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Entanglement between two atoms in the system of Schrödinger cat state interacting with two entangled atoms*

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By the negative eigenvalues of partial transposition of density matrix, this paper investigates the time evolution of entanglement of the two entangled atoms in the system of two atoms interacting with Schrödinger cat state. The result shows that the two atoms are always in the entanglement state, and the degree of entanglement between the two atoms exhibits ordinary collapses and revivals at 0.2 degree of entanglement, when the light field is large enough. On the other hand, the reinforcement of three different light fields on the degree of entanglement between two atoms is not evident.

Keywords: negative eigenvalues, entanglement state, Schrödinger cat state
PACC: 4250, 0365

1. Introduction

Quantum entanglement is one of the most striking features of quantum mechanics. Entanglement states of two or more particles have not only played a vital role in discussion of nonlocal quantum correlations,[1] but also formed the basis of quantum communication and information processing, such as quantum computation,[2] quantum cryptography,[3] quantum teleportation,[4] and quantum dense coding.[5] A number of entanglement measures have been discussed in the literatures,[6−16] such as the von Neumann reduced entropy of the entanglement. The quantum entropy has been used to study the dynamic characteristics of the two-level atom interacting with a light field. It is shown that the quantum entropy is an useful physical quantity for measuring pure degree of the entanglement state. Recently, two two-level atoms in the entanglement state have aroused huge research interest. By means of negativity and concurrence, the time evolution of entanglement of the two atoms and the entanglement between the two modes in the system of two atoms interacting with two mode fields are investigated.[17] The entanglement time evolution of two entangled two-level atoms that interact resonantly with a single-mode field in the Fock state is researched.[18]

In this paper, the entanglement between the two atoms is investigated. We consider the situation of two entangled two-level atoms interacting with Schrödinger cat state. The influence of the strength of a light field and the phase angle between two coherent states on the entanglement between the two atoms is discussed by using full quantum theory.

2. The model and its solution

Assume that two entangled two-level atoms A and B are simultaneously injected into a light cavity, the dipole interaction between atoms can be neglected. There are same couplings of the two atoms as interacting with a field, under these conditions. The Hamiltonian of the system can be written as ($\hbar = 1$)

$$H = \omega a^{+}a + \omega_0 \sum_{j=\mathrm{A}}^{\mathrm{B}} S_{\mathrm{z}}^{j} + g \sum_{j=\mathrm{A}}^{\mathrm{B}} (a^{+}S_{-}^{j} + aS_{+}^{j}), \quad (1)$$

where a^{+} and a are the creation and annihilation operators of the monochromatic photons with frequency

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ω respectively, ω_0 denotes the atomic transition frequency, S_z^j and $S_\pm^j (j = \mathrm{A}, \mathrm{B})$ denote the inversion and transition operators of the jth atom respectively, and g describes the coupling constant of the field and the atom.

If we consider that at time $t = 0$ the two atoms are initially in an entanglement state:

$$|\varphi_\mathrm{a}(0)\rangle = \frac{1}{\sqrt{2}}(|e_\mathrm{A}, g_\mathrm{B}\rangle + |g_\mathrm{A}, e_\mathrm{B}\rangle), \tag{2}$$

the field is initially in Schrödinger cat state:[19]

$$|\varphi_\mathrm{f}(0)\rangle = |\alpha, \varphi\rangle = \{2[1 + \cos\varphi \exp(-2\,|\alpha|^2)]\}^{-1/2}[|\alpha\rangle + \mathrm{e}^{\mathrm{i}\varphi}\,|-\alpha\rangle], \tag{3}$$

where

$$|\alpha\rangle = \sum_{n=0}^{\infty} F_n\,|n\rangle\,, \quad F_n = \exp(-\,|\alpha|^2/2)(|\alpha|^{2n}/n!)^{1/2}\exp(\mathrm{i}n\phi),$$

$$|-\alpha\rangle = \sum_{n=0}^{\infty} F'_n\,|n\rangle\,, \quad F'_n = \exp(-\,|\alpha|^2/2)(-\,|\alpha|^{2n}/n!)^{1/2}\exp(\mathrm{i}n\phi).$$

Here, $|n\rangle$ is the n-particle eigenstate, φ is the phase angle between the two coherent states $|\alpha\rangle$ and $|-\alpha\rangle$, $\alpha = |\alpha|\,\mathrm{e}^{\mathrm{i}\phi}$ is a complex parameter, ϕ and $|\alpha|$ represent the direction angle of the excitation and the strength of the light field respectively (for simplicity we suppose $\phi = 0$), and coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ are 180° out of phase with respect to each other.

