preparing and controlling the entangled state by using the two-mode coherent fields interacting with a moving atom in experiment.

2. The model and its solution

The present cavity quantum electrodynamic experiments[22] are equipped with an atomic beam passing along the axis of the rectangular or cylindrical cavity so that one can investigate the interaction of an atom in motion with different field modes. The system considered here consists of a two-level atom in motion interacting with a quantized field via the two-quantum transition processes. The effective Hamiltonian in the rotating-wave approximation[17] is

$$H = \omega_0 S_z + \sum_{j=1}^{2} \omega_j a_j^+ a_j + g f_1(z) f_2(z)(a_1^+ a_2^+ S_- + a_1 a_2 S_+), \quad (1)$$

where z, $+$, and $-$ are the usual pseudo-spin operators of the two-level atom; $a_j^+(a_j)$ is the photon creation (annihilation) operator of the field mode with a frequency $w_j (j \equiv 1, 2)$; g is the coupling constant between the atom and the two-mode coherent fields; $f_i(z)(i = 1, 2)$ denotes the mode structure of the ith-mode coherent fields. We restrict our study to the atomic motion along the z axis so that the z-dependence of the field-mode function must be taken into account.The atomic motion can be incorporated in a usual way[23]

$$f_i(z) \to f_i(vt), \quad (2)$$

where υ denotes the atomic motion speed. In order to make the later discussion easy, a TEM$_{mnp}$ mode is defined as

$$f_i(vt) = \sin\left(\frac{p_i \pi v t}{L}\right), \quad (3)$$

where p_i represents the number of half wavelengths of the ith field mode inside a cavity of the length L.

For simplicity, we consider the two-photon resonance case, in which $\omega_0 = \omega_1 + \omega_2$ and $p_1 = p_2 = p$, and that the atom enters the cavity and leaves the cavity after passing p half-wavelengths of electric field. Using the standard techniques,[24] in the interaction picture and two-dimensional atomic space the time evolution Dyson operator can be written as

$$U_{\rm I}(t) = \begin{bmatrix} \hat{u}_{11} & -{\rm i}\hat{u}_{12} \\ -{\rm i}\hat{u}_{21} & \hat{u}_{22} \end{bmatrix}, \quad (4)$$

where

$$\hat{u}_{11} = \cos(\hat{A} g\theta(t)), \quad \hat{u}_{21} = a_1^+ a_2^+ \frac{\sin(\hat{A} g\theta(t))}{\hat{A}},$$
$$\hat{u}_{22} = \cos(\hat{B} g\theta(t)), \quad \hat{u}_{12} = a_1 a_2 \frac{\sin(\hat{B} g\theta(t))}{\hat{B}},$$
$$\hat{A} = \sqrt{(a_1 a_2 a_1^+ a_2^+)}, \quad \hat{B} = \sqrt{(a_1^+ a_2^+ a_1 a_2)},$$

$$\theta(t) = \int_0^t [f(vt')]^2 \mathrm{d}t' = \int_0^t \left[\sin\left(\frac{p\pi vt'}{L}\right)\right]^2 \mathrm{d}t' = \frac{1}{2}\left[t - \frac{L}{2p\pi vt}\sin(2pgt)\right]. \quad (5)$$

We consider at the time $t = 0$ the atom is in a coherent superposition state of the excited state $|1\rangle$ and the ground state $|0\rangle$

$$|\psi_{\mathrm{A}}(0)\rangle = \cos(\xi/2)|1\rangle + \exp(\mathrm{i}\varphi)\sin(\xi/2)|0\rangle, \quad (6)$$

where $0 < \xi < \pi$, and $0 < \varphi < 2\pi$ is the phase of the atom dipole.

The field is in the two-mode coherent state

$$|\psi_{\mathrm{F}}(0)\rangle = |\alpha_1, \alpha_2\rangle = \sum_{n_1,n_2=0}^{\infty} F(n_1)F(n_2)|n_1, n_2\rangle,$$
$$F(n_j) = \exp\left(-\frac{\bar{n_j}}{2}\right)\frac{\alpha_j^{n_j}}{\sqrt{n_j!}} \quad (7)$$

with $F(n_j)$ denoting the atomic distribution; $\alpha_j = \sqrt{n_j}\exp(\mathrm{i}\psi_j)$, where $\bar{n}_j$ and ψ_j represent the initial average photon number and the direction angle of the excitation for $j(j \equiv 1, 2)$mode respectively.

