Vol 16 No 12, December 2007
1009-1963/2007/16(12)/3610-06
Chinese Physics
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Finite entropy of Schwarzschild anti-de Sitter black hole in different coordinates*

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(Received 18 January 2007; revised manuscript received 8 February 2007)

This paper studies the finite statistical-mechanical entropy of the Schwarzschild anti-de Sitter (AdS) spacetime arising from quantum massless scalar field by using the 'brick wall' approach in the Painlevé and Lemaitre coordinates. At first glance, it seems that the results would be different from that in the Schwarzschild-like coordinate since both the Painlevé and the Lemaitre spacetimes do not possess the event horizon obviously. However, this paper proves that the entropies in these coordinates are exactly equivalent to that in the Schwarzschild-like coordinate.

Keywords: black hole, entropy, Painlevé coordinate, Lemaitre coordinate
PACC: 0470, 0462, 9760

1. Introduction

In quantum field theory, we can use a timelike Killing vector to define particle states. Therefore, in static spacetimes we know that it is possible to define positive frequency modes by using the timelike Killing vector. However, in these spacetimes there could arise more than one timelike Killing vector which make the vacuum states inequivalent. This means that the concept of particles is not generally covariant in curved spacetime.

Bekenstein and Hawking[1,2] found that, by comparing black hole physics with thermodynamics and from the discovery of black hole evaporation, black hole entropy is proportional to the area of the event horizon. The discovery is one of the most profoundness in modern physics. However, the issue of the exact statistical origin of the black hole entropy has remained a challenging one. Recently, much effort has been concentrated on this problem.[3−34] The 'brick wall' model (BWM) proposed by 't Hooft[11] is an extensively used way to calculate the entropy in a variety of black holes, black branes, de Sitter spaces, and anti-de Sitter spaces.[11−34] In this model the Bekenstein–Hawking entropy of the black hole is identified with the statistical-mechanical entropy arising from a thermal bath of quantum fields propagating outside the event horizon.

As we mentioned above that in quantum field theory, the concept of particles is not generally covariant and depends on the coordinate chosen to describe the particular spacetime.[35,36] It is therefore interesting to study the following question: can we obtain the same statistical mechanical entropies of the black hole in the Painlevé and the Lemaitre coordinates by employing the BWM due to the fact that this question arises naturally after Shankaranarayanan *et al*[37,38] studied the Hawking temperature of the Schwarzschild black hole in the Painlevé and the Lemaitre coordinates by using the method of complex paths and they showed that the results are equal to those in the Schwarzschild-like coordinate. In both the Painlevé and the Lemaitre coordinates, the metrics have no coordinate singularity as those in the Schwarzschild-like coordinate. However, they both acquire singularity at the event horizon in the action function. Therefore, there could be particles production in these spacetimes and we can use the knowledge of the wave modes of the quantum field in these coordinate settings to calculate the statistical-mechanical entropies. The purpose of this paper is to investigate the question by using the well-known Schwarzschild AdS black hole carefully. We consider the AdS black hole, which had been investigated carefully in Refs.[28–30,32], for two reasons: (a) the AdS has a negative cosmological constant which regularizes the contribution to the entropy from in-

*Project supported by the National Natural Science Foundation of China (Grant No 10675045) and the Foundation for the Author of National Excellent Doctoral Dissertation of China (Grant No 200317) and the SRFDP (Grant No 20040542003).
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finity; (b) black holes in asymptotically AdS spacetime can be in stable equilibrium with a thermal heat bath of radiation at the Hawking temperature provided that the black hole is sufficiently large.

To compare the results obtained in this article with the entropy of the anti-de Sitter black hole in the Schwarzschild-like coordinate, we first introduce the entropy for the Schwarzschild-like coordinate. The Schwarzschild AdS spacetime in the Schwarzschild-like coordinate is described by

$$\mathrm{d}s^2 = f\mathrm{d}t_{\mathrm{s}}^2 - f^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2, \tag{1}$$

with

$$f(r) = 1 - \frac{2m}{r} - \frac{\lambda}{3}r^2,$$

where m is the mass of the black hole and λ is cosmological constant which we will assume it to be negative. We find that the function f can be rewritten as

$$f = -\frac{\lambda}{3r}(r - r_{\mathrm{H}})\left(r^2 + r_{\mathrm{H}}r - \frac{6m}{\lambda r_{\mathrm{H}}}\right),$$

where r_{H} is the coordinate singularity and the event horizon of the AdS black hole, which is

$$\begin{aligned} r_{\mathrm{H}} = & \sqrt[3]{-\frac{3}{\lambda}\left(m + \sqrt{m^2 - \frac{1}{9\lambda}}\right)} \\ & + \sqrt[3]{-\frac{3}{\lambda}\left(m - \sqrt{m^2 - \frac{1}{9\lambda}}\right)}. \end{aligned} \tag{2}$$

