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Cluster-size dependent randomization traffic flow model*

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In order to exhibit the meta-stable states, several slow-to-start rules have been investigated as modification to Nagel–Schreckenberg (NS) model. These models can reproduce some realistic phenomena which are absent in the original NS model. But in these models, the size of cluster is still not considered as a useful parameter. In real traffic, the slow-to-start motion of a standing vehicle often depends on the degree of congestion which can be measured by the clusters' size. According to this idea, we propose a cluster-size dependent slow-to-start model based on the speed-dependent slow-to-start rule (VDR) model. It gives expected results through simulations. Comparing with the VDR model, our new model has a better traffic efficiency and shows richer complex characters.

Keywords: traffic flow, slow-to-start, jam, phase-transition, hysteresis effect
PACC: 6470,0520

1. Introduction

Recently, traffic dynamics has attracted much attention of researchers in diverse fields.[1−12] Among the novel models, the cellular automata (CA) based models are preferred, because they have a simple structure and adapt the variables of the discrete time and the discrete space. Because of these advantages, it can possibly simulate the dynamical processes of a huge number of interacting vehicles. In 1992, Nagel and Schreckenberg [13] suggested a probabilistic CA model to simulate a single-lane highway traffic, called NS model later. It is the simplest CA model describing the traffic flow correctly. In the NS model, due to the variables of discrete time and space, the cars update their positions and velocities according to the following rules:

1) Acceleration:

$$v_j(t) \to v_j\left(t+\frac{1}{3}\right) = \min(v_j(t)+1, v_{\max});$$

2) Braking:

$$v_j\left(t+\frac{1}{3}\right) \to v_j\left(t+\frac{2}{3}\right) = \min\left(v_j\left(t+\frac{1}{3}\right), d_j(t)\right);$$

3) Randomization:

$$v_j\left(t+\frac{2}{3}\right) \to v_j(t+1) = \max\left(0, v_j\left(t+\frac{2}{3}\right)-1\right)$$

with probability p;

4) Driving: Car j moves $v_j(t+1)$ cells ahead.

The $v_{\max}$ above is the maximal speed of a car, and $d_j(t)$ is the number of empty cells in front of the car j. In their simulations the road is divided into cells of length 7.5 m - the normal space headway of two successive cars in real traffic. Each cell can either be empty or occupied by just one vehicle at a certain time. Meanwhile, the time is divided into time steps of one second. Thus, a speed of n means $n \times 7.5$ m/s.

This minimal model, with just one stochastic parameter p, can reproduce many macroscopic effects observed in real traffic. Each feature of the rules is

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essential for the model and cannot be ignored. Simplicity is one of the model's excellent features. But in such simple rules, some phenomena in real traffic, for example, the occurrence of metastable states,[14] cannot be reproduced. In order to make up this oversight, many new models have been established. Most of these models are derived from the NS model; some new updating rules have been introduced. With these models, hysteresis and other phenomena absent in the NS model can be shown and simulated.

Experimental observations indicate that a reduction of the flow from a jam, compared with the maximum possible flow, is an important ingredient for the occurrence of metastable states with long lifetime.[15] Such a reduction can be produced by the so-called slow-to-start (s2s) rules,[16] in which standing cars accelerate with a low probability compared with moving cars. The s2s rules can conduce to not only metastability and, consequently, hysteresis effect, but also phase separated states at high densities. Considering this kind of s2s rules, researchers proposed some modified versions of the NS model to capture the hysteresis effect and the metastable states in real traffic. To recapitulate, the existing s2s models are as follows:

1) The TT model: The first CA model with s2s rules was suggested by Takayasu and Takayasu in 1993. In this version, the first item in the rules of NS model is modified as the standing cars accelerating with a probability $q_t = 1 - p_t$, in front of which there is only one empty cell, while the rest cars accelerate deterministically.[17,18]

2) The BJH model: In this version, a car follows s2s rules according to the state of the car in front of it. When a car shuts down not because of randomization in the current time step, the car behind it will shut down with a probability p_s in the next time step.[14,18,19]

3) The VDR model: The NS model with a speed-dependent s2s rule (VDR) was proposed by Barlovic, Schadschneider, and Schreckenberg in 1998. In this model, the randomization probability is related to the speed of car. A standing car will randomize with a higher probability than a moving car.[14]

These models have similar characters to each other. Figure 1 shows the fundamental diagram of the VDR model. Fundamental diagrams for the other two are quite similar. In the diagram we can find hysteresis curves clearly, which is an obvious advancement to the NS model. But these models are far from perfect. None of them has taken the congestion degree as a remarkable ingredient. In order to exhibit cognizable s2s effects, we must remarkably distinguish the difference between moving cars and standing cars, regardless of the traffic conditions around them. For example, in the VDR model, usually a moving car randomizes with a probability $p = 1/64$, while a standing car must randomize with a probability as high as $p_0 = 0.75$. As a result, even if the traffic state around a standing car is quite smooth, the car must take quite a long time to restart, which is unrealistic. Especially in the process of resolving a cluster, the resolving process should be more fluent increasingly, because the traffic should become smoother with the smaller cluster. But none of the existing s2s models can describe this phenomenon. So it is necessary to establish a new model which takes both s2s effect and the congestion-degree dependent motions into account. In the probabilistic descriptions of traffic flow, the rate for a car to escape from a cluster depends on the cluster's size,[20−22] which enlightens us to study a modified NS model whose randomization probability depends on the size of clusters.

