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Neutrino energy loss by electron capture on strongly screened iron group nuclei*

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The influences on the neutrino energy loss rates in iron group nuclei at the same density are investigated in the presence of strong electron screening and in the absence of electron screening. The results show that at a temperature of 15×10^9 K, the neutrino energy loss rates which come from the electron capture process for most iron group nuclei decrease no more than 2 orders of magnitude but for the others (such as ^{53,55,56,57,58,59,60}Co, ^{56,59}Ni) they can decrease about 3 orders of magnitude due to strong electron screening (SES), whereas, at a temperature of $10^9 K$ the neutrino energy loss rates of the most iron group nuclei can be diminished greatly due to the SES. For example, ^{61}Fe, ^{60}Fe, and ^{62}Ni the neutrino energy loss rates decrease about 4, 15 and 16 orders of magnitude and for ^{57}Cr, ^{58}Cr, and ^{60}Cr decrease about 18, 12, and 10 orders of magnitude respectively. According to our calculations the neutrino energy loss rates of nuclei ^{58}Mn, ^{59}Mn, ^{60}Mn, and ^{62}Mn may decrease about 13 orders of magnitude at a temperature of 10^9 K due to the SES.

Keywords: neutrino energy loss, strong electron screening, electron capture
PACC: 1460G, 2340

1. Introduction

During the stellar evolution and nucleosynthesis process, the poignant supernovae explosions could lead up because of the unstable nuclear burning and iron nuclear collapse. When the temperature increases up to 3.5×10^9 K or more, the nuclear burning could turn to the stage of the silicon burning. At this time, the course of nucleosynthesis would push the element speedily to the iron group nuclei. Some researches have shown that the electron capture (EC) plays an important role in this process, and it is the resource and factory which can produce a large number of neutrinos. To our surprise, lots of messages are taken away from the star by the neutrinos, and the energy of the stellar core and the neutrino could lose much. Therefore the researches on the neutrino and the neutrino energy loss (NEL) rates have been a hotspot and frontier-border subject in the astrophysics and particle-physics. In the last decades, some author have investigated extensive results deduced from their calculations of the neutrino energy loss rates, such as Fuller, Fowler and Newman.[1−4] Based on a simple shell model, they did much pioneering work, in which they made the detailed calculation of the NEL rates originating from the EC on a lot of nuclei of the stars. Itoh *et al*[5,6] analysed the NEL rates from the pair, photo-, and plasma bremsstrahlung, and recombination neutrino processes based on the Weinberg–Salam theory. Esposito *et al*[7] reviewed the purely leptonic neutrino emission processes contributing to the energy loss rate of the stellar plasma. However all the authors above did not pay any attention to the NEL rates of a great many of iron group nuclei in strong electron screening (SES). Some researches have shown that it is extremely important and necessary to calculate accurately the screening corrections to the NEL rates from the EC process in dense stars by the relativistic degenerate electron liquid. The plasma can affect the chemical potential of the nuclei at higher densities and temperatures, and the Fermi energy of the electron gas may go up to 10 MeV. Hence it is very important and necessary to discuss in particular the changes of the threshold energy and the electron screening due to the EC. This issue has already been studied by some authors, such as Gutierrez *et al.*[8] Bravo and Garcia-Senz,[9] Luo and Peng.[10−13] In 2002, a new screening potential was calculated by using the linear response theory and discussed by Itoh *et al.*[14]

In this paper, based on the p–f shell model, we focus on the iron group nuclei and investigate the NEL

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rates by using a new strong screening potential derived from the linear response theory. The remainder of this paper is organized as follows: in Section 2, we analyse the NEL rates in the presence of SES in the stars. In Section 3 the influence of SES on NEL rates for most iron group nuclei is discussed and compared the NEL rates in the presence of SES with those in the absence of electron screening. And some conclusions are given in Section 4.

