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Evolutionary snowdrift game with disordered environments in mobile societies*

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We investigate an evolutionary snowdrift game on a square $N = L \times L$ lattice with periodic boundary conditions, where a population of n_0 ($n_0 \leq N$) players located on the sites of this lattice can either cooperate with or defect from their nearest neighbours. After each generation, every player moves with a certain probability p to one of the player's nearest empty sites. It is shown that, when $p = 0$, the cooperative behaviour can be enhanced in disordered structures. When $p > 0$, the effect of mobility on cooperation remarkably depends on the payoff parameter r and the density of individuals ρ ($\rho = n_0/N$). Compared with the results of $p = 0$, for small r, the persistence of cooperation is enhanced at not too small values of ρ; whereas for large r, the introduction of mobility inhibits the emergence of cooperation at any $\rho < 1$; for the intermediate value of r, the cooperative behaviour is sometimes enhanced and sometimes inhibited, depending on the values of p and ρ. In particular, the cooperator density can reach its maximum when the values of p and ρ reach their respective optimal values. In addition, two absorbing states of all cooperators and all defectors can emerge respectively for small and large r in the case of $p > 0$.

Keywords: snowdrift game, disorderd lattice, mobility, cooperation
PACC: 0250, 0175, 0270

1. Introduction

Understanding the emergence and persistence of cooperation is one of the central problems in evolutionary biology and socioeconomics.[1,2] Evolutionary game theory[2,3] has become a powerful means to investigate this problem. Especially two simple games, the prisoner's dilemma (PD)[4] and its variation, the snowdrift game (SG),[5] have attracted most attention. In the spatial extensions of PD, it is widely accepted that spatial structure benefits the evolution of cooperation.[6−10]

The SG (also known as the hawk-dove or chicken game) can be illustrated with a situation in which two drivers are caught in a blizzard and trapped on either side of a snowdrift. They can either get out and start shovelling (cooperate) or stay in the car (defect). If both cooperate, they have the benefit b of getting home while sharing the labour c. Thus each driver obtains the net benefit $R = b - c/2$. If both defect, they do not get anywhere and obtain zero benefit $P = 0$. If only one of them cooperates, both can get home, but the defector avoids the labour cost and obtains benefit $T = b$, whereas the cooperator's benefit is reduced by the workload, i.e. $S = b - c$. In the SG, $b > c > 0$, leading to $T > R > S > P$. Therefore, the best action depends on the co-player: to defect if the other cooperates, but to cooperate if the other defects. This leads to stable coexistence of cooperators and defectors in well-mixed populations. Without loss of generality, it is useful to assign $R = 1$ so that the payoffs can be characterized by a single parameter r, where $r = c/(2b - c)$ is the cost-to-benefit ratio of mutual cooperation. In terms of $0 < r < 1$, we have $T = 1 + r$, $R = 1$, $S = 1 - r$, and $P = 0$. According to the replicator dynamics,[11] the equilibrium frequency of cooperators in the snowdrift game is $1 - r$.

In recent years, many of networks of interacting entities or agents such as humans, computers, animal species, proteins, and neurons have been investigated vigorously.[12−14] And previous works on SG have focused on the effects of connectivity in structures such as lattices[15] and fully connected networks,[11,15] and other network structures.[16−18] However, unexpectedly, spatial structure reduces the proportion of cooperators for a wide range of parameters in the context of SG.[15] Recently, Vainstein *et al* investigated the PD game in site diluted lattices[19] and found that

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the fraction of cooperators in the population is enhanced due to disorder. And then, this group considered the mobility of individuals in PD game for the payoff parameter $b = 1.4$[20] and showed that the cooperative behaviour can be enhanced for a certain mobile probability p as compared with the 'never-move' case. In fact, in the real world, not all individuals interact the same number of times, either due to the nonsynchronous character of the interaction or to the environmental structure that prevents some of the contacts. In addition, real systems are far from being static: individuals are really dynamic and continuously come into contact, and lose contact, with other individuals. For example, the farmworkers in Sinkiang do not usually occupy well-defined territorial regions, they always move far from their present places of work. Hence, the interactions between them are rather instantaneous and time dependent. However, the factors of the disordered environments and the mobility of individuals have not been considered in SG up to now.

