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Quantum search via superconducting quantum interference devices in a cavity*

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We propose a scheme for implementing the Grover search algorithm with two superconducting quantum interference devices (SQUIDs) in a cavity. Our scheme only requires single resonant interaction of the SQUID-cavity system and the required interaction time is very short. The simplicity of the process and the reduction of the interaction time are important for restraining decoherence.

Keywords: Grover search algorithm, superconducting quantum interference devices
PACC: 0365, 4250

1. Introduction

The Grover search algorithm[1] is an efficient quantum algorithm which can offer a quadratic speed-up to the classical search algorithm. Moreover, many techniques based on the search algorithm are widely used in our life, and it may have utility for a wide range of problems,[2] so the quantum search algorithm becomes more and more important. The Grover search algorithm solves the following problem:[1] Given a search space of size $N = 2^n$, for which no prior knowledge about the structure of the information in it was known, if we want to look for one certain item in this unsorted database, then how many times does it take to find the target in this search space? Classically, this problem requires approximately N operations. Even the most efficient algorithm still needs on average $\frac{N}{2}$ operations. Whereas, the Grover quantum search requires only $O(\sqrt{N})$ queries to accomplish the same task. Furthermore, it has been verified experimentally that the algorithm is efficient in the few-qubit case via nuclear magnetic resonance (NMR)[3,4] and linear optics[5,6] techniques.

Firstly, let us briefly review the essential procedure of Grover's search algorithm. The task is to find one item among the database described by a quantum register with n qubits which have $N = 2^n$ possible states. The database can be accessed by an "oracle". For a special instance of the search problem, it can expediently be represented by a function f, which takes the input, an integer x ranging from 0 to $N-1$, as a variable. By definition, $f(x) = 1$ when x is a solution to the search problem, and $f(x) = 0$ if x is not a solution to the search problem. In order to realize the oracle, we use n data qubits and one auxiliary working qubit. The oracle is a unitary operator denoted by O, the action of which can be written as $|x\rangle \xrightarrow{O} (-1)^{f(x)}|x\rangle$. The search process can be summarized as the following seven steps:[7]

1. Prepare the $n+1$ qubits in the state of $|0\rangle^{\otimes n}|1\rangle$.

2. Perform Hadamard transformations on each of the $n+1$ qubits.

3. Apply the oracle. The auxiliary working qubit can be thrown after this step.

4. Perform Hadamard transformations on each of the n data qubits.

5. Apply a phase shift to the data qubits except

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when they are in the state $|0\rangle^{\otimes n}$, which can be described by the unitary operator $2|0\rangle^{\otimes n}\langle 0| - I$, where I is the $N \times N$ identity matrix.

6. Perform Hadamard transformations on each of the n data qubits.

7. Repeat steps 3 to 6 with a finite number of times, and then measure the n data qubits. The number of repetitions [8] for obtaining an item is $R = CI \left[\dfrac{\arccos\sqrt{\dfrac{1}{N}}}{2\arccos\sqrt{\dfrac{N-1}{N}}} \right]$, above-bounded by $\pi\sqrt{\dfrac{N}{4}}$.

Recently, many physical systems have been suggested as possible realizations of qubits and quantum logic gates.[9] As we know, theoretically, the manipulation of a few qubits has made great progress.[10,11] From experimental aspect, it has already been demonstrated in, such as, cavity QED,[12] trapped ions,[13] NMR[14] and photonic systems.[15] Despite recently exciting experimental progress, the physical realization of a scalable quantum computer remains a great challenge. The building blocks of quantum computers are two-qubit logic gates. Few systems have demonstrated controlled qubits and qubit coupling between pairs taken from more than four qubits. It is difficult to couple different subsystems in a controlled manner, while at the same time shielding the system from the influence of its environment. One particularly attractive possibility is to use matter qubits to serve as hardware as they are static and potentially long lived, and an optical coupling mechanism can create suitable entanglement.