The initial state of the system can be written as

$$|\psi_{\mathrm{FA}}(0)\rangle = |\psi_{\mathrm{F}}(0)\rangle \otimes |\psi_{\mathrm{A}}(0)\rangle. \quad (8)$$

At any time $t > 0$ the state vector of the system is given by

$$|\psi_{\mathrm{FA}}(t)\rangle = U_{\mathrm{I}}(t)|\psi_{\mathrm{FA}}(0)\rangle = |D\rangle|1\rangle + |T|0\rangle, \quad (9)$$

with

$$|D\rangle = \sum_{n_1,n_2=0}^{\infty} F(n_1)F(n_2)(\cos(\xi/2)\exp(\mathrm{i}[n_1\psi_1 + n_2\psi_2])\cos(g\theta(t)\gamma_{n_1+1,n_2+1})|n_1, n_2\rangle - \mathrm{i}\sin(\xi/2)\exp(-\mathrm{i}[\varphi - n_1\psi_1 - n_2\psi_2])\sin(g\theta(t)\gamma_{n_1,n_2})|n_1 - 1, n_2 - 1\rangle), \quad (10)$$

$$|T\rangle = \sum_{n_1,n_2=0}^{\infty} F(n_1)F(n_2)(\cos(\xi/2)\exp(\mathrm{i}[n_1\psi_1 + n_2\psi_2])\sin(g\theta(t)\gamma_{n_1+1,n_2+1})|n_1 + 1, n_2 + 1\rangle + \mathrm{i}\sin(\xi/2)\exp(-\mathrm{i}[\varphi - n_1\psi_1 - n_2\psi_2])\cos(g\theta(t)\gamma_{n_1,n_2})|n_1, n_2\rangle), \quad (11)$$

where $\gamma_{n_1,n_2} = \sqrt{n_1, n_2}$.

The density operator for the system is given by

$$\rho_{\mathrm{FA}}(t) = |\psi_{\mathrm{FA}}(t)\rangle\langle\psi_{\mathrm{FA}}(t)| = \begin{bmatrix} |D\rangle\langle D| & |D\rangle\langle T| \\ |T\rangle\langle D| & |T\rangle\langle T| \end{bmatrix}, \quad (12)$$

while the reduced two-mode coherent field density operator for the system is expressed as

$$\rho_{\mathrm{F}}(t) = \mathrm{Tr}_{\mathrm{A}}(\rho_{\mathrm{FA}}(t)) = |D\rangle\langle D| + |T\rangle\langle T|. \quad (13)$$

The eigenvalues and the eigenstates of the reduced two-mode coherent field density operator $\rho_{\mathrm{F}}(t)$ are obtained to be

$$\lambda_{\mathrm{F}}^{\pm} = \langle D|D\rangle \pm \exp(\mp\delta)|\langle D|T\rangle| = \langle T|T\rangle \pm \exp(\pm\delta)|\langle D|T\rangle|, \quad (14)$$

$$|\psi_{\mathrm{F}}(t)\rangle = \frac{1}{\sqrt{2\lambda_{\mathrm{F}}^{\pm}\cosh\delta}}(\exp[1/2(\mathrm{i}\alpha \pm \delta)]|D\rangle \pm \exp[-1/2(\mathrm{i}\alpha \pm \delta)]|T\rangle), \quad (15)$$

where

$$\sinh(\delta) = \sinh^{-1}[\frac{1}{2|\langle D|T\rangle|}(\langle D|D\rangle - \langle T|T\rangle)], \quad (16)$$

$$\alpha = \tanh^{-1}\left[\frac{\mathrm{Im}\langle D|T\rangle}{\mathrm{Re}\langle D|T\rangle}\right]. \quad (17)$$

Similarly, we can obtain the reduced atom density operator for the system as follows:

$$\rho_{\mathrm{A}}(t) = \begin{bmatrix} \langle D|D\rangle & \langle T|D\rangle \\ \langle D|T\rangle & \langle T|T\rangle \end{bmatrix}. \quad (18)$$