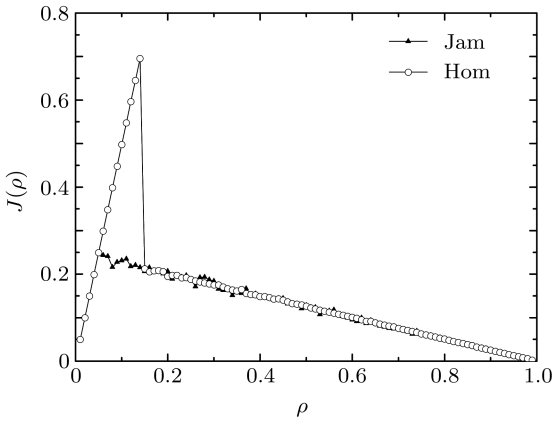


Fig.1. The fundamental diagram of the VDR model. The parameter is chosen as $p_0 = 0.75$, and $p = 1/64$. Obtained by starting from two different initial states: a completely jammed state (low branch) and a homogeneous state (upper branch). Averaged over 100 individual runs.

In Section 2, shown is a cluster-size dependent s2s model. In subsection 2.1, the model is described in detail and its rationality is discussed. In subsection 2.2, we use the model to study the metastability and compare the results with those from the existing s2s models. In subsection 2.3, the characters of jams are simulated by using this model. In subsection 2.4, we use this model to study some complex traffic phenomena. Some conclusions drawn from the present study are given in Section 3.

2. An s2s model with cluster-size dependent randomization

In the NS model, the randomization probability p is a crucial parameter. It is the only parameter inducing random process. The randomization reflects several important features in real traffic actions, including the fluctuations at maximum speed, the retarded acceleration and the overreactions at braking.[23,24] The s2s is just a point of view that standing cars accelerate more difficultly than moving cars. We can roughly consider it as an effect of inertia and friction force. Besides, the restart process in real traffic is also influenced by the traffic conditions around. The more congested the traffic is, the harder it will be for a driver to restart his car unhindered. Considering this, the drivers will tend to retard acceleration when the traffic is busy. Therefore, it is indispensable to take the degree of congestion into account.

Kaupuzs *et al* have depicted a traffic model with discrete space and continuous time.[22] They related their model to the size-dependence of domain currents and a quantitative criterion by considering the Zero-Range Process as a generic model for domain dynamics. They believed that the probability per unit time for each car to move to the next cell was given by a certain transition rate depending on the actual configuration of empty and occupied cells. The latter is characterized by the cluster distribution. The first car in each cluster is allowed to move forward by one cell. The transition rate of this stochastic event depends on the size of the cluster to which the car belongs. Kaupuzs *et al* declared that the larger the cluster, the smaller the transition rate. Under this hypothesis, they obtained satisfying simulation results.

Enlightened by their work, we use the size of cluster as a parameter to measure the congestion degree, and then suggest an NS model with cluster-size dependent randomization (CDR for short). We carry out Kaupuzs' idea in a complete CA model and study some traffic problems with it.

2.1. Description of the CDR model

In the CDR model, the randomization probability of a standing car depends on the size of the cluster containing this car. This is the only term different from those in the NS model. In the CDR model, $p_0 = p(L)$, where L is the cluster size. Here we define a cluster with size L as an uninterrupted string of L occupied cells bounded by an empty cell on each side. The function $p(L)$ should have the following characters: (i) $0 \leq p \leq 1$; (ii) $p(L)$ should be an ascending function of L and have an upper boundary. In this paper, the function is chosen as $p(L) = P \arctan(L)$, and $P = 0.5$. Moving cars are randomized with a probability $p = 1/64$. The whole length of the road is set to have 10000 cells in this paper, and ρ represents the global density of cars on the road. In the simulations, periodical boundary conditions are adopted.

The CDR model is also rational and logical. Some people may question how the drivers can know the size of the cluster. However, it must be emphasized that the pivotal idea of this paper is that the randomization probability depends on the congestion degree. Cluster size is only a quantification of the traffic conditions. We suppose that the drivers can observe the surroundings and then have get a rough estimate of the congestion. According to the estimate, they make a decision to restart with a certain rate.

Actually, only the car at the head of a cluster has a chance to accelerate immediately. So the value of L which can impact the driver's decision is the length of string at the back of the car indeed. But this will not cause much discrepancy. Kaupuzs *et al* have described this process as the influence of waiting time. They have declared that the driver of the first car in a cluster does not know how many cars are staying behind him, but he knows his time spent in the jam. This waiting time contains the indirect information of the cluster size. The larger the cluster is, the longer the cars should be frozen in it, and the more congested the traffic should be. Thus, it will take more time for a standing car to restart. Under this assumption, they used the cluster size as the parameter to determine the restarting rate of the head car[22] and obtained expected results.