2. The NEL rate in the presence of SES

The NEL rate (in $m_{\rm e}c^2{\rm s}^{-1}$) subjected to the EC on the kth nuclei (Z,A) in thermal equilibrium at a temperature T is given by a sum over the initial parent state i and the final daughter state f[15]

$$\lambda_k^{\nu}=\sum_i\frac{(2J_i+1)\,{\rm e}^{-E_i/kT}}{G\,(Z,A,T)}\sum_f\lambda_{if}^{\nu}, \tag{1}$$

where J_i and E_i are the spin and excitation energy of the parent state, $G(Z,A,T)$ is the nuclear partition function. The NEL rate associated with the EC from one of the initial states to all possible final states is λ_{if}^{ν}, $\lambda_{if}^{\nu}=\dfrac{\ln 2}{(ft)_{if}}f_{if}$ with the relation $\dfrac{1}{(ft)_{if}}=\dfrac{1}{(ft)_{if}^{\rm F}}+\dfrac{1}{(ft)_{if}^{\rm GT}}$. The ft is related to the Gamow–Teller and Fermi transition matrix elements separately by the following relationships:[16]

$$\frac{1}{(ft)_{if}^{\rm GT}}=\frac{10^{3.596}}{|M_{\rm GT}|_{if}^2},\quad\text{and}\quad\frac{1}{(ft)_{if}^{\rm F}}=\frac{10^{3.79}}{|M_{\rm F}|_{if}^2}. \tag{2}$$

We can obtain the phase space factor in the absence of SES from Ref.[1], and it is defined as

$$f\,(Q_{if})=\int_{\varepsilon_0}^{\infty}\varepsilon p\,(Q_{if}+\varepsilon)^3\times\frac{F\,(Z,\varepsilon)}{1+\exp\left(\dfrac{\varepsilon-U_F-1}{kT}\right)}{\rm d}\varepsilon, \tag{3}$$

where k is the Boltzmann constant; T is the electron temperature; $U_{\rm F}$ is the electron chemical potential; $Q_{if}=Q_{00}+E_i-E_f$ is the EC threshold energy, with $Q_{00}=M_{\rm p}c^2-M_{\rm d}c^2$, $M_{\rm p}$ and $M_{\rm d}$ being the mass of the parent nucleus and the daughter nuclei respectively, E_i and E_f being the excitation energies of the nucleus ith state and fth state respectively; ε is the total rest mass and kinetic energy; Note that in this paper all of the energies and momenta are in units of $m_{\rm e}c^2$ and $m_{\rm e}c$, with $m_{\rm e}$ being the electron mass and c being the speed of light; $F\,(Z,\varepsilon)$ is the Coulomb wave correction which is the ratio of the square of the electron wavefunction distorted by the Coulomb scattering potential to the square of the wave function of free electron,[1,17] as shown below:

$$F(Z,\varepsilon)=2(1+\gamma)\left(\frac{2p_{\rm e}R}{\hbar}\right)^{-2(1-\gamma)}\times{\rm e}^{\pi\nu}\frac{|\Gamma(\gamma+{\rm i}\nu)|^2}{|\Gamma(2\gamma+1)|^2}, \tag{4}$$

where $\nu=Z\alpha(E_{\rm e}/pc)$; $\gamma=[1-(Z\alpha)^2]^{1/2}$; R is the nuclear radius, Γ is the gamma function; Z is the nuclear charge of the parent state; α is the fine-structure constant. In the absence of the electron screening, the threshold energy ε_0 is defined as

$$\varepsilon_0=\begin{cases}Q_{if},\ (Q_{if}<-1),\\ 1,\quad(\text{otherwise}).\end{cases} \tag{5}$$

The electron chemical potential is obtained by inverting the expression for the lepton number density, i.e.

$$n_{\rm e}=\rho/\mu_{\rm e}=\frac{1}{\pi^2N_{\rm A}\lambda_{\rm e}^3}\int_0^{\infty}(f_{-{\rm e}}-f_{+{\rm e}}){\rm d}p, \tag{6}$$