By using the BWM Elizabeth Winstanley[32] found that the finite entropy of a quantum massless scalar field on a Schwarzschild–AdS black hole in the Schwarzschild-like coordinate is

$$S = -\frac{\lambda r_{\mathrm{H}}^2}{1080}(4\xi - 1)^{-3/2}P, \tag{3}$$

with

$$\begin{aligned} P = & (2\xi + 1)(4\xi - 1)^{3/2}\log(\xi + 2) \\ & + (4\xi^4 + 32\xi^3 + 12\xi^2 + 8\xi - 2)\tan^{-1}\left(\frac{\sqrt{4\xi - 1}}{3}\right) \\ & + (5\xi^3 + 4\xi^2 + \xi - 1)\sqrt{4\xi - 1}, \end{aligned} \tag{4}$$

where $\xi = -6m/\lambda r_{\mathrm{H}}^3$.

This paper is organized as follows. In Section 2 the metrics of the Schwarzschild AdS black hole in the Painlevé and Lemaitre coordinates are introduced. In Section 3 the finite statistical-mechanical entropies of the Schwarzschild AdS black hole due to the quantum massless scalar field in the Painlevé and Lemaitre coordinates are investigated. Section 4 is devoted to a summary.

2. Metrics of Schwarzschild AdS spacetime in Painlevé and Lemaitre coordinates

We now introduce the metrics of the Schwarzschild AdS black hole in the Painlevé and Lemaitre coordinates.

2.1. Painlevé coordinate representation for Schwarzschild AdS black hole

The time coordinate transformation from the Schwarzschild-like coordinate Eq.(1) to the Painlevé coordinate[39] is

$$t = t_{\mathrm{s}} + \int \frac{\sqrt{1 - f(r)}}{f(r)}\mathrm{d}r. \tag{5}$$

The radial and angular coordinates remain unchanged. With this transformation, the line element Eq.(1) becomes

$$\begin{aligned} \mathrm{d}s^2 = & \, g_{00}\mathrm{d}t^2 + 2g_{01}\mathrm{d}t\mathrm{d}r + g_{11}\mathrm{d}r^2 \\ & + g_{22}\mathrm{d}\theta^2 + g_{33}\theta\mathrm{d}\phi^2, \end{aligned} \tag{6}$$

with

$$\begin{aligned} & g_{00} = 1 - \frac{2m}{r} - \frac{\lambda r^2}{3}, \quad g_{01} = -\sqrt{\frac{2m}{r} + \frac{\lambda r^2}{3}}, \\ & g_{11} = -1, \quad g_{22} = -r^2, \quad g_{33} = -r^2\sin^2\theta, \end{aligned} \tag{7}$$

which is the Painlevé coordinate representation. The inverse metric is

$$\begin{aligned} & g^{00} = 1, \quad g^{01} = -\sqrt{\frac{2m}{r} + \frac{\lambda r^2}{3}}, \\ & g^{11} = -\left(1 - \frac{2m}{r} - \frac{\lambda r^2}{3}\right), \quad g^{22} = -\frac{1}{r^2}, \\ & g^{33} = -\frac{1}{r^2\sin^2\theta}, \end{aligned} \tag{8}$$

The coordinate has distinguishing features: (a) the spacetime is stationary but not static; (b) the constant-time surfaces is flat; (c) there is now no singularity at $f(r) = 0$. That is to say, the coordinate complies with perspective of a free-falling observer, who is expected to experience nothing out of the ordinary upon passing through the event horizon. However, we will see next that the event horizon manifests itself as a singularity in the expression for the semi-classical action.

2.2. Lemaitre coordinate representation for Schwarzschild AdS black hole

The coordinates that transform from the Painlevé coordinate Eq.(6) to the Lemaitre coordinate $(V,\ U,\ \theta,\ \varphi)$ are given by

$$\tilde{r} = t + \int \frac{\mathrm{d}r}{\sqrt{1-f(r)}}, \quad U = \tilde{r} - t, \quad V = \tilde{r} + t, \tag{9}$$

where t is the Painlevé time and V is the Lemaitre time. The angular coordinates θ and φ remain the same. The line element Eq.(6) in the new coordinate becomes

$$\begin{aligned} \mathrm{d}s^2 \ =\ & \frac{g(U)-1}{4}(\mathrm{d}V^2 + \mathrm{d}U^2) + \frac{g(U)+1}{2}\mathrm{d}V\mathrm{d}U \\ & + y(U)(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2), \end{aligned} \tag{10}$$

with

$$g(U) = 1 - f(r), \qquad y(U) = r^2.$$

The metric in the Lemaitre coordinate has no coordinate singularity just as singularity in the Painlevé coordinates. However, the event horizon also manifests itself as a singularity in the expression for the semiclassical action.