2. SQUID

Among a variety of systems explored for the hardware implementations of quantum computers, cavity quantum electrodynamics system is favoured because of its demonstrated advantages when subjected to coherent manipulations.[12] However, in the cavity–atom system, the strong coupling limit is difficult to meet and individual addressing of the qubits is also a problem-maker. In contrast, the strong coupling limit was realized with superconducting charge[16] and flux qubits[17] in a microcavity. SQUIDs,[18] as well as other solid state circuits, can be perfectly fixed, easily embedded in a cavity and easy to scale up, thus the cavity-SQUID scheme may be more preferable than cavity–atom system. In addition, by placing SQUIDs into a superconducting cavity, the external environment induced decoherence can be greatly suppressed because the cavity can serve as magnetic shield for SQUIDs.

The SQUID considered in our scheme is the radio frequency SQUID (rf-SQUID), which consists of a Josephson tunnel junction in a superconducting loop, the size of which is on the order of 10–100 μm. The rf-SQUID considered here has a three-level Λ-type energy structure, which includes two lower ground states $|0\rangle$, $|1\rangle$ and an upper excited state $|e\rangle$. The two lower states $|0\rangle$ and $|1\rangle$ reside in two distinct potential valleys and serve as logic 0 and 1 in our proposal. The level structure is shown in Fig.1.

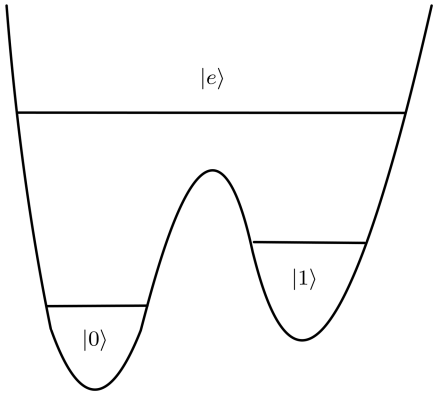


Fig.1. Level structure of the rf-SQUID used in this paper. It has two lower flux states $|0\rangle$, $|1\rangle$ and an upper excited state $|e\rangle$.

Suppose the coupling of $|0\rangle$, $|1\rangle$ and $|e\rangle$ with other levels of the rf-SQUID via the cavity is negligible, which can be readily satisfied by adjusting the level spacings of the rf-SQUID. For a rf-SQUID, the level spacing can be easily changed by varying the external flux ϕ_x or the critical current I_c.[19] Hence, coupling between microwave pulse and any particular rf-SQUID qubit can be obtained selectively via frequency matching.

3. Implementing scheme

Here, we present our scheme for implementing the two-qubit Grover search algorithm with two SQUIDs in a cavity. Firstly, we prepare the two SQUIDs (qubits 1 and 2) in a cavity in the state

$$|\varphi\rangle_{12} = |\varphi\rangle_1|\varphi\rangle_2. \tag{1}$$

Then we perform Hadamard transformations on qubits 1 and 2

$$|0\rangle_i = \frac{1}{\sqrt{2}}(|0\rangle_i + |1\rangle_i), \quad (i = 1, 2) \tag{2}$$

which can be achieved by an additional classical laser pulse with appropriately chosen phase and amplitude. Then the state in Eq.(1) becomes

$$|\varphi\rangle_i = \frac{1}{2}(|0\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2). \quad (3)$$

Obviously, we have obtained the four items ($|0\rangle_1|0\rangle_2$, $|0\rangle_1|1\rangle_2$, $|1\rangle_1|0\rangle_2$ and $|1\rangle_1|1\rangle_2$) stored in the data qubits before applying the Oracle operation. Without loss of generality, we will search the state $|1\rangle_1|0\rangle_2$ from the four states as an example.