where ρ is the density in g/cm^3; $\mu_{\rm e}$ is the average molecular weight; $\lambda_{\rm e}=h/m_{\rm e}c$ is Compton wavelength; $f_{-{\rm e}}=\left[1+\exp\left(\dfrac{\varepsilon_n-U_{\rm F}-1}{kT}\right)\right]^{-1}$ and $f_{+e}=\left[1+\exp\left(\dfrac{\varepsilon_n+U_{\rm F}+1}{kT}\right)\right]^{-1}$ are the electron and positron distribution functions respectively. When the electron screening is taken into consideration, as is known, the election screening can change the Coulomb in-wavefunction of the electron, however, which can be neglected because the screening potential D is much smaller than the average energy of the electrons. And the electron screening decreases the energy of electron to $\varepsilon'=\varepsilon-D$ 'in the capture reaction'. At the same time, the electron screening relatively decreases the number of the electrons with energies higher than the threshold energy of EC. Thus, the threshold energy will increase from ε_0 to $\varepsilon_s=\varepsilon_0+D$. Then the phase space integral with the electron screening is written as

$$f^{\rm s}\,(Q_{if})=\int_{\varepsilon_{\rm s}}^{\infty}\varepsilon'\left(\varepsilon'^2-1\right)^{1/2}(Q_{if}+\varepsilon')^3\times F\,(z,\varepsilon')\,f_{\rm e}\,(\varepsilon)\,{\rm d}\varepsilon'. \tag{7}$$

Itoh *et al*[14] have calculated the electron screening potential which is caused by relativistic degenerate electrons by using the linear response theory, which is widely used in solid-state physics and liquid physics. They provide a more accurate screening potential as follows:

$$D = 7.525 \times 10^{-3} Z \left(\frac{Z\rho_6}{A}\right)^{1/3} J\,(r_{\rm s}, R)\,{\rm MeV}, \quad (8)$$

where ρ_6 is the density in unit of $10^6\,{\rm g/cm^3}$; $J\,(r_{\rm s}, R) = \sum_{i,j=0}^{10} a_{ij} s^i u^j$, with $s = \frac{1}{2}\,(\log_{10} r_{\rm s} + 3)$, $u = \frac{1}{25}\,(R - 25)$, $R = 2K_{\rm F} r_{\rm nuc} = 6.3 \times 10^{-3} Z^{-1/3} \rho_6^{1/3}$, $r_{\rm s} = 1.388 \times 10^{-2} \left(\frac{A}{Z\rho_6}\right)^{1/3}$, a_{ij} cited from Ref.[12], $r_{\rm nus}$ being the nuclear radius and $K_{\rm F} = 2.613 \times 10^{10} \left(\frac{A}{Z\rho_6}\right)^{1/3}$ being the electron Fermi wave number. Formula (8) is valid when $10^{-5} \leq r_{\rm s} \leq 10^{-1}$, and $0 \leq R \leq 50$, which are usually fulfilled in the presupernova environment. And in this paper, ρ_7 is the density in units of $10^7\,{\rm g/cm^3}$, T_9 is the temperature in units of $10^9\,{\rm K}$.

3. The NEL rates of iron group nuclei in the presence of SES

Figures 1 and 2 show the NEL rates of nuclei $^{52-61}$Fe and $^{55-60}$Co as a function of T_9 at a density of $\rho_7 = 10$ in the absence of electron screening. It can be seen from Fig.1 that the temperature only has a slight influence on the NEL rates of the nuclei $^{52-55}$Fe. However, the influence on the other nuclei is enhanced with the temperature T_9 increasing. According to our calculations, the NEL rates of some nuclei (such as ^{56,57,59}Fe) may increase almost about 10 orders of magnitude while the NEL rates of ^{58,60,61}Fe may be increased more and overrun about 20 orders of magnitude. Figure 2 shows that the NEL rates for ^{55,56,57,58,60}Co increase no more than 1 order magnitude. But for ^{59}Co, its NEL rate increases about 2 orders of magnitude. Figures 3 and 4 show the NEL rates of nuclei ^{56,58,59,60,61,62,63}Ni and $^{55-62}$Mn under the same conditions as those Figs.1 and 2. It can be seen from Figs.3 and 4 that the temperature has only a slight influence on the NEL rates of nuclei ^{56,58,59,61,63}Ni and ^{56,58}Mn, and the increments of their NEL rates are within about 1 order of magnitude. We can see from Fig.3 that the NEL rates of ^{60,62}Ni overrun about 6 orders of magnitude. From Fig.4 the NEL rates of ^{55,56}Mn increase about 10 orders of magnitude. But for ^{57,59,61,62}Mn, their NEL rates overrun about 20 orders of magnitude. The NEL rates of nucleus $^{53-60}$Cr and $^{47-50}$V as a function of T_9 under the same conditions as those in Figs.1–4 can be found from Figs.5 and 6. The two figures show that the temperature has only a minor effect on the NEL rates of ^{54}Cr. While the NEL rates of some nuclei can be influenced greatly by temperature T_9 such as ^{53,57}Cr, their NEL rates increase about 20 orders of magnitude and the NEL rates of nuclei ^{55,59}Cr can increase more than 20 orders of magnitude. Figure 6 shows that for ^{48}V, its NEL rate increases about 2 orders of magnitude. However for most nuclei (such as ^{47,49,50}V), their NEL rates may increase more than 2 orders of magnitude with the increase of T_9.