3. Finite entropy of Schwarzschild AdS spacetime in different coordinates

In this section we shall study the finite statistical-mechanical entropy of the Schwarzschild AdS black hole that arises from the quantum scalar field in the Painlevé and Lemaitre coordinates by using the BWM.

3.1. Finite Statistical-mechanical entropy in Painlevé coordinate

The statistical-mechanical entropy can be derived from the free energy of a system, where free energy can be defined in terms of the particle spectrum. One of the ways to calculate the free energy is the BWM proposed by 't Hooft.[11] He argued that black hole entropy is identified with the statistical-mechanical entropy arising from a thermal bath of quantum fields propagating outside the horizon.

Using the Wentzel–Kramers–Brillouin (WKB) approximation with

$$\Phi = \exp(-\mathrm{i}Et + \mathrm{i}W(r))Y_{lm}(\theta,\phi), \tag{11}$$

where $Y_{lm}(\theta,\phi)$ is the usual spherical harmonic function, and substituting the metric Eq.(8) and wave function Φ into the massless Klein-Gordon equation

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial\nu\phi) = 0, \tag{12}$$

we find

$$p_r^{\pm} = \frac{1}{\left(1-\dfrac{2m}{r}-\dfrac{\lambda r^2}{3}\right)}\left[\sqrt{\frac{2m}{r}+\frac{\lambda r^2}{3}}E \pm \sqrt{(1-\frac{2m}{r}-\frac{\lambda r^2}{3})}\sqrt{\frac{E^2}{\left(1-\dfrac{2m}{r}-\dfrac{\lambda r^2}{3}\right)} - \frac{l(l+1)}{r^2}}\right], \tag{13}$$

where $p_r \equiv \partial_r W(r)$ is the momentum of the particles moving in the r direction. The sign ambiguity of the square root is related to the out-going and in-going particles, respectively. In Eq.(13) we utilize the average of the radial momentum (the minus before the p_r^- is caused by a different direction). In this way, the total number of modes is related to all kinds of particles. We have checked that this definition can also be used for all the previous corresponding works. Then the total number of modes with E is given by

$$n_r = \frac{1}{\pi}\int_{r_{\mathrm{H}}+h}^{L}\mathrm{d}r\frac{p_r^+ - p_r^-}{2} = \frac{1}{\pi}\int_{r_{\mathrm{H}}+h}^{L}\frac{\mathrm{d}r\sqrt{E^2 - \dfrac{l(l+1)}{r^2}\left(1-\dfrac{2m}{r}-\dfrac{\lambda r^2}{3}\right)}}{\left(1-\dfrac{2m}{r}-\dfrac{\lambda r^2}{3}\right)}, \tag{14}$$

where h is a small positive quantity and signifies an ultraviolet cutoff, and L is an infrared cutoff. The integral is taken only over those values for which the square root exists. Accordingly, the free energy F at inverse Hawking temperature β is given by the formula

$$\begin{aligned} F &= \frac{1}{\beta}\int(2l+1)\mathrm{d}l\int\ln(1-\exp(-\beta E))\mathrm{d}n_r \\ &= -\frac{1}{\pi}\int_0^{\frac{E^2r^2}{1-\frac{2m}{r}-\frac{\lambda r^2}{3}}}\mathrm{d}[l(l+1)]\int_0^{\infty}\frac{\mathrm{d}E}{\exp(\beta E)-1} \\ &\quad\times\int_{r_\mathrm{H}+h}^{L}\frac{\mathrm{d}r}{\left(1-\dfrac{2m}{r}-\dfrac{\lambda r^2}{3}\right)} \\ &\quad\times\sqrt{E^2-\frac{l(l+1)}{r^2}\left(1-\frac{2m}{r}-\frac{\lambda r^2}{3}\right)}, \end{aligned} \tag{15}$$

and we obtain that

$$F=-\frac{2\pi^3}{45\beta^4}I_r, \tag{16}$$

with

$$I_r=\int_{r_\mathrm{H}+h}^{L}\frac{r^2\mathrm{d}r}{\left(1-\dfrac{2m}{r}-\dfrac{\lambda r^2}{3}\right)^2}. \tag{17}$$