It turns out that the eigenvalues of $\rho_A(t)$ are equal to those of $\rho_{\mathrm{F}}(t)$, that is, $\lambda_{\mathrm{F}}^{\pm} = \lambda_{\mathrm{A}}^{\pm} = \lambda^{\pm}$. The eigenstates of $\rho_{\mathrm{A}}(t)$ are given by

$$\psi_{\mathrm{A}}^{\pm}(t)\rangle = \frac{1}{\sqrt{2\lambda_{\mathrm{A}}^{\pm}\cosh\delta}}(\exp[1/2(\mathrm{i}\alpha \pm \delta)]|e\rangle \pm \exp[-1/2(\mathrm{i}\alpha \pm \delta)]|g\rangle). \quad (19)$$

By using the Schmidt theorem,[25] the system state vector can be rewritten as

$$\Psi_{\mathrm{FA}}(t)\rangle = \sqrt{\lambda^+}|\psi_{\mathrm{F}}^+\rangle|\phi_{\mathrm{A}}^+\rangle + \sqrt{\lambda^-}|\psi_{\mathrm{F}}^-\rangle|\phi_{\mathrm{A}}^-\rangle. \quad (20)$$

The properties of the QE between the two-mode coherent fields and the moving atom, and those between the two-mode coherent fields will be discussed in the next section by using the above results.

3. Numerical results and discussion

The time evolution of the field (atomic) reduced entropy $S(\rho_{\rm F})$ reflects the time evolution of the degree of the entanglement between the field and the atom (DEFA). The higher the reduced entropy is, the greater the DEFA will be. The information concerning the field is inferred by measurements of atomic properties. Following the work by Phoenix and Knight,[25] we can express the reduced entropy $S(\rho_{\rm F})$ of the two-mode coherent fields in terms of the eigenvalue $\lambda_{\rm F}^{\pm}$ of $\rho_{\rm F}(t)$ given by Eq.(14) as

$$S(\rho_{\rm F}) = -\lambda_{\rm F}^{+}\log_2\lambda_{\rm F}^{+} - \lambda_{\rm F}^{-}\log_2\lambda_{\rm F}^{-}. \qquad (21)$$

If $S(\rho_{\rm F})$ takes its minimum value to be zero, the two-mode coherent fields and the atom are disentangled; if $S(\rho_{\rm F})$ takes its maximum value to be 1, the two-mode coherent fields and the atom are in a maximal entangled state.

3.1. Influences of atomic motion and field-mode structure on the entanglement between the fields and the moving atom

Now we examine the influences of the atomic motion and the field-mode structure parameter p on the DEAF $S(\rho_{\rm F})$ as shown in the Fig.1 by the solid line.