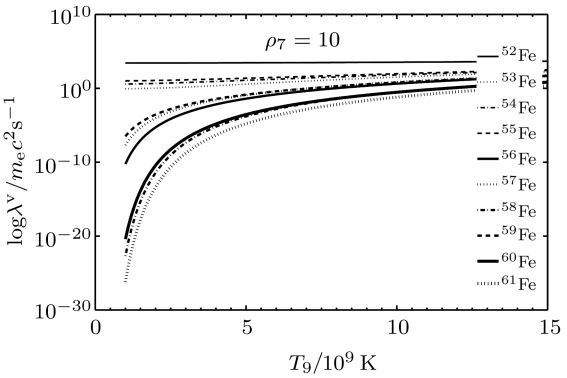


Fig.1. The NEL rats as a function of T_9 in the absence of electron screening at a density of $\rho_7 = 10$.

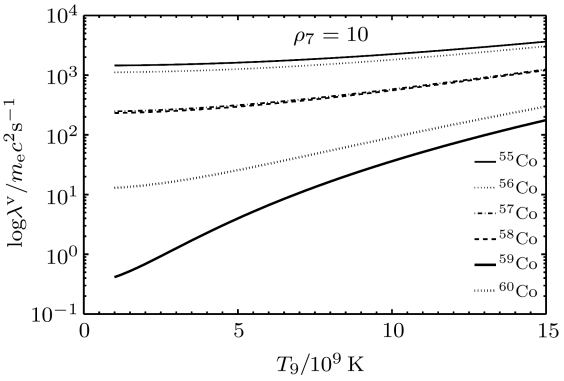


Fig.2. The NEL rats as a function of T_9 in the absence of electron screening at a density of $\rho_7 = 10$.

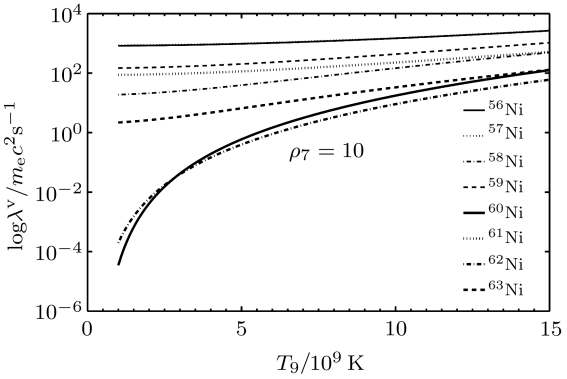


Fig.3. The NEL rats as a function of T_9 in the absence of electron screening at a density of $\rho_7 = 10$.

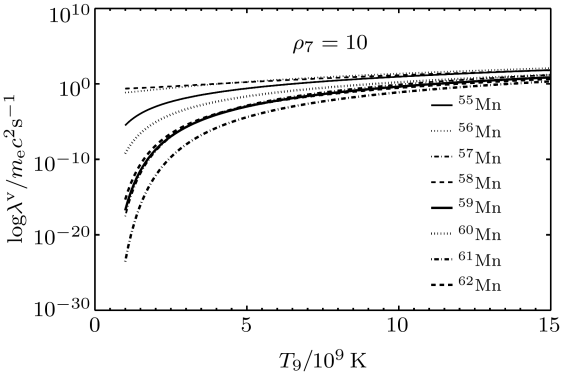


Fig.4. The NEL rats as a function of T_9 in the absence of electron screening at a density of $\rho_7 = 10$.