Let qubit 2 interact with a classical fields tuned to the transition $|1\rangle \leftrightarrow |e\rangle$. After this, the state of qubits 1 and 2 becomes

$$|\varphi\rangle_{12} = \frac{1}{2}(|0\rangle_1|0\rangle_2 + |0\rangle_1|e\rangle_2 + |1\rangle_1|0\rangle_2 + |1\rangle_1|e\rangle_2). \quad (4)$$

Switch on the cavity-SQUIDs interaction, and suppose the cavity mode is resonant with the $|0\rangle \leftrightarrow |1\rangle$ transition but far-off resonant with the transitions of $|0\rangle \leftrightarrow |e\rangle$ and $|1\rangle \leftrightarrow |e\rangle$ of the two rf-SQUIDs, the interaction Hamiltonian can be expressed as (in the interaction picture)[20]

$$\begin{aligned} H \;=\; & \Omega_1(a^+|0\rangle_1\langle 1| + a|1\rangle_1\langle 0|) \\ & + \Omega_2(a^+|0\rangle_2\langle 1| + a|1\rangle_2\langle 0|), \end{aligned} \quad (5)$$

where Ω_1 and Ω_2 are the coupling constants of the SQUIDs 1 and 2 with the cavity, respectively; a^+ and a are the creation and annihilation operators for the cavity mode. It is easy to verify the following evolutions:

$$|0\rangle_1|0\rangle_2|0\rangle \rightarrow |0\rangle_1|0\rangle_2|0\rangle, \quad (6a)$$

$$|0\rangle_1|e\rangle_2|0\rangle \rightarrow |0\rangle_1|e\rangle_2|0\rangle, \quad (6b)$$

$$\begin{aligned} |1\rangle_1|0\rangle_2|0\rangle \rightarrow & \frac{\Omega_1}{\Omega}\Bigg[\frac{1}{\Omega}(\Omega_1\left(\cos(\Omega t) + \frac{\Omega_2^2}{\Omega_1}\right)|1\rangle_1|0\rangle_2|0\rangle \\ & + \frac{\Omega_2}{\Omega}(\cos(\Omega t) - 1)|0\rangle_1|1\rangle_2|0\rangle \\ & - \mathrm{i}\sin(\Omega t)|0\rangle_1|0\rangle_2|1\rangle\Bigg], \end{aligned} \quad (6c)$$

$$\begin{aligned} |1\rangle_1|e\rangle_2|0\rangle \rightarrow & [\cos(\Omega_1 t)|1\rangle_1|0\rangle \\ & - \mathrm{i}\sin(\Omega_1 t)|0\rangle_1|1\rangle]|e\rangle_2, \end{aligned} \quad (6d)$$

where $\Omega = \sqrt{\Omega_1^2 + \Omega_2^2}$. If we choose $\Omega_1 t = \pi$ and $\Omega_2 = \sqrt{3}\Omega_1$, which can be achieved by choosing coupling strengths and interaction time appropriately, we obtain

$$|0\rangle_1|0\rangle_2 \rightarrow |0\rangle_1|0\rangle_2, \quad |0\rangle_1|e\rangle_2 \rightarrow |0\rangle_1|e\rangle_2, \quad (7a)$$

$$|1\rangle_1|0\rangle_2 \rightarrow |1\rangle_1|0\rangle_2, \quad |1\rangle_1|e\rangle_2 \rightarrow -|1\rangle_1|e\rangle_2, \quad (7b)$$

where we have omitted the cavity state, which was left in vacuum state.

Let qubit 2 interact with a classical field again to obtain the transition $|e\rangle_2 \leftrightarrow |1\rangle_2$. The state $|\varphi\rangle_{12}$ becomes

$$|\varphi\rangle_{12} = \frac{1}{2}(|00\rangle_{12} + |01\rangle_{12} + |10\rangle_{12} - |11\rangle_{12}). \quad (8)$$

The above transformations correspond to 2-qubit controlled phase gate (QCPG) operation.

Hereto, we can search the target state from the four states via SQUID.

