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Laser-manipulated the multiphoton transitions of a harmonically trapped particle*

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A single particle magneto-confined in a one-dimensional (1D) quantum wire experiences a harmonic potential, and imposing a sharply focused laser beam on an appropriate site shapes a δ potential. The theoretical investigation has demonstrated that for a sufficiently strong δ pulse the quantum motional stationary state of the particle is one of the eigenstates of the free harmonic oscillator, and it is determined by the site of the laser beam uniquely, namely a quantum state is admissible if and only if the laser site is one of its nodes. The numerical computation shows that all the nodes of the lower energy states with quantum numbers $n \leq 20$, except the coordinate origin, are mutually different. So we can manipulate the multiphoton transitions between the quantum states by adjusting the position of the laser δ pulse and realize the transition from an unknown higher excitation state to a required lower energy state.

Keywords: quantum wire, magnetically trap, laser manipulation, multiphoton transition
PACC: 3280P, 0365G, 7320D, 3290

1. Introduction

With the advance of semiconductor manufacturing technology, the preparation and fabrication technology of quantum wells, quantum wires and quantum dots has tended to maturation,[1−6] as a result one can easily confine a single particle or a few particles in the low-dimensional systems.[7−13] When an individual particle is loaded into a 1D quantum wire and then the applied magnetic field generates a harmonic potential,[14,15] the particle can be trapped in a small spatial region such that the preparation and manipulation of the quantum states become convenient. As is known, the quantum motional state of a particle in a harmonic potential is one of the eigenstates of the free harmonic oscillator.[16] On the other hand, there exist impurities in some semiconductor materials, which form single or multiple obstacles in the quantum wire. The δ potential modelling the short-range interaction between the particles and obstacles is an appropriate model system for the considered problem.[17−21] Experimentally, the single δ pulse as an artificial impurity can be realized by a sharply focused laser beam.[22,23] The array of δ pulses can also be experimentally generated by setting more counter-propagating laser beams of different frequencies to produce a lattice with separated peaks whose widths are very small.[24] When a harmonically trapped particle interacts with a δ potential, Peng and co-authors[25] have shown that for a sufficiently strong δ potential, only the eigenstates of odd parity whose nodes coincide with the site of δ potential can exist. We have also previously demonstrated that adopting the controllable δ pulse[26] or box-like potential[27] as the control signals can manipulate the dynamical behaviour of the system.

In this paper, we study the interaction between a single particle magneto-confined in a 1D quantum wire and a δ pulse with a sufficiently large strength. According to quantum-mechanical theory, when the laser pulse is switched off, the particle obeys the Schrödinger equation of a free harmonic oscillator and has a denumerable infinite number of possible motional stationary states. Our studies reveal that after switching on the laser δ pulse, a stationary eigenstate $\psi_n^{(0)}$ is allowable, the necessary condition is that the laser is focused on a node of the eigenstate. Adopting the iteration relationship of Hermite polynomials, theoretical analysis shows that all neighbouring nodes of the free oscillator eigenstates, except the coordinate origin, do not coincide with each other. Employing the series expression of Hermite polynomial, we numerically calculate all the node values for the

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quantum states with quantum number $n \leq 20$, and confirm a greater difference between the neighbouring nodes, except the origin. So the above-mentioned necessary condition is also sufficient, namely the allowable state of the particle is determined uniquely by the laser site, and all the other quantum states are forbidden. Therefore, we can impose the laser pulse on any node of the required state $\psi_n^{(0)}$ such that the particle transits from an arbitrary possible state to the target state. Then we can achieve the transition from $\psi_n^{(0)}$ to a new state by changing the laser position. These transitions form a kind of non-resonant transition from the multiphoton processes.

2. Matching conditions of the system

We load a single particle into a 1D quantum wire and subject it to a uniform magnetic field for the harmonic trapping. Then we make an artificial impurity formed δ potential which is strong enough by applying a sharply focused laser beam to an appropriate position.[22,23] The Hamiltonian governing the quantum dynamics of the system reads

$$H = -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{1}{2}m\nu^2 x^2 + V_0\delta(x - x_0), \qquad (1)$$

where m is the mass of particle, V_0 and x_0 denote the laser intensity and location respectively. For a charged particle[28] $\nu = \hbar eB/(2mc)$ is the Larmor cyclotron frequency with B being the magnetic field intensity, e and c the electronic charge and the speed of light in vacuum respectively. For a neutral particle ν depends on the magnetic field parameter and internal hyperfine structure of the particle.[15,16] In natural units, the Schrödinger equation corresponding to Eq.(1) is

$$-\frac{1}{2}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + \frac{1}{2}x^2\psi + V_0\delta(x - x_0)\psi = E\psi. \qquad (2)$$

Here, eigenenergy E and laser strength V_0 have been normalized in units of $\hbar\nu$, and coordinate x is in units of the oscillator length $l_\mathrm{h} = \sqrt{\hbar/(m\nu)}$. The dimensionless potential function in Eq.(2) is illustrated in Fig.1.