However, this explanation is not satisfactory somehow. Commenters have doubted whether the waiting time is necessarily related to the cluster size. In order to answer this question, we plot the relationship between the waiting time and the cluster size in Fig.2. We run our model until it reaches a steady state and then adopt the data from 1000 time steps. The global density of vehicles can be chosen freely, and the results are similar. We can see, in steady states, larger clusters induce longer waiting time. It can be imagined that the clusters generally keep sizes fixed in a

globally steady system and move upstream as shown in the spatiotemporal diagrams in later part of this paper. Cars will spend almost the same time crossing a certain jam. For example, the randomization probability of the first car in a jam is $p(L)$, where L is the cluster's size. The averaged time spent in restarting process is

$$\tau = \frac{1}{1-p(L)}.$$

If the cluster keeps the size fixed and τ has a fixed value, the total time for a car to cross the jam is

$$t = L \times \tau = \frac{L}{1-p(L)}.$$

We have restricted $p(L)$ to being an ascending function of L, so t is also an ascending function of L, which is fit for the results shown in Fig.2. In this case, Kaupuzs' explanation does not induce antinomies and can be accepted.

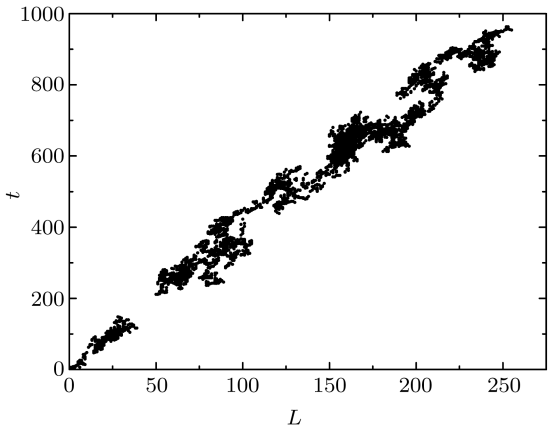


Fig.2. The relationship between the waiting time and the clusters' size. The Y-axis represents the time for a car being frozen in a jam, and the X-axis is for the cluster's size.

The CDR model does not cause conflicting results in steady states. But in precarious states, the dissolving process does exist. According to the rules, the car standing at the head of a dissolving jam requires the longest restarting process, whereas it may not have waited for a long time in the jam. The time for which the car stands on the tail of a jam is the longest one, while the time for restarting is the shortest. A newly formed jam of length 3 and a dissolving jam of length 3 have widely different waiting times but similar evolvements. These transient phenomena seem to be challenging the rationality of our model. It seems that we have to abandon the explanation clarified by waiting times, although our model does not rely on it. However, in the later part we will see the CDR model has a high ability to dissolve jams. It can make the dissolving process swifter than other models, which can shorten the transient process to the least. So this dissolving process is strictly restricted to being a transitory process. After averaging, the influence can be dissolved and cause no antinomies. On the other hand, the case mentioned above is a reasonable phenomenon. Transient process in the CDR model contains intelligent adaptive actions. The restarting motion in a dissolving cluster becomes more and more swift with the cars escaping from it. The dissolving process is an accelerating process. This is quite a realistic character, of which evidences could be found easily in real traffic. Different from steady states, jams' dissolving process can lighten the traffic burdens. With the jams dissolving, the traffic becomes smoother, and it is easier for the cars to restart. When the front car is ready to start, the following car will get ready for starting at the same time. It, as a matter of fact, just reflects restart-preparation motions of the drivers when facing a catabatic congestion. The cluster's influence on s2s will saturate on a finite scale of spatial size. We discuss this process again in subsection 2.3. Although a dissolving jam of length 3 has a much longer waiting time, it is unnecessary for it to have larger randomization parameters than a newly formed jam of length 3.

Schreckenberg *et al* have pointed out that cellular automata models are established in the spirit of modelling complex phenomena in statistical physics, rather than describe the traffic flow accurately on a microscopic level. So in CA models, the motion of a single car might exhibit unrealistic features on short timescales. But these features can be dissolved by averaging over large numbers of cars or long timescales.[14] Through simulation, the CDR model can give several realistic statistical results. Therefore, it is safe to state that our model is rational.