Step 1 Let qubit 2 successively interact with two classical pulses tuned to the transitions $|0\rangle_2 \leftrightarrow |e\rangle_2$ and $|1\rangle_2 \leftrightarrow |0\rangle_2$, respectively. Choosing the amplitude and phase of the classical field appropriately, one obtains the transitions $|0\rangle_2 \leftrightarrow |e\rangle_2$ and $|1\rangle_2 \leftrightarrow |0\rangle_2$, respectively. Then the state becomes

$$|\varphi\rangle_{12} = \frac{1}{2}(|00\rangle_{12} + |0e\rangle_{12} + |1e\rangle_{12} - |10\rangle_{12}). \quad (9)$$

Step 2 Perform operations on the two qubits as in Eq.(7), and then we will have

$$|\varphi\rangle_{12} = \frac{1}{2}(|00\rangle_{12} + |01\rangle_{12} - |11\rangle_{12} - |10\rangle_{12}). \quad (10)$$

Step 3 Performing Hadamard gate operations on qubits 1 and 2, we obtain the state of the two qubits

$$|\varphi\rangle_{12} = |1\rangle_1|0\rangle_2. \quad (11)$$

Finally, we measure the state of qubits 1 and 2 by detectors. Obviously, the state of qubits 1 and 2 is the result that we want to search.

4. Discussion and conclusion

Finally, we will briefly discuss the experimental feasibility of our scheme. The cavity-SQUID system, as well as other solid state circuits, is favoured because it is easy to be embedded in electronic circuits and scaled up to large registers,[21,22] and the control and measurement techniques are quite advanced.[23] The interaction time can be perfectly controlled by external control. The time constants involved are long enough to realize all the required manipulations.[17] Finally, the coupling between microwave pulses and any particular rf-SQUID qubit can be obtained selectively via frequency matching. Thus our scheme is realizable within current experimental technology.

For the sake of definiteness, let us estimate the experimental feasibility of realizing the logic gates using SQUIDs with the parameters already available in present experiment.[18,22,24,25] Suppose the quality factor of the superconducting cavity is $Q = 1 \times 10^6$, the cavity mode frequency is $\omega_c = 50\,\mathrm{GHz}$, and the cavity decay time is $\kappa^{-1} = \dfrac{Q}{\omega_c} = 20\,\mu\mathrm{s}$. The realistic length of the cavity is $l = 1.4\,\mathrm{mm}$. When each of the two SQUIDs is located at one of the antinodes of the cavity, the distance of them, D, is equal to the length of the cavity. For SQUIDs of size $d = 40\,\mu\mathrm{m}$, we have $D/d = 35$ which satisfies perfectly the requirement of $D \gg d$. The upper state energy relaxation constant is $\gamma_e^{-1} = 2.5\,\mu\mathrm{s}$. For a superconducting standing-wave cavity and a SQUID located at one of the antinodes of the magnetic field, the coupling constant is $g = 1.8 \times 10^8\,\mathrm{Hz}$; thus the resonant interaction time for our scheme is $T_\gamma = \dfrac{\pi}{g} \approx 1.7 \times 10^{-8}\,\mathrm{s}$. Meanwhile, the strong coupling condition can be perfectly achieved as $\dfrac{g^2}{\gamma_e \kappa} = 1.6 \times 10^6 \gg 1$. The Rabi frequency for the interaction of the rf pulse and the SQUID is $\Omega = 8.5 \times 10^7\,\mathrm{Hz}$, thus the interaction time for the single photon detection is $T_l = \dfrac{\pi}{2\Omega} \simeq 1.8 \times 10^{-8}\,\mathrm{s}$. Both interaction times T_γ and T_l are much shorter than the cavity decay time and the relaxation time of the upper state. In addition, the above estimation is very conservative when compared to present experimental achievements. In fact, a superconducting cavity with $Q = 3 \times 10^8$ has already been reported.[26]

In conclusion, we have realized the Grover search algorithm with two SQUIDs in a cavity. The scheme implements the operation of controlled phase gate via the cavity-SQUIDs interaction and executes single-bit operation by a rf pulse on the rf-SQUID. Our scheme is achieved without any type of measurement, does not use the cavity mode as the data bus and only requires a single resonant interaction of the SQUID-cavity system. Thus the presented scheme is very simple and the required interaction time is very short. The simplicity of the procedure and the reduction of the operation time are important for suppressing decoherence.