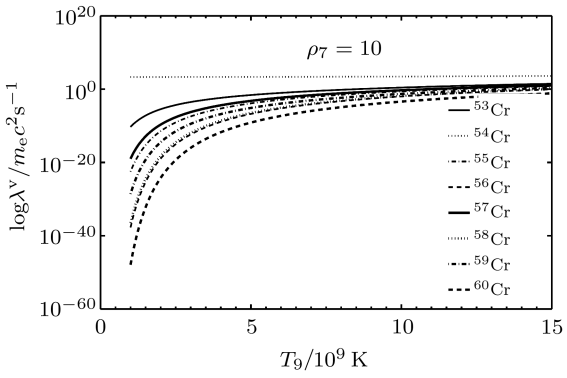


Fig.5. The NEL rats as a function of T_9 in the absence of electron screening at a density of $\rho_7 = 10$.

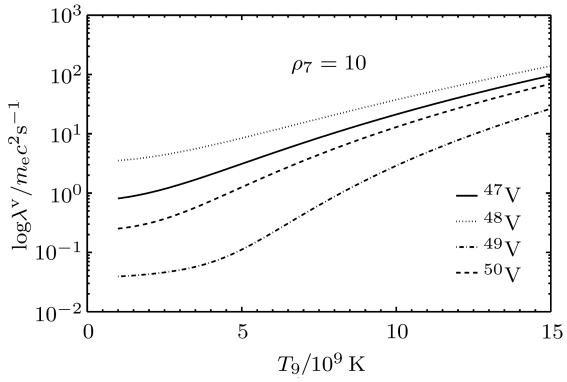


Fig.6. The NEL rats as a function of T_9 in the absence of electron screening at a density of $\rho_7 = 10$.

Figures 7 and 8 show the influence of the SES on the NEL rates for the nuclei $^{52-61}$Fe and $^{55-60}$Co as a function of T_9 at a density of $\rho_7 = 10$ in the presence of strong electron screening. It can be seen from Fig.7 that the temperature has only a slight influence on the NEL rates for nuclei of ^{52,53,54,55}Fe. Whereas the NEL rates of the most nuclei can be influenced greatly by T_9, such as for ^{56,57}Fe, and their NEL rates may increase about 10 orders of magnitude and for ^{58,59,60,61}Fe the NEL rates their NEL rates can overrun 20 orders of magnitude. The influence of temperature T_9 on the NEL rates for nuclei $^{55-60}$Co in the presence of SES can be seen from Fig.8. The NEL rates for nuclei ^{55,56,57,58,60}Co may increase about 2 orders of magnitude, but for ^{59}Co the NEL rate overruns 4 orders of magnitude. Figures 9 and 10 show the NEL rates of nuclei ^{56,58,59,60,61,62,63}Ni and $^{55-62}$Mn under the same condition as those in Figs.7 and 8. It can be seen from Figs.9 and 10 that the temperature has only a slight influence on the NEL rates of nuclei ^{56,57,58,59,61}Ni and ^{56}Mn. We can see from Fig.9 that the NEL rates of ^{62,63}Ni overrun about 20 orders of magnitude but for ^{60}Ni its NEL rate increases about 10 orders of magnitude. Figure 10 shows the increment of NEL rate of ^{55}Mn is within about 10 orders of magnitude and for the NEL rates of ^{57,58}Mn their increments are within about 20 orders of magnitude.

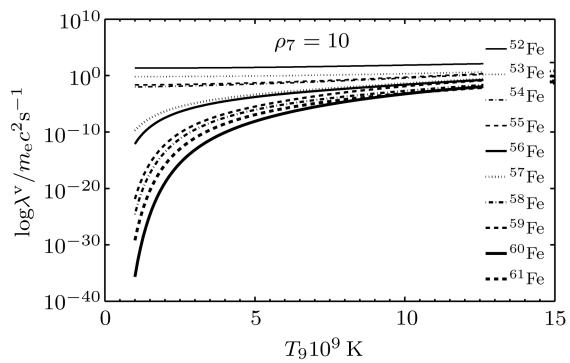


Fig.7. The NEL rats as a function of T_9 in the presence of strong electron screening at a density of $\rho_7 = 10$.