In the present work, we explore the SG with disordered environments in mobile societies. We find that, in the case of 'never-move', depending on the amount of disorder, cooperation can be enhanced. And we also find that the introduction of mobility has a significant effect on the cooperation. For small r, the persistence of cooperation is enhanced due to the introduction of individuals' mobility; whereas for large r, the individuals' mobility inhibits the emergence of cooperation; for the intermediate value of r, the cooperative behaviour is sometimes enhanced and sometimes inhibited, depending on the values of p and ρ. In particular, the fraction of cooperators can reach its maximum when p and ρ reach their optimal values. In addition, the system can exist in two absorbing states of all cooperators and all defectors respectively for small and large r in the case of $p > 0$.

2. The model

We consider the spatial version of SG,[15] placing the interacting individuals on the vertices of an $N = 100 \times 100$ square lattice with periodic boundary conditions. Initially, a population of n_0 ($n_0 \leq N$) players labelled by $1, 2, ..., n_0$ are randomly distributed on the sites of the lattice. Each player follows one of the two simple strategies: cooperation (C) or defection (D), represented by the variable $S_i = \pm 1$ respectively. During the process of interactions, each player interacts at most with its four nearest neighbours (self-interactions are excluded). The players collect their payoffs depending on the payoff-matrix parameters, and the total payoff of a certain player is the sum over all its interactions. A player's payoff is a measure of its reproductive success. In every generation, the players update their strategies according to the deterministic rule:[6,19−21] i.e., each individual will adopt the strategy of the player with the highest payoff among its nearest neighbours and itself in the next round. After each generation, every player moves with a certain probability p ($p \in [0, 1]$) to one of the nearest empty sites (if it is possible) and there cannot be more than one player on the same site.

To characterize the macroscopic behaviour of the system, we introduce an order parameter. Let $\rho_C(t)$ represent the fraction of cooperators at a given time,

$$\rho_C(t) = \frac{1}{2n_0} \sum_{i=1}^{n_0} (S_i + 1),$$

where n_0 is the total number of individuals. Since we are interested in the long time regime, and the results depend on the choice of the frozen empty sites, we define the order parameter as the average over time ($\langle ... \rangle$) and over the realizations of the disorder $\overline{(...)}$ of the asymptotic cooperator density $\rho_C = \overline{\langle \rho_C(\infty) \rangle}$. $\rho_C = 0$ means that the population is full of defectors, and $\rho_C = 1$ denotes the population is occupied by cooperators. For an intermediate case $0 < \rho_C < 1$, both strategists coexist.

3. Simulation results and analysis

Using Monte Carlo simulations, we study the key quantity of cooperator density ρ_C for different values of the individual density ρ and the moving probability p. Initially, the two strategies of C and D are randomly distributed among the individuals with equal probability 1/2. Eventually, the system reaches a dynamic equilibrium state where the fraction of cooperators ρ_C fluctuates stably around a certain value. The simulation results were obtained by averaging over the last 2000 Monte Carlo (MC) time steps of the total 20000. Each data point results from an average over 20 simulations for the same type of network structures.

In Fig.1 the cooperator density ρ_C is plotted against the total individual density ρ on the lattice for several different values of r in the case of $p = 0$. If the total density of individuals ρ is near zero, almost

all players are isolated and do not change strategy since there is no combat cluster at all; the cooperator density is the same as the initial one: $\rho_C \simeq \rho_C(0)$.

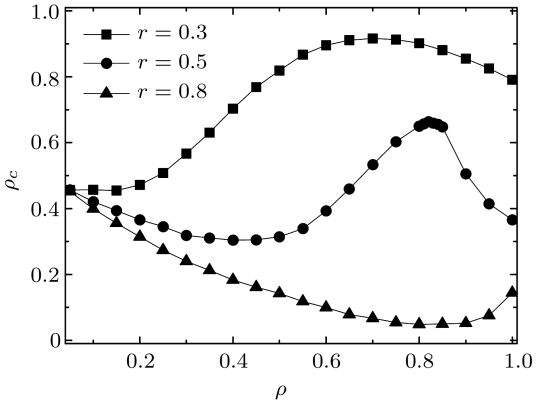


Fig.1. Monte Carlo results for the cooperator density ρ_C in the steady state versus the lattice occupation ρ for several values of r. The mobile probability of individuals $p = 0$.