Clearly, Eq.(2) is exactly solvable. Applying the integral-equation method[29,30] to it produces the equivalent integral equation

$$\begin{aligned}\psi_n \;=\; & A_n\psi_n^{(0)} + B_n\tilde{\psi}_n^{(0)} \\ & +2V_0\tilde{\psi}_n^{(0)}\int^x \psi_n^{(0)}\delta(x - x_0)\psi_n\mathrm{d}x \\ & -2V_0\psi_n^{(0)}\int^x \tilde{\psi}_n^{(0)}\delta(x - x_0)\psi_n\mathrm{d}x, \qquad (3)\end{aligned}$$

where A_n and B_n are constants determined by the infinite boundary conditions and the matching conditions at laser site x_0. Function $\psi_n^{(0)}$ is one of the normalized eigenstates of a free harmonic oscillator with energy $E = E_n^{(0)} = n + 1/2$, and $\tilde{\psi}_n^{(0)} = \psi_n^{(0)}\int(\psi_n^{(0)})^{-2}$ represents an unbounded solution of the same energy.[30]

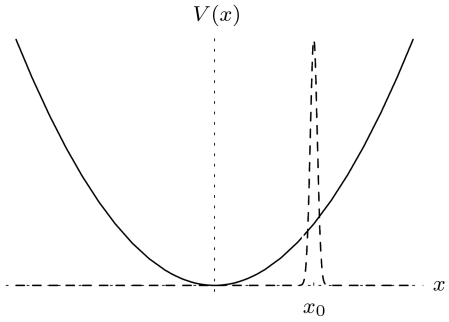


Fig.1. The dimensionless potential functions in an arbitrary range of parameters. The solid curve is corresponding to the harmonic oscillator potential and the dashed curve to the δ potential. The potential strength and coordinate are in units of $\hbar\nu$ and the oscillator length respectively.

This integral equation can be directly proved by taking the second derivative on both sides of Eq.(3). According to the integral formula of the δ function, from Eq.(3) we easily obtain the exact solution of Eq.(2) as

$$\begin{aligned}\psi_n(x \leq x_0) \;=\; & A_{nl}\psi_n^{(0)}(x) + B_{nl}\tilde{\psi}_n^{(0)}, \\ \psi_n(x \geq x_0) \;=\; & [A_{nr} - 2V_0\tilde{\psi}_n^{(0)}(x_0)\psi_n(x_0)]\psi_n^{(0)}(x) \\ & +[B_{nr} + 2V_0\psi_n^{(0)}(x_0)\psi_n(x_0)]\tilde{\psi}_n^{(0)}(x).\end{aligned} \qquad (4)$$

Here, A_{nl} and B_{nl} denote the superposition constants for the left region of the δ potential, and A_{nr} and B_{nr} the superposition constants for the right region. Because $\tilde{\psi}_n^{(0)}$ is an unbounded solution which tends to infinity at the positive and negative infinite boundaries, while the wavefunction in Eq.(4) should be zero at the boundaries, we have

$$B_{nl} = 0, \quad \text{and} \quad B_{nr} + 2V_0\psi_n^{(0)}(x_0)\psi_n(x_0) = 0. \qquad (5)$$

Making use of Eqs.(4) and (5), the continuity of ψ_n at x_0 results in

$$A_{nl} = A_{nr} - 2V_0\tilde{\psi}_n^{(0)}(x_0)\psi_n(x_0). \qquad (6)$$

Inserting Eqs.(5) and (6) into Eq.(4) and noticing the normalization of $\psi_n^{(0)}$, we obtain the solution

$$\psi_n(x) = A_{nl}\psi_n^{(0)}(x) = N_n H_n(x)\mathrm{e}^{-x^2/2}, \qquad (7)$$

which is valid for all the spatial region. Here $N_n = [1/(\sqrt{\pi}2^n n!)]^{1/2}$ is the normalized constant, and $H_n(x)$ the Hermite polynomials. Integrating Eq.(2) over a small spatial region from $x_0 - \varepsilon$ to $x_0 + \varepsilon$, we obtain the jumping condition of the first derivative at laser site x_0 as[28]

$$\psi_{nx}(x_0 + \varepsilon) - \psi_{nx}(x_0 - \varepsilon) = 2V_0\psi_n(x_0), \quad 0 < \varepsilon \ll 1. \tag{8}$$