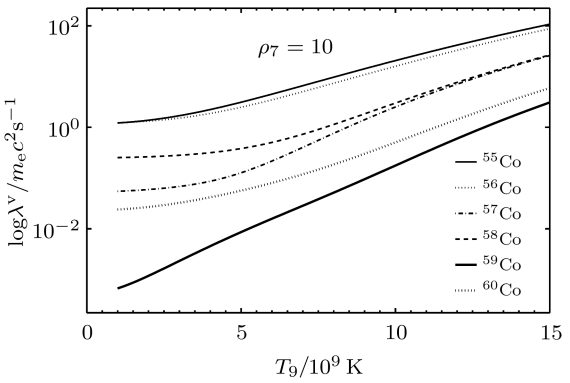


Fig.8. The NEL rats as a function of T_9 in the presence of strong electron screening at a density of $\rho_7 = 10$.

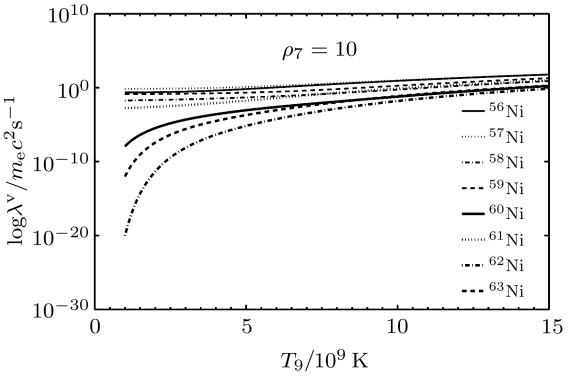


Fig.9. The NEL rats as function of T_9 in the presence of strong electron screening at a density of $\rho_7 = 10$.

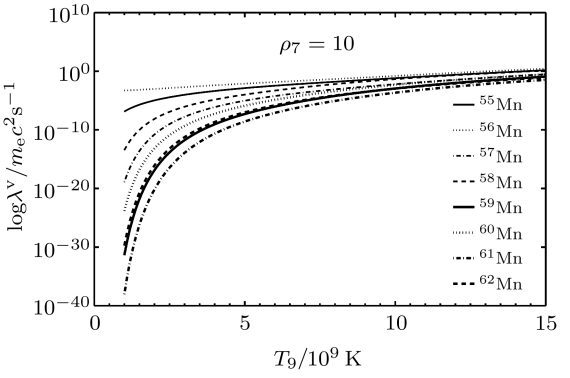


Fig.10. The NEL rats as a function of T_9 in the presence of strong electron screening at a density of $\rho_7 = 10$.

The NEL rates of the nuclei $^{53-60}$Cr and $^{47-50}$V as a function of T_9 under the same conditions as those in Fig.7–10 can be seen from Figs.11 and 12. Figure 11 shows that the temperature has only a mirror effect on the NEL rate for nucleus of ^{54}Cr. But the NEL rates of some nuclei can be influenced greatly by temperature T_9, for example, for nuclei ^{53,55}Cr, the increments of their NEL rates are within about 20 orders of magnitude. For $^{47-50}$V, the increments of their NEL rates, are within 2 orders of magnitude with temperature T_9 increasing, which is shown in Fig.12.

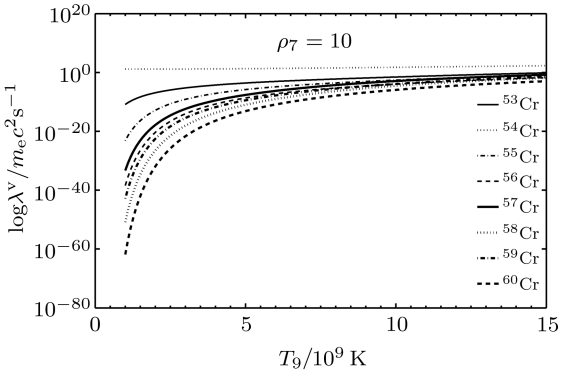


Fig.11. The NEL rats as function of T_9 in the presence of strong electron screening at a density of $\rho_7 = 10$.