2.2. Metastabilities and hysteresis curves simulated by the CDR model

The metastable states can be established by our CDR model as well. The metastable states will be exhibited mainly in the fundamental diagrams. To accomplish this, there are two basic strategies. Note that the hysteresis effect is directly related to the oc-

currence of metastability, which makes the flow at a certain density dependent on the history. We usually distinguish the occurrence of metastable states by the hysteresis phenomenon. Two methods are used to reach the branches in the hysteresis curves. In the first method, we change the density of cars by adding cars to or removing cars from the traffic flow on the road. Starting from a free flowing state (low density) and then adding cars continuously to the road, we obtain a flow-density diagram that becomes the upper branch of the hysteresis curve. And starting from a jamming state (high density) and then removing cars continuously from it, we obtain the lower branch. In the second method, the global density of cars is kept unchanged. Starting from a homogeneous initial state, in which cars are distributed in equal distance, we have the upper branch. Then starting from a complete jamming state, which comprises a large compact cluster of standing cars, we obtain the low branch.[14,25−27]

Figure 3 shows the fundamental diagrams simulated by the CDR model with the two methods above separately. Figure 3(a) is obtained by the first method. The upper branch in the figure is obtained by adding cars to the system, and the lower branch by removing cars from it. We have averaged the results from over 100 individual runs. In this diagram, we can find emblematical hysteresis curve which covers quite a large scale of densities. It is remarkable that in each s2s model, the hysteresis curves obtained by this method are all similar. Figure 3(b) shows the hysteresis curve obtained by the second method, which is more common in use. In Fig.3(b), as the character of the VDR model, the diagram is convex everywhere and has no infection point. It is because these two models do not simply depend on spatial conditions.[18] There is a steep falling-down portion from the peak on the upper branch, as expected in all s2s models. Different from Fig.1, there is also such an obvious decline on the lower branch in Fig.3(b), which is not conspicuous in the results of other s2s models. That means, in the CDR model, the discontinuous phase-transition effect is more intense, which represents a more prominent s2s character. Nevertheless, unlike the VDR model, in our new model, the difference in randomization probability between moving cars and standing cars is not always large. We can consider this new s2s model as a more effective one than the other existing ones.

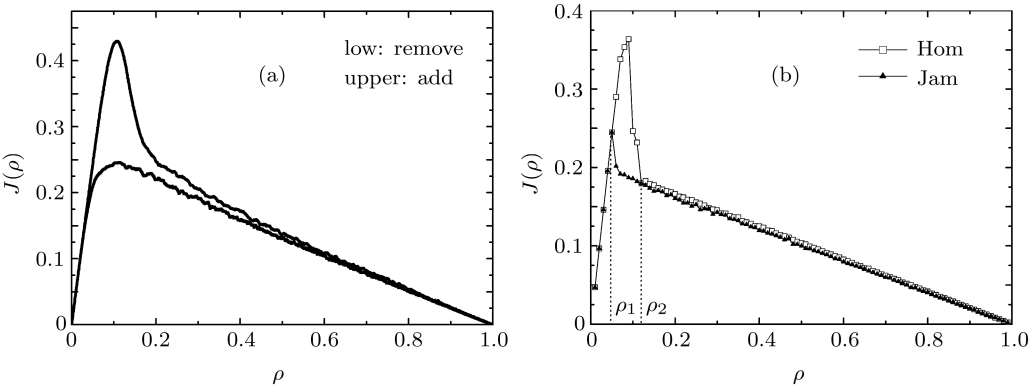


Fig.3. Fundamental diagrams of the CDR model, obtained by two different methods: (a) by changing vehicle densities, i.e., adding cars to the system (curve of add) or removing cars from it (curve of remove), and (b) from different initial states i.e. by starting from homogeneous (curve of Hom) or completely jammed (curve of Jam) initial states separately.

Comparing the two diagrams above, we find some differences. Figure 3(a) is the direct representation of metastability and hysteresis effects. When we change the car density on the road, the traffic flow does not change subtly with it, but evolves into multiple traffic states. These states can have long lifetimes, and they may be considered as metastable states. This process can occur in quite a large range of densities, as shown in Fig.3(a). But not all of these states can survive after a long time of evolution. At low densities, the steady state is homogeneous and free flow, while at high densities, congested traffic is the final outcome. So in these ranges, multiple states will resolve into uniform states in the end. Only in a certain range of densities can metastable states have quite long lifetimes and then induce multiple final states. This character is exhibited as sensitivity to initial states in macroscopic observations just as shown in Fig.3(b). So Fig.3(b) is the representation of the sensitivity to initial states, which can describe metastability and hysteresis effects

in an oblique way.

The two phase-transition points in diagrams obtained by the second method have essential significance. The density where the two curves bifurcate is denoted as ρ_1, and the density where the two curves converge again is denoted as ρ_2 as shown in Fig.3(b). These two phase-transition points are boundaries of different macroscopic characters of traffic flow. In the range $\rho < \rho_1$, homogeneous states are steady. Jams will dissolve soon and the traffic will turn into a homogeneous free flow over evolvement. In the range $\rho > \rho_2$, jamming states are steady. Little jams will assemble into large ones and the traffic will be steady on a phase-separated state in the end. There are one or more large jams and several free flow regimes in the final system. In the range between ρ_1 and ρ_2, metastable states can be maintained lengthily and finally evolve into different steady states determined by the initial conditions. These are clearly reflected in Fig.3(b).

The above discussion can be substantiated in Fig.4. Figure 4 is the spatiotemporal diagrams for $\rho = 0.10$. Figure 4(a) shows the macroscopic character of the traffic flow starting from a homogeneous initial state, while Fig.4(b) shows the result from a jamming initial state. At equal global densities, different initial states produce different final systems. The spatiotemporal diagrams show results accordant with the discussion above and offer us an intuitional impression. This is also due to the model's clarity in simulating traffic systems.