This can be proved by taking the first derivative from Eq.(4) and inserting them into Eq.(8), where the Wronskian relation[29] $\tilde{\psi}^{(0)}_{nx}(x_0)\psi^{(0)}_n(x_0) - \tilde{\psi}^{(0)}_n(x_0)\psi^{(0)}_{nx}(x_0) = 1$ will be used. The substitution of Eq.(7) into Eq.(8) gives the final matching condition

$$\psi_n(x_0) = \psi^{(0)}_n(x_0) = 0, \tag{9}$$

as the first derivative $\psi^{(0)}_{nx}(x)$ is continuous for any x. Combining Eq.(7) with Eq.(9), we conclude that the wavefunction satisfying the infinite boundary conditions (5) and the matching conditions (6) and (8) must be one of the eigenstates of a free oscillator and vanishes at x_0 for the δ potential being strong enough.[25] In other words, one of the nodes for the admissible state must coincide with the laser position. Thus only when a laser pulse is imposed on a node of an eigenstate, the system can occupy the eigenstate.

3. Laser-manipulated the multiphoton transitions

It is well known that the nodes of the wavefunctions in Eq.(7) are the zero points of the Hermite polynomials, and $H_n(x)$ has n zero points which are symmetric on the origin. So the zero points of a highly excited state are very dense. Now we prove, with the exception of the origin of coordinate, all the nodes of the neighbouring wavefunctions are different from each other. Using the iteration relation between the neighbouring Hermite polynomials[28] $H_{n+1}(x) - 2xH_n(x) + 2nH_{n-1}(x) = 0$, we know that the zero point of $H_1(x)$ is not the zero point of $H_2(x)$, as $H_0(x) = 1$ is a nonzero constant. Similarly, $H_3(x)$ is not equal to zero at the zero points of $H_2(x)$. Consequently, for any $n \in [0, \infty)$ $H_n(x)$ and $H_{n+1}(x)$ do not have the same zero points, except the origin. The above theoretical analysis shows that except the origin, all nodes of neighbouring eigenstates do not coincide mutually.

Applying the series expression of Hermite polynomial, from $H_n(x_z) = 0$ we numerically calculate zero points x_z of the eigenstates with quantum number $n \leq 20$. The results are listed in Table 1, where x_z is in units of harmonic oscillator length. Table 1 shows that all of the zero points of lower energy states are different from each other, and there is a greater distance between any node and its neighbouring node. So, when we inject a laser pulse modelled by a δ potential at a position being a node of a lower energy state, the transition probability to this state is much greater than to other lower energy states. Because the nodes of highly excited states have a large density, any node of a lower energy state may be very close to some nodes of the highly excited states. When the laser pulse is injected at a node of a lower energy state, it can affect not only the transition to this lower energy state but also the transition to these highly excited states. However, as the quantum particle tends to occupy its low-energy state, the resulting transition occurs with a greater probability to this low-energy state. Particularly, after precooling the system to the lower energy states of $n \leq 20$, one can more precisely control the particle to a required state. Because the Hermite polynomial H_0 has no zero point and the unique zero point of $H_1(x)$ is the origin which coincides with the zero points of some high-energy states, the lowest-energy state which can be fixed by the above transitions is the second excited state.

From Table 1 we know the corresponding laser position $x_0 = x_z = 0.707106078$ for $n = 2$ as shown in Fig.2(a). In Fig.2(b), we display that when the laser site is set as $x_0 = x_z = 1.2247449$, the final low-energy state of the system is uniquely determined as the third excited state with $n = 3$. Differing from the common transition mechanism, we refer these transitions to a kind of non-resonance transition from the multiphoton process. In a quantum transition, if the energy of initial state is greater than that of the final state, the system will release energy whose value equals the energy difference between the initial and final states. When the initial-state energy is less than the end-state one, the system will absorb the difference energy from the laser field. When the laser pulse is imposed on a node of the initial state, the system will has no level transition, thus no emission or absorption of energy. Particularly, the multiphoton transition mechanism enables us to control the quantum transition from any unknown initial state to a required final state by using the laser pulse.

Table 1. The nodes (x_z/l_h) of the eigenstates with quantum number $n \leq 20$.