References

[1] Grover L K 1997 *Phys. Rev. Lett.* **79** 4709
Grover L K 1998 *Phys. Rev. Lett.* **80** 4329

[2] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)

[3] Chuang I L, Gershenfeld N and Kubinec M 1998 *Phys. Rev. Lett.* **80** 3408

[4] Jones J A, Mosca M and Hansen R H 1998 *Nature* **393** 344

[5] Bhattacharya N, van Linden van den Heuvell H B and Spreeuw R J C 2002 *Phys. Rev. Lett.* **88** 137901

[6] Walther P, Resch K J, Rudolph T, Schenck E, Weinfurter H, Vedral V, Aspelmeyer M and Zeilinger A 2005 *Nature* **434** 169

[7] Cao Z L and Dong P 2007 *J. Phys.* A **40** 348

[8] Boyer M, Brassard G, yer P Ho and Tapp A 1998 *Fortschr. Phys.* **46** 493

[9] Braunstein Eds S and Lo H K 2000 *Fortschr. Phys.* **48** Nos. 9-11

[10] Zheng S B 2005 *Chin. Phys.* **14** 2222

[11] Kuang L M and Wu Q Q 2006 *Chin. Phys.* **15** 2593

[12] Raimond J M, Brune M and Haroche S 2001 *Rev. Mod. Phys.* **73** 565

[13] García-Ripoll J J, Zoller P and Cirac J I 2003 *Phys. Rev. Lett.* **91** 157901

[14] Negrevergne C, Mahesh T S, Ryan C A, Ditty M, Cyr-Racine F, Power W, Boulant N, Havel T, Cory D G and Laflamme R 2006 *Phys. Rev. Lett.* **96** 170501

[15] Knill E, Laflamme R and Milburn G J 2001*Nature* **409** 46

[16] Wallraff A, Schuster D I, Blais A, Frunzio L, Huang R S, Majer J, Kumar S, Girvin S M and Schoelkopf R J 2004 *Nature* **431** 162

[17] Yang C P, Chu S and Han S 2004 *Phys. Rev. Lett.* **92** 117902

[18] Xue Z Y, Zhang G, Dong P, Yi Y M and Cao Z L 2006 *Eur. Phys. J.* B **52** 333

[19] Rouse R, Han S and Lukens J E 1995 *Phys. Rev. Lett.* **75** 1614
Friedman J R, Patel V, Chen W, Tolpygo S K and Lukens J E 2000 *Nature* **406** 43

[20] Yang C P, Chu S and Han S 2003 *Phys. Rev.* A **67** 042311

[21] Vion D, Aassime A, Cottet A, Joyez P, Pothier H, Urbin C a, Esteve D and Devoret M 2002 *Science* **296** 886

[22] Yu Y, Han S, Chu X, Chu S and Wang Z 2002 *Science* **296** 889

[23] Makhlin Y, Sch G and Shnirman A 2001 *Rev. Mod. Phys.* **73** 357

[24] Chiorescu I, Nakamura Y, Harmans C J P M and Mooij J E 2003 *Science* **299** 1869

[25] Yu Y, Nakada D, Lee J C, Singh B, Crankshaw D S, Orlando T P, Berggren K K and Oliver W D 2004 *Phys. Rev. Lett.* **92** 117904

[26] Osnaghi S, Bertet P, Auffeves A, Maioli P, Brune M, Raimond J M and Haroche S 2001 *Phys. Rev. Lett.* **87** 037902