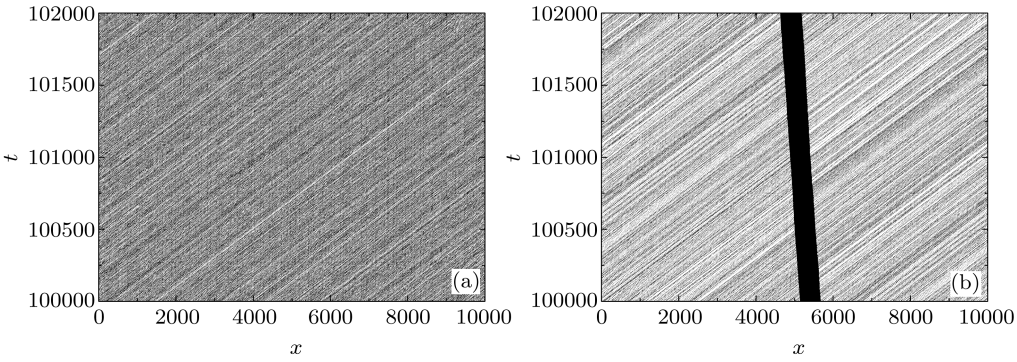


Fig.4. Spatiotemporal diagrams for $\rho = 0.10$, starting from homogeneous (a) or jamming (b) initial state.

In the $\rho_1 < \rho < \rho_2$ range, metastable characters can also be embodied in the spatiotemporal diagrams directly. Figure 5 shows a spatiotemporal diagram at $\rho = 0.12$. The traffic system starts from a homogeneous initial state. In this case, the steady state should be a homogeneous and free flow. We focus on the ($0 \leq x \leq 2000$, and $10000 \leq t \leq 10600$) area to discern the local regimes. In the figure, most of the range is covered by a traffic flow with relatively high local densities. A narrow gap of sparse flow is distinguishable in it, and moves downstream with time. However, in the thickness area, vehicles do not congest. They are still free but have a smaller spatial headway. Gradual transition can be observed at the boundaries. This phenomenon can be observed in real traffic sometimes. This state has quite a long lifetime (more than 100000 time steps) before the gap dissolves and the system evolves into a globally homogeneous state finally. During the evolution, the parameters such as the flow and averaged velocity are kept invariable. So Fig.5 can be understood as a description of metastable states embodied on a macroscopic scale.

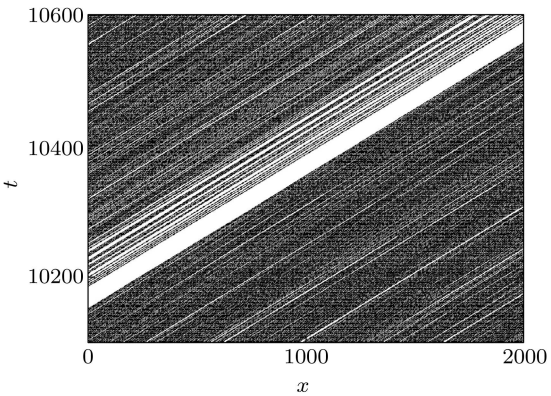


Fig.5. Metastable states on a macroscopic scale shown in the spatiotemporal diagram of the CDR model for $\rho = 0.12$, starting from a homogeneous initial state.

2.3. Characters of jams simulated by the CDR model

We have mentioned that in the CDR model, the dissolving process of a jam is an accelerating process. This leads to a result that in this model, jams dissolve much faster than in the VDR model when the parameters are similar.

The researchers of the VDR model present the time-dependent length of the initial jam.[14] The starting is from a completely jammed state. Then we plot the length of the biggest cluster on the road versus time. It is found that in the range of $\rho < \rho_1$, jams are in unstable states, which will sooner or later dissolve with time. Even in the area in which the parameter ρ is a little smaller than ρ_1, the steady state of the traffic system is not jamming or multiple, but homogeneous. No big jam can persist in the traffic flow. In their paper, Fig.5 in Ref.[14] shows the process. At the beginning, the length $L(t)$ declines steeply in the figure, and then it strongly fluctuates without any obvious systematic time-dependent regime for about 80000 time steps. Finally it dissolves to a steady state where the length $L(t)$ equals 1 perpetually. The averaged length $< L(t) >$ over several individual samples shows an obvious exponential decay in time. It takes about 50000 time steps on an average to arrive at the steady state.

The same process is studied in the CDR model. Figure 6 shows the time dependence of the maximum length of jams in the system. The density $\rho = 0.045$ is also chosen to be just a little smaller than ρ_1. Figure 6(a) shows the result of a single progress and Fig.6(b) shows the averaged result over 500 individual samples. In Fig.6(a) it is quite easy to find that $L(t)$ decays almost linearly with time t at the beginning, and then it fluctuates randomly. Finally, the traffic system dissolves to a steady homogeneous state. It takes only 5000 time steps to accomplish this evolution, much shorter than in the VDR model. In Fig.6(b), the averaged length $< L(t) >$ decays linearly at the first 500 time steps, then approaches to 1 gradually. It takes about 20000 time steps to accomplish the declining process, which is also much shorter than in VDR model. So the CDR model's traffic efficiency is much better, especially in the process of dissolving jams.