n	1	2	3	4	5
	0	0.70710678	0	0.52464762	0
$\lvert x_z\rvert$			1.2247449	1.6506801	0.95857246
					2.0201829
n	6	7	8	9	10
	0.43607741	0	0.38118699	0	0.34290133
	1.3358491	0.81628788	1.1571937	0.72355102	1.0366108
$\lvert x_z\rvert$	2.3506050	1.6735516	1.9816568	1.4685533	1.7566836
		2.6519614	2.9306374	2.2665806	2.5327317
				3.1909932	3.4361591
n	11	12	13	14	15
	0	0.31424038	0	0.29174551	0
	0.65680957	0.94778839	0.60576388	0.87871379	0.56506958
	1.3265571	1.5976826	1.2200550	1.4766827	1.1361156
$\lvert x_z\rvert$	2.0259480	2.2795071	1.8531077	2.0951833	1.7199926
	2.7832901	3.0206370	2.5197357	2.7484707	2.3257325
	3.6684708	3.8897249	3.2466090	3.4626569	2.9671669
			4.1013376	4.3044486	3.6699504
					4.4999907
n	16	17	18	19	20
	0.27348105	0	0.25826775	0	0.24534071
	0.82295145	0.53163300	0.77668292	0.50352016	0.73747373
	1.3802585	1.0676487	1.3009209	1.0103684	1.2340762
	1.9517880	1.6129243	1.8355316	1.5241706	1.7385377
$\lvert x_z\rvert$	2.5462022	2.1735028	2.3862991	2.0492317	2.2549740
	3.1769992	2.7577629	2.9613775	2.5911338	2.7888061
	3.8694479	3.3789321	3.5737691	3.1578488	3.3478546
	4.6887389	4.0619467	4.2481179	3.7621874	3.9447640
		4.8713452	5.0483640	4.4285328	4.6036824
				5.2202717	5.3874809

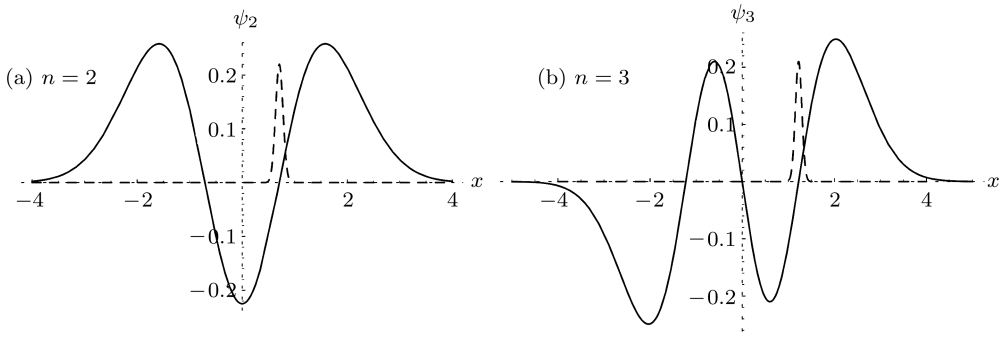


Fig.2. The two low-energy states identified by the laser δ pulse: (a) eigenstate of $n = 2$ determined by the laser position $x_0 = 0.70710678$; (b) eigenstate of $n = 3$ associated with the laser position $x_0 = 1.2247449$. The dashed curves indicate the δ potential. Here the coordinate is in units of oscillator length l_h, and the wavefunctions is in units of $\sqrt{l_\mathrm{h}^{-1}}$.

4. Conclusions and discussion

We have investigated how to use a laser δ pulse with a sufficiently large strength to manipulate the quantum states of a single particle trapped by a magnetic field in a 1D quantum wire, and established a necessary and sufficient condition for determining the quantum states by setting the laser position. The results show that if the stationary eigenstate $\psi_n^{(0)}$ is an allowable quantum state of the system, the laser pulse must be injected on a node of the state. Our theoretical analysis and numerical calculation confirm that except the origin the nodes are mutually different for all the low-energy states with $n \leq 20$. So, for a δ pulse with a fixed site and sufficiently large strength only one low-energy state is admissible, and the other quantum states are forbidden approximately. Therefore, the above necessary condition is also sufficient, namely the quantum state of the system is determined uniquely by the site of the laser δ pulse. Thus, we can impose a laser pulse on a node of $\psi_n^{(0)}$ to make the transition from any possible but unknown state to the $\psi_n^{(0)}$ state. Then we can realize the transition from $\psi_n^{(0)}$ to a new quantum state by changing the laser position to the node of the new state. Such transitions are a kind of novel multiphoton processes, which depend only on the laser site and are independent of the laser strength and frequency.

It is well known that a harmonically trapped particle system can also be easily prepared in an ion trap.[16,31−34] Subjecting a single ion confined in a linear trap to a laser field modelled by a δ pulse, the considered system (1) can be formed as an interaction system of the trapped-ion and laser. All the above results can be smoothly extended to the new system thereby.

Although realizing a laser δ pulse with a zero width is impossible, such a δ potential can be approximately obtained by a sharply focused laser beam.[22,23] The equivalent system can also be realized approximately by selecting an impurity in a quantum wire to form a δ potential with a fixed site and by applying a magnetic trap to a variable central position.[35] For such a system one can control the central site of the magnetic trap by changing the currents through the coils of the magnetic trap[35] that is completely equivalent to the adjustment of the laser site in the above system.

It is quite difficult but important to manipulate an unknown quantum state of a particle. We have demonstrated the feasibility of such manipulations for the considered system. Our multiphoton transition mechanism enables us to control the quantum transition from any unknown initial state to a required final state by using the laser pulse. This manipulation method may be useful for the trapped particles in practical applications.

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