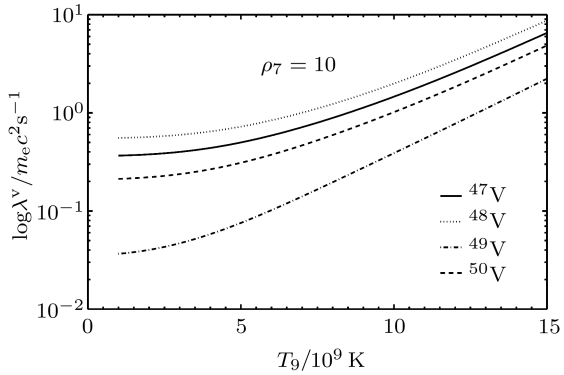


Fig.12. The NEL rats as a function of T_9 in the presence of strong electron screening at a density of $\rho_7 = 10$.

Comparing with the NEL rates from Figs.1 and 7, from Figs.2 and 8, from Figs.3 and 9, from Figs.4 and 10, from Figs.5 and 11, Figs.6 and 12, in the absence of electron screening and in the presence of SES, gives the influences of T_9 on NEL rates at a density of $\rho_7 = 10$, which are described as follows. Figs.1 and 7 show that the influence of the NEL rates for $^{52-61}$Fe decrease no more than 2 orders of magnitude at a temperature of $T_9 = 15$, but the NEL rates for ^{61}Fe and ^{60}Fe decrease about 4 and 15 orders of magnitude respectively at $T_9 = 1$ due to the SES. The changes in NEL rates of the nuclei $^{55-60}$Co can be seen from Figs.2 and 8. The two figures show that the NEL rates for the nuclei $^{55-58}$Co decrease about 1 order magnitude, however for the nuclei ^{59,60}Co decrease about 2 orders of magnitude at a temperature of $T_9 = 15$. The NEL rates for the nuclei ^{59}Co at $T_9 = 1$ decrease about 2 orders of magnitude and for others decrease more than 3 order magnitudes due to the SES. The

influences of temperature T_9 on the NEL rates of nuclides ^{56,58,59,60,61,62,63}Ni and $^{55-62}$Mn between Figs.3 and 9, and between Figs.4 and 10 are compared. The four figures shows that the influences of SES lead the NEL rates of nuclei $^{55-62}$Mn and ^{58,60,61,62,63}Ni to decrease about 2 orders of magnitude and those of nuclide ^{56,59}Ni to decrease about 3 orders of magnitude at a temperature of $T_9 = 15$. The influences of SES cause the NEL rates of nuclei ^{56,58,59,61,63}Ni and ^{55,56,57}Mn to decrease about 3 orders of magnitude at a temperature of $T_9 = 1$, whereas the SES influences lead the NEL rates of nuclide ^{58,59,60,62}Mn to decrease about 13 orders of magnitude and for nuclei ^{62}Ni and^{61}Mn their NEL rates decrease even more than 16 orders of magnitude due to the SES. Figures 5 and 11; Figs.6 and 12 show that the SES exerts an influence on the NEL rates of nuclei $^{53-60}$Cr and $^{47-50}$V. From the four figures, we can see that the NEL rates of nuclei $^{53-60}$Cr decrease no more than 3 orders of magnitude and those of nuclei $^{47-50}$V decrease no more than 1 order of magnitude at a temperature of $T_9 = 15$. Whereas the NEL rates of nuclei $^{53-56}$Cr decrease no more than 2 orders of magnitude at a temperature of $T_9 = 1$ and those of nuclei ^{57}Cr, ^{58}Cr, and ^{60}Cr decrease about 18, 12, and 10 order of magnitude respectively. From Figs.6 and 12, at a temperature of $T_9 = 1$ the NEL rates of nuclei ^{47}V, ^{48}V, ^{49}V, and ^{50}V decrease about 40%, 70%, 10%, and 10% respectively.