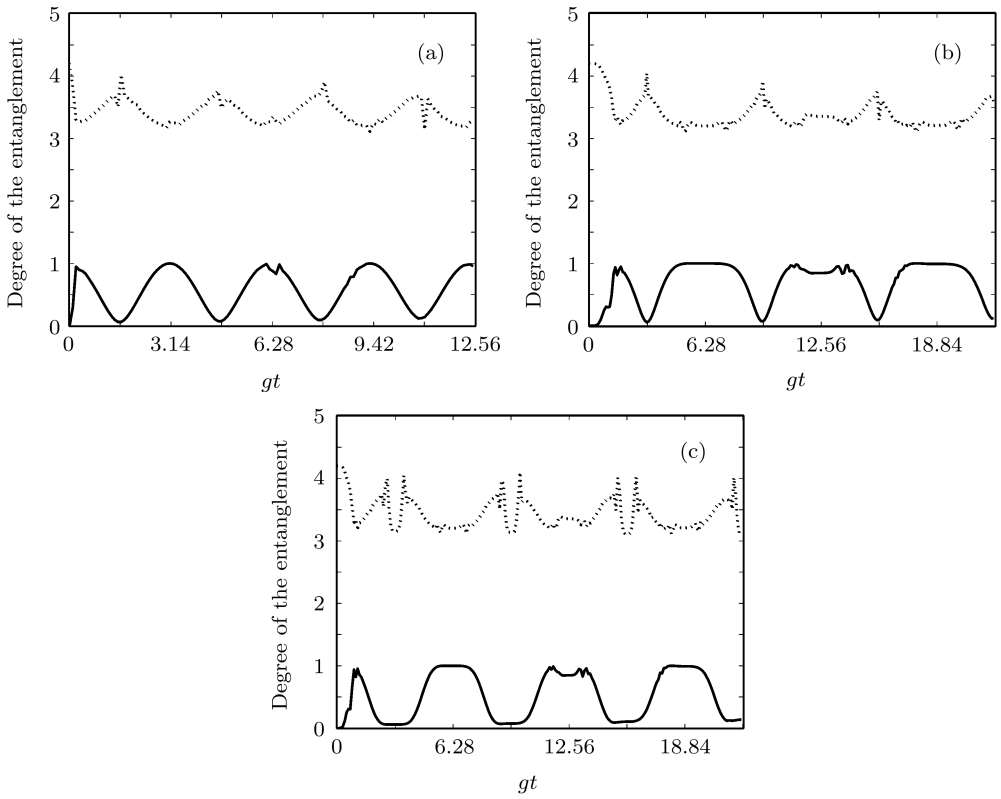


Fig.1. The time evolution of the degree of the entanglement. $S(\rho_{\rm F})$ is shown by the solid line and $E_{\rm R}(\rho_{\rm F})$ by the dotted line with $\xi = 0$, $\beta = 0$, $\bar{n}_1 = \bar{n}_2 = 20$; Subfigure (a) is for the atomic motion neglected; (b) is for the atom moving at a speed $v = gL/\pi$ and $p = 1$; (c) for the atom moving at a speed $v = gL/\pi$ and $p = 2$.

First, the atomic motion is neglected for the case in which $\xi = 0$, $\bar{n}_1 = \bar{n}_2 = 20$, and $\beta = 0$, as shown in Fig.1(a). The $S(\rho_{\rm F})$ evolves following a sin wave with a period of π except at initial stage of time. This corresponds to the result obtained in Ref.[26].

Furthermore, Figs.1(b) and 1(c) illustrate the time evolution of $S(\rho_{\rm F})$ when the atom is moving at a speed $v = gL/\pi$, and have field-mode structure parameters $p = 1$ and $p = 2$ respectively, the time evolution of $S(\rho_{\rm F})$ appears as an approximate rectangle

wave. At time $gt_{\rm d} = (2n+1)\pi$, where $t_{\rm d}$ refers to the time when the two-mode coherent fields is disentangled from the atom, $S(\rho_F)$ takes its minimum value. But $S(\rho_{\rm F})$ exhibits maximal values at time $gt_{\rm e} = 2m\pi (m = 1, 2, 3, \cdots)$, where $t_{\rm e}$ denotes the time required to prepare the maximal entangled state between the two-mode fields and the moving atom.

This point can be quantitatively interpreted by using Eqs.(14) and (20). Under a strong field (both $\bar{n}_1$ and $\bar{n}_2$ are very large), after a straightforward calculation, we can obtain $\lambda^+ \approx 1$ and $\lambda^- \approx 0$ in this case. The system state $|\psi_{\rm FA}\rangle$ can be expressed as

$$|\psi_{\rm FA}\rangle = |\psi_{\rm F}^+\rangle \otimes |\psi_{\rm A}^+\rangle. \tag{22}$$

Obviously, this is a disentangled direct product state. Therefore, $S(\rho_{\rm F})$ is equal to zero. When $gt_{\rm e} = 2m\pi$, the value of $S(\rho_{\rm F})$ tends to 1, which means that the DEAF is very large. This can also be explained from Eqs.(14) and (20). The state vector of the system can be written by using the above mentioned condition as

$$|\Psi_{\rm FA}\rangle \approx \frac{1}{\sqrt{2}}(|\psi_{\rm F}^+\rangle|\psi_{\rm A}^+\rangle + |\psi_{\rm F}^-\rangle|\psi_{\rm A}^-\rangle). \tag{23}$$

This state is just the EPR (Einstein-Podolsky-Rosen) entanglement state and applied widely in quantum information (QI) processing.