Indeed, for small ρ, the curves for different values of r merge. When $\rho > 0.05$, the cooperative behaviour will be determined by the dynamics, and the curves for different values of r will depart from each other: the higher the r is, the more favourable the defector is; and vice versa. In fact, in the region of $0 < r < 0.34$, the curves first rise with ρ and then they drop slowly after the maximal values of ρ_C occur at large ρ but $\rho < 1$ (see the curve for $r = 0.3$ in Fig.1). For $0.65 < r < 1$, the change of curves with ρ shows the reverse trend as that of $0 < r < 0.34$, but the maximal values of ρ_C occur in the limit $\rho \to 0$ (see the curve for $r = 0.8$ in Fig.1). For the intermediate values of r ($0.34 < r \leq 0.65$), the curves first drop, and then they rise with ρ when the values of ρ_C reach their minimum, and then they will drop with ρ after the largest values of ρ_C appear (see the curve for $r = 0.5$ in Fig.1). From the above results, we can see that, for any value of the payoff parameter r, the maximum of the cooperator density ρ_C occurs at the optimal value of the individual density ρ ($\rho < 1$). Therefore, we can conclude that, due to disorder, the cooperative behaviour is enhanced in the context of SG. It is pointed out that the main difference between PD and SG is that: In the context of PD, for small payoff parameter b, the cooperator density ρ_C increases monotonically with the total density of individuals ρ,[19] i.e., the cooperation is not enhanced in disordered structures in the space of small payoff parameters; while as for our results, in the context of SG, the fraction of cooperators ρ_C has a maximal value at certain value of the individual density ρ ($\rho < 1$). Furthermore, we would point out that the cooperative behaviour, being enhanced due to the disorder in a wide range of parameters, is a general result for both PD and SG. Interestingly, the disordered structures promote the emergence and persistence of cooperation for a wide range of the payoff parameter r as compared with the result $1 - r$ in well-mixed situation in the context of SG.

In the following, we will consider the case of mobility. Figure 2 shows three plots of the cooperator density ρ_C as a function of ρ for different values of the mobile probability p and the payoff parameter r. We

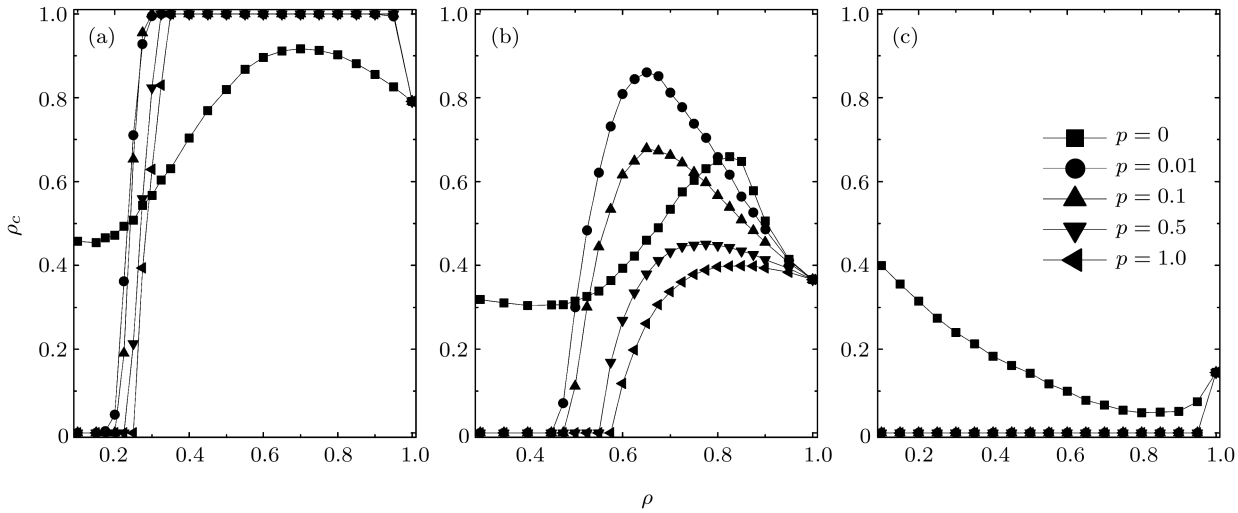


Fig.2. Monte Carlo result for cooperator density ρ_C as a function of the lattice occupation ρ for different values of mobility p and payoff parameter r: (a) $r = 0.3$, (b) $r = 0.5$ and (c) $r = 0.8$.