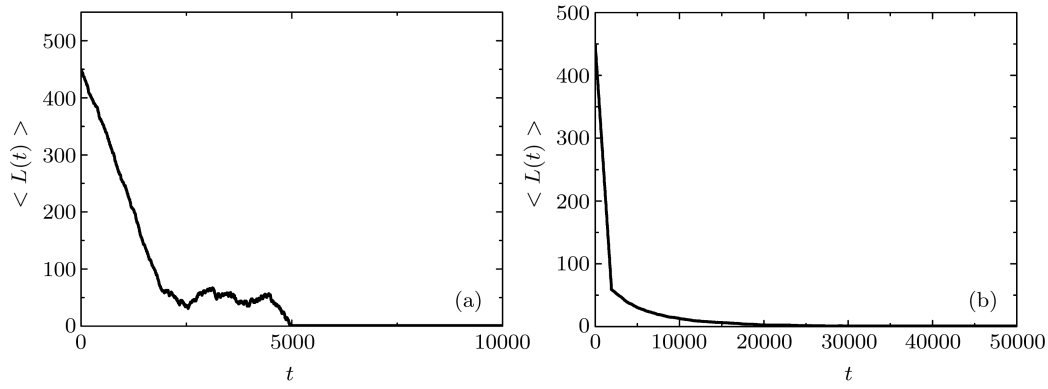


Fig.6. Time-dependent maximum length of jams in the system for $\rho = 0.045$, starting from a complete jamming initial state. (a) is for the result for a single sample. (b) for the averaged result over 500 individual runs.

The reason why the CDR model can perform a more efficiently dissolving process is as follows. First, the dissolving process in the CDR model is an accelerating process. As the cluster becomes smaller, the vehicles restart more easily. This can avoid the meaningless waiting induced by the high randomization probability in the VDR model. Second, in the CDR model, randomization is a dynamic motion considering the congestion conditions. This allows us to adjust the randomization probability to being quite high in serious congested traffic and relatively low in unblocked traffic, which in turn makes an intelligent adaptive system. At the beginning, the cluster is large, and congestion is serious. Under this condition, this model adopts a high randomization probability to prevent the cars from escaping too rashly. When a car escapes from the jam, the car behind it will wait for a while rather than restarts immediately. This keeps a certain headway and avoids jams' emerging again soon somewhere downstream. In fact, the main difference in dissolving process between the CDR model and the VDR model is the time spent in fluctuating. The

CDR model has a much shorter fluctuation period, while jams reform somewhere frequently and induce a long fluctuation time to dissolve all of them in VDR model.

On the other hand, on the right-hand side of the phase-transition point, homogeneous states are unstable. Jams emerge spontaneously with time and then become steady states. It has been approved that in the NS model, the cluster-size distribution shows an exponential character near the phase-transition point. Big jams are very rare and unlikely to be maintained lengthily in the system. And no phase-separation phenomena can be observed.

Figures 7(a), 7(b) and 7(c) show the cluster-size distributions near the phase transition point for the NS model, the VDR model and the CDR model, respectively. In each diagram, the X-axis represents the cluster size L, and the Y-axis, the number of vehicles frozen in jams with certain sizes. Starting from a homogeneous state, we simulate 100 individual samples with 10100 time steps each. The results for the initial 10000 time steps are removed. All the considered results are accumulated together to describe the averaged distribution of the cluster sizes. In Fig.7(a), we can see clearly that the diagram for the NS model shows an exponential character.