Obviously, if $\varphi = 0$, then Eq.(3) describes the even coherent state:[19]

$$|\alpha\rangle_\mathrm{e} = \{2[1 + \exp(-2\,|\alpha|^2)]\}^{-1/2}(|\alpha\rangle + |-\alpha\rangle),$$

when $\varphi = \pi$, then Eq.(3) is equal to the odd coherent state:

$$|\alpha\rangle_\mathrm{o} = \{2[1 - \exp(-2\,|\alpha|^2)]\}^{-1/2}(|\alpha\rangle - |-\alpha\rangle),$$

and, finally, for $\varphi = \pi/2$, Eq.(3) is equal to the Yurke–Stoler coherent state:[20]

$$|\alpha\rangle_\mathrm{YS} = (2)^{-1/2}(|\alpha\rangle + \mathrm{i}\,|-\alpha\rangle).$$

Then the initial state vector of the system is

$$|\varphi_\mathrm{s}(0)\rangle = \frac{1}{\sqrt{2}}(|e_\mathrm{A}, g_\mathrm{B}\rangle + |g_\mathrm{A}, e_\mathrm{B}\rangle) \otimes |\alpha, \varphi\rangle\,. \tag{4}$$

We only consider resonance situation, namely, transition frequency of the atom and effective action resonance frequency of the field satisfying $\omega_a = \omega$. Then, along with time evolution, the state vector of the system at any time $t > 0$ is given by

$$\begin{aligned}|\varphi_\mathrm{s}(t)\rangle = \sum_{n=0}^{\infty} &[A_n(t)\,|e_\mathrm{A}, e_\mathrm{B}, n-1\rangle + B_n(t)\,|e_\mathrm{A}, g_\mathrm{B}, n\rangle \\ &+C_n(t)\,|g_\mathrm{A}, e_\mathrm{B}, n\rangle + D_n(t)\,|g_\mathrm{A}, g_\mathrm{B}, n+1\rangle].\end{aligned} \tag{5}$$

Based on the initial condition, we can easily gain the solution of Schrödinger equation in the interaction representation given by

$$\begin{aligned}A_n(t) &= -\mathrm{i}\sqrt{\frac{n}{2n+1}}\sin[\sqrt{2(2n+1)}gt](F_n + F'_n\mathrm{e}^{\mathrm{i}\varphi})\{2[1 + \cos\varphi\exp(-2\,|\alpha|^2)]\}^{-1/2}, \\ B_n(t) &= C_n(t) = \frac{1}{\sqrt{2}}\cos[\sqrt{2(2n+1)}gt](F_n + F'_n\mathrm{e}^{\mathrm{i}\varphi})\{2[1 + \cos\varphi\exp(-2\,|\alpha|^2)]\}^{-1/2}, \\ D_n(t) &= -\mathrm{i}\sqrt{\frac{n+1}{2n+1}}\sin[\sqrt{2(2n+1)}gt](F_n + F'_n\mathrm{e}^{\mathrm{i}\varphi})\{2[1 + \cos\varphi\exp(-2\,|\alpha|^2)]\}^{-1/2}.\end{aligned} \tag{6}$$

The properties of the quantum entanglement between two entangled atoms will be discussed in the next section by using above results.

3. The entanglement between the two two-level atoms

For two subsystems in the mixed state, we may use the negative eigenvalues of partial transposition of density matrix as a measurement of the degree of entanglement between the two subsystems. Consider a density matrix $\rho(t)$ and its partial transposition $\rho^{\mathrm{T}}(t)$ for a two spin-1/2 system, the measure of the entanglement $E_{\mathrm{AB}}(t)$ is then defined by[21]

$$E_{\mathrm{AB}}(t) = -2\sum_i \lambda_i^-, \tag{7}$$

where λ_i^- is a negative eigenvalue of $\rho^{\mathrm{T}}(t)$. Then $E_{\mathrm{AB}} = 0$ which denotes that two atoms are separated. Then $E_{\mathrm{AB}} = 1$ which denotes that two atoms are in the maximum entanglement state. Then $0 < E_{\mathrm{AB}} < 1$ which denotes that two atoms are in entanglement state. In the following, let us only consider the time behaviour of the degree of entanglement between the two atoms $E_{\mathrm{AB}}(t)$. In order to derive a calculation formalism of the degree of entanglement between the two atoms, we must obtain the eigenvalues of the reduced atom density operator. From Eq.(5), we have the reduced atom density operator