We find that I_r can also be expressed as

$$I_r=\frac{9}{\lambda^2}\int_{r_\mathrm{H}+h}^{L}\frac{r^4\mathrm{d}r}{(r-r_\mathrm{H})^2\left(r^2+r_\mathrm{H}r-\dfrac{6m}{\lambda r_\mathrm{H}}\right)^2}. \tag{18}$$

Let $r=r_\mathrm{H}x,\ h=\hat{h}r_\mathrm{H},\ L=\hat{L}r_\mathrm{H}$, where $\hat{h}$, $\hat{L}$ and x are some dimensionless positive parameters dependent on the geometry, Eq.(18) becomes

$$I_r=\frac{9}{\lambda^2 r_\mathrm{H}}\int_{1+\hat{h}}^{\hat{L}}\frac{x^4\mathrm{d}x}{(x-1)^2(x^2+x+\xi)^2}, \tag{19}$$

where $\xi=-6m/\lambda r_\mathrm{H}^3$. Let $\hat{L}\to\infty$ and $\hat{h}\to 0$ to give our final finite answer, which is

$$I_r=\frac{9}{\lambda^2 r_\mathrm{H}}(\xi+2)^{-3}(4\xi-1)^{-3/2}P, \tag{20}$$

with

$$\begin{aligned} P &= (2\xi+1)(4\xi-1)^{3/2}\log(\xi+2)+(4\xi^4+32\xi^3 \\ &\quad+12\xi^2+8\xi-2)\tan^{-1}\left(\frac{\sqrt{4\xi-1}}{3}\right) \\ &\quad+(5\xi^3+4\xi^2+\xi-1)\sqrt{4\xi-1}. \end{aligned} \tag{21}$$

The final result for the entropy can now be calculated from using the value of the inverse temperature $\beta=-\dfrac{12\pi}{\lambda r_\mathrm{H}}(\xi+2)^{-1}$ and inserting free energy into the relation $S=\beta^2\partial F/\partial\beta$, which is given by

$$S=-\frac{\lambda r_\mathrm{H}^2}{1080}(4\xi-1)^{-3/2}P. \tag{22}$$

We find that it is in agreement with the Winstanley result Eq.(3). That is to say, the finite entropy calculated in the Painlevé coordinate is exactly equal to that in the Schwarzschild-like coordinate.

By the equivalence principle and the standard quantum field theory in flat space, for constructing a vacuum state for the massless scalar field in the Painlevé spacetime we should leave all the positive frequency modes empty. Kraus[40] pointed out that for the metric (6) it is convenient to work along a curve $\mathrm{d}r+\sqrt{1-g(r)}\mathrm{d}t=0$, then the condition is simply a positive frequency with respect to t near this curve. It is easy to prove that the modes used to calculate the entropy are essentially the same as that in the Schwarzschild-like coordinate.

3.2. Finite Statistical-mechanical entropy in Lemaitre coordinate

We can also use the WKB approximation with

$$\Phi=\exp(-\mathrm{i}EV/2+\mathrm{i}W(U))Y_{lm}. \tag{23}$$

In order to compare the result of the Lemaitre coordinate with that of the Schwarzschild-like coordinate, we take $\exp[-\mathrm{i}EV/2]$ in the wave function Φ since

$$\begin{aligned} V &= \tilde{r}+t \\ &= 2t+\int\frac{\mathrm{d}r}{\sqrt{1-f(r)}} \\ &= 2t_\mathrm{s}+2\int\frac{\sqrt{1-f(r)}}{f(r)}\mathrm{d}r+\int\frac{\mathrm{d}r}{\sqrt{1-f(r)}} \\ &= 2t_\mathrm{s}-2\mathrm{i}F(r), \end{aligned} \tag{24}$$

and then $\exp[-\mathrm{i}EV/2]=\exp[-\mathrm{i}Et_\mathrm{s}+F(r)]$.

Then we substitute the metric Eq.(10) and wave function Φ Eq.(23) into Klein-Gordon equation Eq.(12), we have

$$p_U^{\pm}=\frac{g}{1-g}\left[\frac{1+g}{g}E\pm\sqrt{\frac{1-g}{g}}\sqrt{\frac{E^2}{1-g}-\frac{l(l+1)}{y}}\right], \tag{25}$$

where $p_U\equiv\partial_U W(U)$ is the momentum of the particle moving in direction U. The sign ambiguity of the

square root is related to the out-going (p_U^+) or in-going (p_U^-) particles. Therefore, the number of modes is