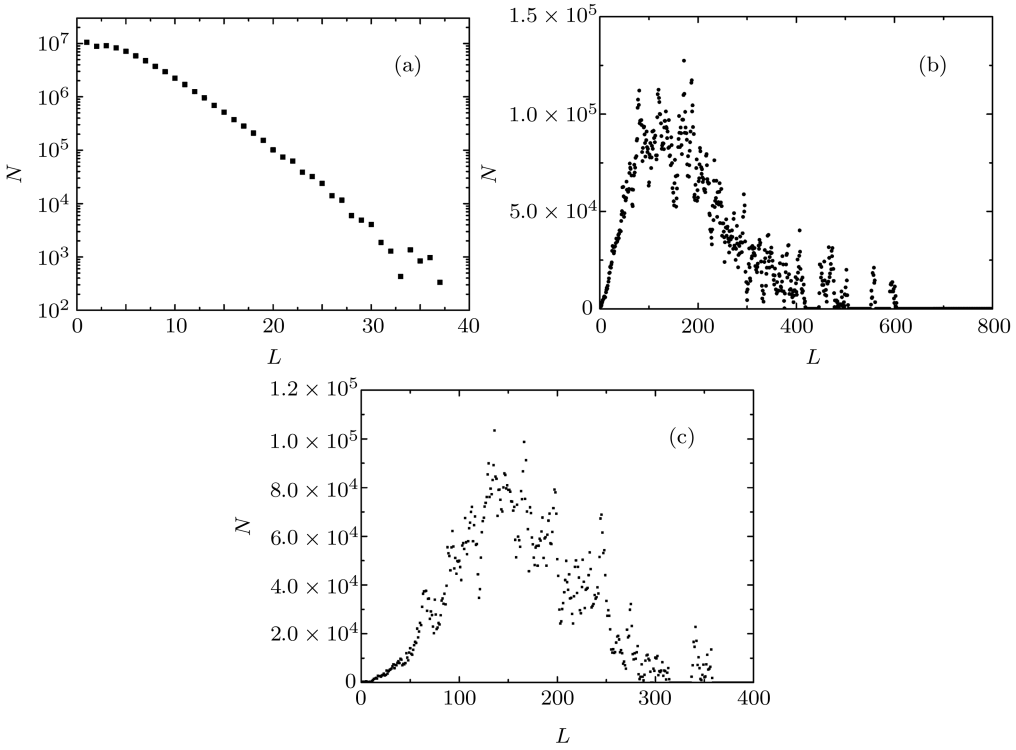


Fig.7. Cluster-size distributions near the phase-transition point, accumulating over 100 samples, separately with (a) the NS model at $\rho = 0.12$ and $p = 0.02$, (b) the VDR model at $\rho = 0.15$, and (c) the CDR model at $\rho = 0.15$, fixating on the $L > 1$ region.

But Figs.7(b) and 7(c) are quite different from Fig.7(a). They both show a distribution far from exponential form. We have removed the isolated tiptop point at $L = 1$ and fixates in the $L > 1$ range in both diagrams. Figure 7(b) seems similar to a Poisson distribution. And Fig.7(c) is characteristic of a normal distribution. There exist little jams with different sizes in the system. They seem to be homogeneous in a macroscopic view, which can be seen from the fact that clusters with size 1 occupy the largest proportion. But in a microscopic view, we can find many small clusters with various sizes obeying the distribution described in the figures. This scenario will be shown more clearly in the spatiotemporal diagram later.

We can also see that the positions of the peaks in Figs.7(b) and 7(c) are similar. But in the latter case, the distribution is restricted within a range of small values of L compared with the former case. So our the CDR model produces small jams compared with the VDR model under the same conditions. It is beneficial for the traffic fluency.

At higher densities, the CDR model can produce more amplitude jamming characters. Figure 8 shows the cluster-size distribution at $\rho = 0.60$. We can see

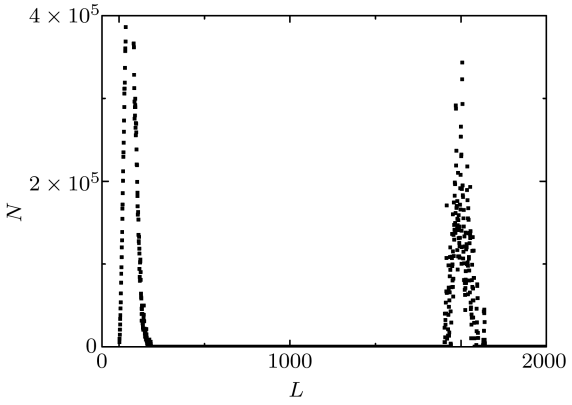


Fig.8. Cluster-size distribution of the CDR model for $\rho = 0.60$, fixating in the $L > 1$ region.

an interesting configuration with two peaks in the diagram. In order to show them in a distinguishable way, we have also removed the isolated tiptop point at $L = 1$ as has been done in Fig.7. But the removed tiptop point has reflected that clusters of size 1 still amounts to the largest proportion at such a high density. Most cars could keep empty cells on the both sides of it, and the traffic is not congested seriously anywhere. This model is trying its best to keep the traffic expedite. The configuration with two peaks implies a serious phase-separation regime of the traffic system. This will be shown in the spatiotemporal diagrams later.

2.4. Complex characters exhibited in the spatiotemporal diagrams

Spatiotemporal diagram is a commonly used tool in the study of the macroscopic character of traffic systems. It can give intuitional impressions, and is easy to associate with real traffic phenomena.

Figure 9 shows an interesting phase-transition phenomenon in the spatiotemporal diagrams. We plotted the spatiotemporal diagram for $\rho = 0.14$ in Fig.9(a) which is to the left of the phase-transition point, and the diagram for $\rho = 0.145$ in Fig.9(b) which is to the right of the phase-transition point. Both of the traffic systems shown in the figure start from homogeneous initial states. The two densities are adjacent but the macroscopic characters are quite different. In Fig.9(a), the road is covered by a homogeneous traffic flow, while in Fig.9(b), congested regimes agglomerate from the homogeneous flow clearly. These jams move upstream with time; their sizes correspond to the peak in Fig.7(c), and sizes of little jams in the homogeneous regime correspond to other parts in the distribution. Figure 9 indicates an intuitional scenario of phase-transition and phase-separation regimes.