From these figures we can draw the following conclusions. (1) Atomic motion leads the evolution period T of the DEEA to be the double of the evolution period in the case without the atomic motion, that is $T = 2\pi$, see Figs.1(a) and 1(b). (2) Atomic motion leads to the extension of time $t_{\rm e}$, see Fig.1(b). (3) The increase of the field-mode structure parameter p causes the range of $t_{\rm e}$ to decrease, and the range of time $t_{\rm d}$ to increase. So, the field-mode structure parameter p can control the ranges of times $t_{\rm e}$ and $t_{\rm d}$, that is, p can control the time range of maximal entanglement and disentangled between the two- mode coherent fields and moving atom. see Figs.1(b) and 1(c).

Physically, all these features can be attributed to the change in atom-field interaction time due to atomic motion. This is due to the fact that the difference in entropy $S(\rho_{\rm F})$ between with and without the atomic motion results from the time factor. The time factor is the scaled time gt: when the atomic motion is neglected, it is $g\theta(t)$, and when the atomic motion is taken into account it is $g\theta(t) = \frac{1}{2}\left[gt - \frac{1}{2p}\sin(2pgt)\right]$. This is made up of two parts: one is linear part $1/2gt$, and the other is nonlinear part $\frac{1}{4p}\sin(2pgt)$ and its amount is very small ($\leq 1/4p$), so $g\theta(t) \approx 1/2gt$ holds approximately, which just leads to the double of the period of squeezing factors, but the nonlinear part determines the range of $t_{\rm e}$. It is shown that the range of $t_{\rm e}$ can be controlled. Once time $t_{\rm e}$ is controlled, we can prepare a maximal entangled state between the two-mode coherent fields and a moving atom in time $t_{\rm e}$ by choosing the parameters of atomic motion and field-mode structure. Our results are important for the experimental realization of the preparation and the control of maximal entangled states.

3.2. Influences of atomic motion and field-mode structure on the entanglement between the two-mode coherent fields

The above results show that the Hamiltonian given by Eq.(1) leads to the entanglement between the two-mode coherent fields and the atom, so that the state of the two-mode coherent fields may evolve into a mixed state. In this case, the degree of the entanglement between two-mode coherent fields cannot be measured by the reduced entropy. However, the relative entropy of the entanglement is a good measure of the degree of the entanglement of two subsystems in a mixed state, which is defined as[11]

$$E_{\rm R}(\rho) = {\rm main}_{\sigma\in D} S(\rho||\sigma), \tag{24}$$

where $S(\rho||\sigma) = {\rm Tr}[\rho(\log_2\rho - \log_2\sigma)]$ is the relative entropy. The minimum is taken over D, the set of all disentangled states. For a pure state, the relative entropy of the entanglement reduces to the reduced entropy of the entanglement, while for a mixed state, it is usually difficult to calculate the relative entropy of the entanglement except for some specific states. The following theorem on the relative entropy of the entanglement has been proved,[27,28] and it turns out to be quite suitable for the present analysis.

Using the result in Ref.[26], the relative entropy of the entanglement of the state Eq.(13) is written as

$$\begin{aligned} E_{\rm R}(\rho_{\rm F}) &= -\sum_n a_{n,n}\log_2 a_{n,n} - S(\rho_{\rm F}) \\ &= -\sum_n a_{n,n}\log_2 a_{n,n} + \lambda^+\log_2\lambda^+ \\ &\quad +\lambda^-\log_2\lambda^-, \end{aligned} \tag{25}$$

where $a_{n,n} = \langle n, n|\rho_{\rm F}|n, n\rangle$. Consequently, it is possible to compute numerically the relative entropy of the entanglement given by Eq.(25).

The time evolutions of the relative entropy of the entanglement $E_{\rm R}(\rho_{\rm F})$ are shown by using the dotted line in Figs.1(a)–1(c), which reflect the time evolution of the degree of the entanglement between the two-mode coherent fields (DETMCF) when atomic motion is neglected. The DETMCF increases monotonically with the relative entropy. Comparing the dotted line with the solid line in Figs.1(a) and 1(b), we can arrive at the following conclusions. (1) The time evolution of the DETMCF is just the opposite to DEFA, that is, as the latter increases with time, the former decreases with time, which comes from the fact that DEFA can impair the DETMCF due to the atom interacting with the two-mode coherent fields. (2) Atomic motion leads the evolution period T of the DETMCF to be the double of the evolution period in the case without the atomic motion, that is $T = 2\pi$, see Figs.1(a) and 1(b). In particular, when $\beta = 0$, and $\bar{n}_1 = \bar{n}_2$, at $t_{\rm d}$ it can be obtained that $\lambda^+ = 1$ and $\lambda^- = 0$. The states of the system can be written in the same form as expression (22), which means that the two-mode coherent fields and the atom are disentangled, while Eq.(15) can be further reduced to