$$\begin{aligned} n_U &= \frac{1}{\pi}\int_{U_++\tilde{h}}^{\tilde{L}} \mathrm{d}U \frac{p_U^+ - p_U^-}{2} \\ &= \frac{1}{\pi}\int_{U_++\tilde{h}}^{\tilde{L}} \mathrm{d}U \sqrt{\frac{g}{1-g}}\sqrt{\frac{E^2}{1-g} - \frac{l(l+1)}{y}}, \end{aligned} \tag{26}$$

where we make use of the average of the U-direction momentum. The integral in the second line is taken only over those values for which the square root exists. Then the free energy is

$$\begin{aligned} F &= \frac{1}{\beta}\int (2l+1)\mathrm{d}l \int \ln(1-\exp(-\beta E))\mathrm{d}n_U \\ &= -\frac{1}{\pi}\int_0^{\frac{E^2 y}{1-g}} \mathrm{d}l(l+1)\int_0^{\infty}\frac{\mathrm{d}E}{\exp(\beta E)-1} \\ &\quad \times \int_{U_++\tilde{h}}^{\tilde{L}} \mathrm{d}U \sqrt{\frac{g}{1-g}}\sqrt{\frac{E^2}{1-g} - \frac{l(l+1)}{y}} \\ &= -\frac{2\pi^3}{45\beta^4}\int_{U_++\tilde{h}}^{\tilde{L}} \frac{y\sqrt{g}dU}{(1-g)^2}. \end{aligned} \tag{27}$$

Taking out the integration, and inserting free energy into the relation $S = \beta^2 \partial F/\partial\beta$, we obtain the finite statistical-mechanical entropy of the Schwarzschild AdS black hole in the Lemaitre coordinate

$$S = -\frac{\lambda r_{\mathrm{H}}^2}{1080}(4\xi - 1)^{-3/2}P. \tag{28}$$

Comparing with Eqs.(3) and (22), we find that it equals to the entropy calculated in the Painlevé and Schwarzschild-like coordinates.

It is interesting that although both the Painlevé and the Lemaitre spacetimes do not possess the event horizon obviously, the entropies calculated in them are the same with that calculated in the Schwarzschild-like coordinate. It is well known that the wave modes obtained by using semiclassical techniques, in general, are the exact modes of the quantum system in the asymptotic regions. Thus, if the asymptotic structure of the spacetime is the same for the two coordinates, then the semiclassical wave modes associated with these two coordinate systems will be the same. From Eq.(9) we know that the differential relationship between the Lemaitre time V and the Painlevé time t can be expressed as $\mathrm{d}V = \mathrm{d}t + \mathrm{d}\tilde{r} = 2\mathrm{d}t + \mathrm{d}r/\sqrt{1-g(r)}$. Now let us also work along the curve $\mathrm{d}r + \sqrt{1-g(r)}\mathrm{d}t = 0$, we obtain $\mathrm{d}V = \mathrm{d}t$. It is shown that the two definitions of positive frequency—with respect to V in the Lemaitre spacetime and with respect to t in the Painlevé spacetime—do coincide. Therefore, it should not be surprised that the entropies driven from the modes in the Lemaitre and Painlevé coordinates are the same.

4. Summary

We have investigated the finite statistical-mechanical entropies of the Schwarzschild AdS black hole arising from the quantum massless scalar field in the Painlevé and Lemaitre coordinates by using the 't Hooft BWM. At first glance, we might have anticipated that the results are different from that of the Schwarzschild-like coordinate due to two reasons: (a) both the Painlevé and Lemaitre spacetimes possess a distinguishing property: the metrics do not possess singularity; (b) it is not obvious that the time V in the Lemaitre spacetime tends to the time t in the Painlevé spacetime. Nevertheless, for either the Painlevé or Lemaitre coordinate, the event horizon manifests itself as a singularity in the action function and then there could be particles production. Hence we can use the knowledge of the wave modes of the quantum field to calculate the statistical-mechanical entropies. By comparing our results Eqs.(22) and (28), which are worked out exactly, with the well-known result Eq.(3) we find that, up to a subleading correction, the finite statistical-mechanical entropies of the Schwarzschild AdS black hole arising from the quantum scalar field in both the Painlevé and Lemaitre coordinates are equivalent to that in the standard Schwarzschild-like coordinate. When we construct a vacuum state for the massless scalar field in the Painlevé spacetime we take the condition $\mathrm{d}r + \sqrt{1-g(r)} = 0$, and then we find that the modes used to calculate the entropies in both the Painlevé and Lemaitre coordinates are essentially the same as that in the Schwarzschild-like coordinates since both V and t tend to the Schwarzschild time t_{s} as r goes to infinity under this condition. Therefore, it should not be a surprise that the entropies driven from the modes in the Lemaitre, Painlevé, and Schwarzschild-like coordinates are the same.

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