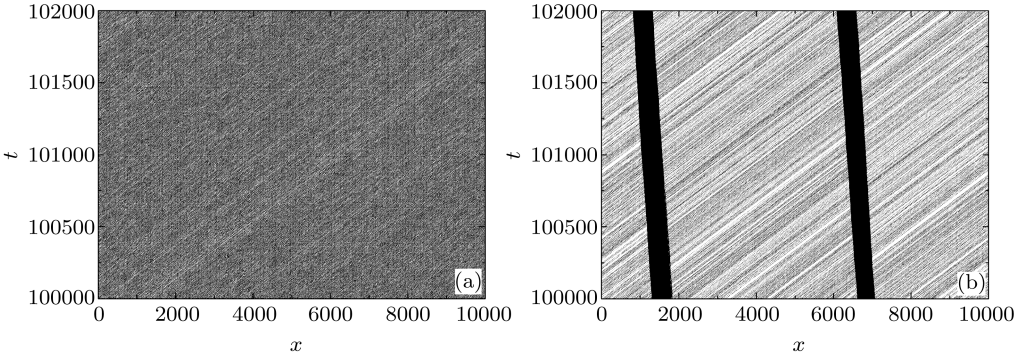


Fig.9. Phase-transition exhibited in the spatiotemporal diagrams for $\rho = 0.14$ (a) and for $\rho = 0.145$ (b). Both start from homogeneous initial states.

In certain circumstances, the CDR model can simulate more complex traffic characters. In Fig.10, we plot the spatiotemporal diagram at $\rho = 0.60$. We can find stop-and-go waves in the diagram. There is a large jam in the system corresponding to the right peak in Fig.8, and a good many of smaller jams corresponding to the left peak. The diagram shows a complex traffic congestion system. Figure 11 simulates a shock in the traffic induced by an obstruct for 100 time steps, of which the situation is like that in front of a traffic light. Vehicles accumulate near the obstruct and dissolve gradually after the traffic has resumed as shown by the narrow shadow in the figure. This process has been studied by the velocity-effect

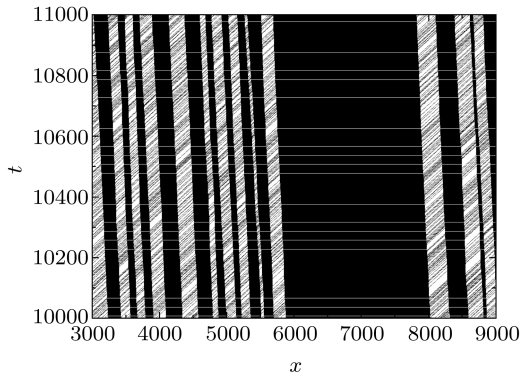


Fig.10. Spatiotemporal diagram of the CDR model for $\rho = 0.60$.

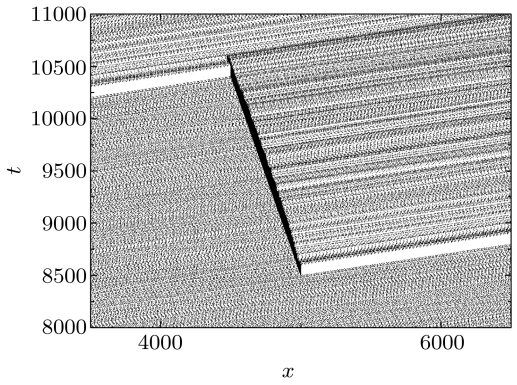


Fig.11. Shock in the traffic induced by an obstruct for 100 time steps.

(VE) model before.[28] In their simulation, the result appears as a black triangle in the spatiotemporal diagram. Compared with the VE model, our model is so steady that no other than longer shock time can induce such a process. The congested area in the CDR model is much narrower and has less influence on the traffic. The strong point of the VE model in this process is the prompt reaction it manifests. We believe that our model would be even better combining with other models. We view it as a usable and valuable outcome.

3. Conclusions

To summarize, a new model called CDR is proposed in this paper to study the cluster-size-dependence of the s2s. In the CDR model, the randomization probability depends on the velocity of the car and the size of jam containing that car. It can produce metastability and hysteresis effect as produced by the other s2s models, in addition, it can also capture some other features in real traffic. The restart rule makes the dissolving process of a jam intelligently adaptive. It is an accelerating process, which reflects the drivers' preparation motion for restarting beforehand and induces a higher traffic efficiency than the VDR model. In simulation, the CDR model can exhibit amplitude complex effects accordant with real traffic. The traffic system simulated by the CDR model is more robust than the existing models. The CDR model tends to establish a high-efficient light-congested traffic system. Therefore, it is safe to state that this model can represent more amplitude congestion scenarios and more fit for reproducing complex phenomena in real traffic. By suitable modification of the update rules, CA models can reproduce some important features of real traffic flow. There is still much room for further investigation, both analytically and computationally.

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