$$|\psi_{\rm F}\rangle = \sqrt{2}\sum_{n=0}^{\infty} F(2n+1)F(2n+1)(-1)^{n+1} \times(\cos(\xi/2)|2n+1, 2n+1\rangle -{\rm i}(\sin(\xi/2)|2n, 2n\rangle). \quad (26)$$

Obviously, when the two-mode coherent fields are in the state with an odd or even photon number, the state is respectively expressed as

$$|\psi_{\rm F}\rangle = \sqrt{2}\sum_{n=0}^{\infty} F(2n+1)F(2n+1) \times(-1)^{n+1}|2n+1, 2n+1\rangle(\xi = 0),$$

or

$$|\psi_{\rm F}\rangle = {\rm i}\sqrt{2}\sum_{n=0}^{\infty} F(2n+1)F(2n+1) \times(-1)^{n}|2n, 2n\rangle(\xi = \pi). \quad (27)$$

When $\xi = \pi/2$, the maximal entangled state between the two-mode fields can be produced, that is,

$$|\psi_{\rm F}\rangle = \sum_{n=0}^{\infty} F(2n+1)F(2n+1) \times(-1)^{n+1}(|2n+1, 2n+1\rangle -{\rm i}|2n, 2n\rangle). \quad (28)$$

The states given by Eqs.(27) and (28) have been applied widely in QI processing.[29,30] (3) The increase in field-mode structure parameter p causes the frequency $F_{\rm f}$ of the maximal entangled state between the two-mode fields to increase in $t_{\rm d}$, see Fig.1(c). (4) Our result further shows that under the condition of a strong field(both $\bar{n}_1$ and $\bar{n}_2$ are very large) and when the atom is in a coherent trapping state, a sustained entangled state between the two-mode coherent fields, which is independent of atomic motion and the evolution time, can be obtained as shown in Figs.2(a) and 2(b).

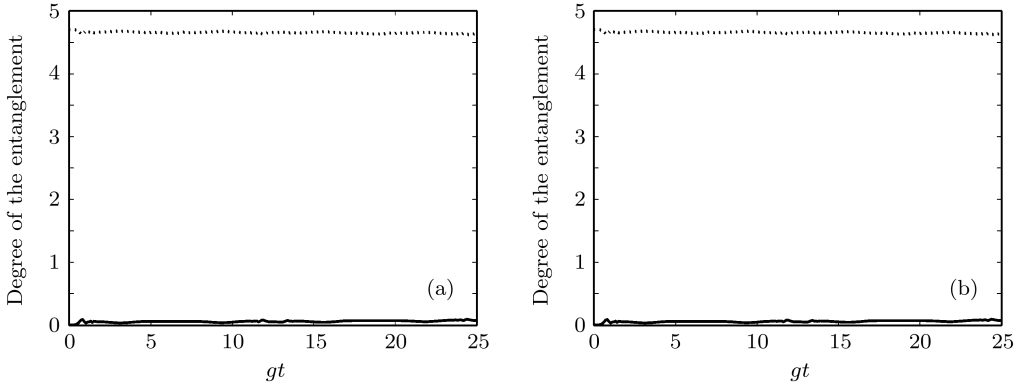


Fig.2. The time evolution of the degree of the entanglement. $S(\rho_{\rm F})$ is shown by the solid line and $E_{\rm R}(\rho_{\rm F})$ by the dotted line with $\xi = \pi/2$, $\beta = 0$, and $\bar{n}_1 = \bar{n}_2 = 40$; Subfigure (a) is for the atomic motion neglected; (b) for the atom moving at a speed $v = gL/\pi$ and $p = 1$.