can see from Fig.2(a) that, when $\rho \leq 0.225$, the emergence of cooperation is inhibited due to the introduction of mobility. In the region of $0.225 \leq \rho \leq 0.3$, for each curve of $p > 0$, there exists a threshold of ρ above which the persistence of cooperation is enhanced as compared with the result of $p = 0$. The thresholds of ρ are about $0.25, 0.25, 0.275, 0.3$ for $p = 0.01, 0.1, 0.5, 1.0$ respectively. And for large ρ, there appears a homogeneous state of all cooperators for any $p > 0$. There are two reasons for this: small r is in favour of the persistence of cooperation; due to the mobility of individuals, the chance of interactions between the players increases remarkably, which enhances the persistence of cooperation. For large r, due to the introduction of the individuals' mobility, the emergence of cooperation is remarkably inhibited, and the homogeneous state of all defectors appears for any $p > 0$ (see Fig.2(c)). The reason is that, large r benefits the defectors, and when $p > 0$, the chance of interactions among individuals obviously increases, which inhibits the emergence of cooperation. From Figs.2(a) and 2(c), we can conclude that in a wide range of ρ, when the payoff parameter r benefits the cooperators, the introduction of mobility enhances the persistence of cooperation; whereas, when the payoff parameter benefits the defectors, the introduction of mobility inhibits the emergence of cooperation. These results were not discussed in the work of Vainstein.[20]

In the intermediate region (see Fig.2(b)), compared with the results of $p = 0$, the cooperative behaviour is sometimes enhanced and sometimes inhibited, depending on the magnitude of p and ρ. The maximal value of ρ_C occurs at the optimal value of the mobile probability p for a certain value of ρ. It is pointed out that the optimal values of ρ are not equal to one, which indicates that the emergence and persistence of cooperation is enhanced due to disorder in the case of mobility. Next, we will show that the cooperator density changes with the mobile probability p for several values of ρ in this parameter region (see Fig.3).

In Fig.3, the curves of cooperator density ρ_C against mobile probability p for several values of ρ when $r = 0.5$ are plotted. From Fig.3, we can see that, for small ρ ($\rho = 0.45$), any amount of mobility could inhibit the emergence of cooperation. In the intermediate density of individuals (see the curves for $\rho = 0.55$, 0.65, 0.75), when the mobile probability p is small, the fraction of cooperators can reach its maximal value. When the total density of individuals ρ is large (not included here), the introduction of mobility can inhibit the persistence of cooperation, but the values of ρ_C are not zero (see Fig.2(b)). We can conclude that the effect of the individuals' mobility on the cooperative behaviour depends remarkably on the value of the individual density ρ and the mobile probability p in this payoff parameter space.

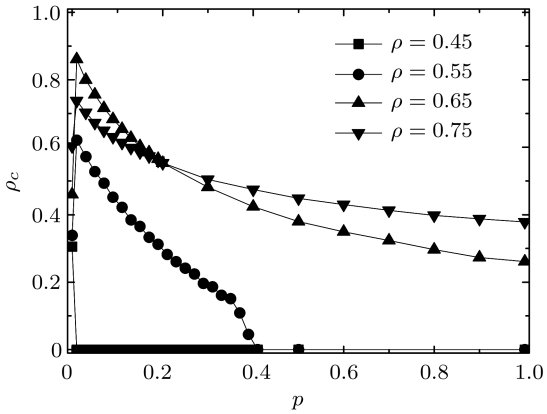


Fig.3. The cooperator density ρ_C as a function of the mobile probability p for several densities $\rho = 0.45, 0.55, 0.65, 0.75$ when $r = 0.5$.

4. Conclusions

In summary, we have studied the evolution of cooperation in the context of SG with mobile agents on site diluted square lattices. It was shown that, in the case of $p = 0$, the fraction of cooperators in the population can be enhanced due to disorder. The introduction of individuals' mobility has a significant effect on the cooperative behaviour: in a wide range of parameter ρ, the introduction of mobility enhances the persistence of cooperation when the payoff parameter r benefits the cooperators (small r); whereas the emergence of cooperation is inhibited due to the introduction of mobility when r benefits the defectors (large r); for the intermediate value of r, the cooperative behaviour is sometimes enhanced and sometimes inhibited, depending on the values of p and ρ. In particular, the fraction of cooperators can reach its maximum when the values of p and ρ reach their optimal values. In addition, the system appears to have two homogeneous states of all cooperators and all defectors respectively for small and large r in a wide range of ρ in the case of introduction of mobility. To some extent, the introduction of the mobility could be of particular interest in the evolutionary game dynamics and may shed some lights on the evolution of cooperation in the society.

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