From Figs.1–12, we can see that at the same density of $\rho_7 = 10$, the temperature has different influences on NEL rates of the iron group nuclei in the presence of SES and in the absence of electron screening. At the same density, the lower the temperature is, the larger its influence on NEL is, because the electron energy is so low at the lower temperatures that the temperature can considerably affect the NEL rates. At the same time the number of the neutrinos which are produced in the electron capture process is so large that the NEL rats can be affected greatly. On the other hand, as is known, the electron screening can change the Coulomb in-wave function of the electrons which however can be neglected because the screening potential D is much less than the average energy of the electrons. And the electron screening decreases the energy of the electrons in the capture reaction to $\varepsilon' = \varepsilon - D$. Simultaneously, the electron screening relatively decreases the number of the electrons with energies higher than the threshold energy of electron capture. Thus, the threshold energy will increase. It maybe greatly affect NEL rates which comes from the EC process.

4. Conclusions

In this paper, the influences of temperature on NEL of iron group nuclei at the same density are investigated in the presence of SES and in absence of electron screening. We can draw the following conclusions that at a temperature of $T_9 = 15$, NEL rates which come from the EC process for most iron group nuclei decrease no more than 2 orders of magnitude and for others (such as ^{53,55,56,57,58,59,60}Co, ^{56,59}Ni) the NEL rates can decrease about 3 orders of magnitude due to the SES. Whereas, at a temperature of $T_9 = 1$ the NEL rates of the most iron group nuclei would be diminished greatly due to the SES, such as ^{61}Fe, ^{60}Fe, and ^{62}Ni, the NEL rates decrease about 4, 15, and 16 orders of magnitude respectively and for ^{57}Cr, ^{58}Cr, and ^{60}Cr their NEL rates decrease about 18, 12, and 10 orders of magnitude respectively. We can see from our calculations that the NEL rates of nuclei ^{58}Mn, ^{59}Mn, ^{60}Mn, and ^{62}Mn may decrease about 13 orders of magnitude at a temperature of $T_9 = 1$ due to the SES. In a word, the conclusions we obtained maybe have a significant influence on the further study of nuclear astrophysics and neutrino astrophysics, especially late stellar evolution and r-process ('r' for 'rapid') nucleosynthesis.

References

[1] Fuller G M, Fowler W A and Newman M J 1980 *AP. J.* **42** 447

[2] Fuller G M, Fowler W A and Newman M J 1982 *AP. J.* **252** 715

[3] Fuller G M, Fowler W A and Newman M J 1982 *AP. J.* **48** 279

[4] Fuller G M, Fowler W A and Newman M J 1985 *AP. J.* **293** 1

[5] Itoh N, Nishikawa A and Kohyama Y 1996 *AP. J.* **470** 1015

[6] Itoh N, Hayashi H, Nishikawa A and Kohyama Y 1996 *AP. J.* **102** 411
[7] Esposito S, Mangano G and Miele G 2003 *Nuclear Physics.* B **658** 217
[8] Gutierrez J, Garcia-Berro E, Iben I, Isern J, Labay J and Canal R 1996 *ApJ* **459** 701
[9] Bravo E and Garcia-Senz D 1999 *MNRAS* **307** 984
[10] Luo Z Q and Peng Q H 1996 *Science in China* **26** 665 (in Chinese)
[11] Luo Z Q and Peng Q H 1997 *ChAA* **1** 28
[12] Luo Z Q and Peng Q H 2001 *ChAA* **25** 1
[13] Luo Z Q and Peng Q H 2005 *Acta Astronomical Sinica* **46** 253
[14] Itoh N, Tomizawa N, Tamamura M, Yasuharu Kohyama and Satoshi Nozawa 2002 *ApJ* **579** 380
[15] Pruet J J and Fuller G M 2003 *AP. JS* **149** 189
[16] Luo Z Q, Liu M Q, Lin L B and Peng Q H 2006 *Chin. Phys.* **14** 1272
[17] Zhang J, Liu M Q and Luo Z Q 2006 *Chin. Phys.* **15** 1477