4. Conclusion

We have investigated the preparation and control of entangled states in a system with two-mode coherent fields interacting with a moving two-level atom via the two-photon transition. We discuss the quantum entanglement between the two-mode coherent fields and the two-level atom by using the quantum reduced entropy and that between the two-mode coherent fields by using the quantum relative entropy. We can draw the following conclusions. (1) Two types of maximal entangled states can be prepared, that is, a maximal atom-fields entangled state and a maximal two-mode coherent field entangled state, and the period T of the two kinds maximal-entangled state is determined by choosing the parameters of atomic motion and interaction time. (2) The duration and the frequency of the two kinds of maximal entangled states can be determined by choosing the field-mode structure parameter p. (3) A sustained entangled state between the two-mode coherent fields, which is independent of atomic motion and the evolution time, can be obtained under the condition of a strong field and the atom is in a coherent trapping state. Our results show that we can prepare and control three kinds of entanglement states by choosing parameters of atomic motion, field-mode structure and the system. The system of the two-mode coherent fields interacting with a moving atom via two-photon process is a valuable resource of entanglement in quantum information processing. Our results are important for the experimental realization of the preparation and the control of entangled states by using the system we have considered here.

References

[1] Bemett C H and Divincenzo D P 2000 *Nature* **404** 247
[2] Cheng M T, Liu S D, Wang X and Wang Q Q 2007 *Chin. Phys.* **56** 4924 (in Chinese)
[3] You J, Li J H and Xie X T 2005 *Chin. Phys.* **14** 1329
[4] Song T Q 2004 *Acta Phys. Sin.* **53** 3358 (in Chinese)
[5] Xiang S H and Song K H 2005 *Acta Phys. Sin.* **54** 1190 (in Chinese)
[6] Wu Y, Chu C M and Leung T P 1999 *Phys. Rev.* A **59** 3032
[7] Song K H 2006 *Chin. Phys.* **25** 286
[8] Nogues G 1999 *Nature* (London) **400** 239
[9] Hui K S 2000 *Acta Phys. Sin* **49** 441 (in chinese)
[10] Furusawa A, Sorensen J L, Braunstein S L, Fuchs C A, Kimble H J and Polzik E S 1998 *Science* **282** 706
[11] Wang C Z and Fang M F 2002 *Acta Phys. Sin.* **51** 1989 (in Chinese)
[12] Wu Y and Deng L 2004 *Opt. Lett.* **29** 1144
[13] Wu Y and Yang X 2006 *Phys. Rev.* D **73** 067701
[14] Wu Y, Payne M G, Hagley E W and Deng L 2004 *Phys. Rev.* A **70** 063812
[15] Wu Y and Yang X 2005 *J. Opt.* B **7** 54
[16] Wu Y and Yang X 1997 *Phys. Rev.* A **56** 2443
[17] Schlicher R R 1989 *Opt. Commun.* **70** 97
[18] Joshi A and Lawande S V 1992 *Int. J. Mod. Phys.* B **6** 3539
[19] Zhou B J, Liu X J, Zhou Q P and Liu M W 2007 *Chin. Phys.* **16** 0420
[20] Liu X J, zhou B J, Fang M F and Zhou Q P 2006 *Acta Phys. Sin* **55** 0704 (in Chinese)
[21] Tan X X, Yun J and Zhang C Q 2006 *Chin. Phys.* **15** 2263
[22] Wu Y 1996 *Phys. Rev.* A **54** 1586
[23] Hu Y H, Fang M F and Liao X P 2007 *Chin. Phys.* **16** 1344
[24] Wu Y and Yang X 1997 *Phys. Rev. Lett.* **78** 3086
[25] Phoenix S J D and Knight P L 1988 *Ann. Phys.* (N.Y.) **186** 381
[26] Liu X J and Fang M F 2003 *Chin. Phys.* **12** 0971
[27] Rains E 1999 *Phys. Rev.* A **60** 179
[28] Wu S and Zhang Y e-print quant-ph/0004018
[29] Zhang Q, Li F L and Li H R 2006 *Acta Phys. Sin.* **55** 2275 (in Chinese)
[30] Jiang C L, Fang M F and Wu Z Z 2006 *Acta Phys. Sin.* **55** 4647 (in Chinese)