$$\rho_{\mathrm{atom}}(t) = \mathrm{Tr}_{\mathrm{field}}[\rho(t)] = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} \end{pmatrix}, \tag{8}$$

where

$$\begin{aligned}
&\rho_{11} = \sum_{n=1}^{\infty} |A_n(t)|^2, \quad \rho_{12} = \rho_{13} = \sum_{n=0}^{\infty} A_{n+1}(t)B_n^*(t), \quad \rho_{14} = \sum_{n=0}^{\infty} A_{n+2}(t)D_n^*(t), \\
&\rho_{21} = \rho_{12}^*, \quad \rho_{22} = \rho_{23} = \sum_{n=0}^{\infty} |B_n(t)|^2, \quad \rho_{24} = \sum_{n=0}^{\infty} B_{n+1}(t)D_n^*(t), \\
&\rho_{31} = \rho_{21}, \quad \rho_{32} = \rho_{33} = \rho_{22}, \quad \rho_{34} = \rho_{24}, \\
&\rho_{41} = \rho_{14}^*, \quad \rho_{42} = \rho_{24}^*, \quad \rho_{43} = \rho_{42}, \quad \rho_{44} = \sum_{n=0}^{\infty} |D_n(t)|^2.
\end{aligned} \tag{9}$$

Based on the defined partial transposition of density matrix, $\rho_a^{\mathrm{T}}(t)$ is written as

$$\begin{aligned}
\rho_a^{\mathrm{T}}(t) &= \sum\nolimits_{\langle i|} \rho_{\mathrm{a}}(t)\,|j\rangle \otimes |j\rangle\,\langle i| \\
&= \begin{pmatrix} \rho_{11} & \rho_{12} & -\rho_{12} & \rho_{22} \\ -\rho_{12} & \rho_{22} & \rho_{14} & -\rho_{24} \\ \rho_{12} & \rho_{14} & \rho_{22} & \rho_{24} \\ \rho_{22} & \rho_{24} & -\rho_{24} & \rho_{44} \end{pmatrix}.
\end{aligned} \tag{10}$$

Consequently, with the eigenvalue equation of the density operator $\rho_a^{\mathrm{T}}(t)$, the eigenvalues of the partial transposition of density matrix of the atom are given by

$$\begin{aligned}
\lambda_a^1(t) &= \rho_{22} + \rho_{14}, \\
\lambda_a^2(t) &= \frac{\theta_1}{3} - \frac{2}{3}(\theta_1^2 - 3\theta_2)^{1/2}\cos\beta, \\
\lambda_a^3(t) &= \frac{\theta_1}{3} + \frac{1}{3}(\theta_1^2 - 3\theta_2)^{1/2}(\cos\beta + \sqrt{3}\sin\beta), \\
\lambda_a^4(t) &= \frac{\theta_1}{3} + \frac{1}{3}(\theta_1^2 - 3\theta_2)^{1/2}(\cos\beta - \sqrt{3}\sin\beta),
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
\beta &= \frac{1}{3}\arccos\left(\frac{\xi}{\varsigma}\right), \\
\xi &= 9\theta_1\theta_2 - 2\theta_1^2 - 27\theta_3, \\
\varsigma &= 2(\theta_1^2 - 3\theta_2)^{3/2},
\end{aligned} \tag{12}$$

with

$$\begin{aligned}
\theta_1 &= -(\rho_{11} + \rho_{22} + \rho_{44} - \rho_{14}), \\
\theta_2 &= \rho_{11}\rho_{22} + \rho_{11}\rho_{44} + 2\rho_{12}^2 + 2\rho_{24}^2 + \rho_{22}\rho_{44} \\
&\quad -\rho_{22}^2 - \rho_{11}\rho_{14} - \rho_{14}\rho_{44}, \\
\theta_3 &= -\rho_{11}\rho_{22}\rho_{44} - 2\rho_{12}^2\rho_{44} - 2\rho_{11}\rho_{24}^2 + \rho_{14}\rho_{22}^2 \\
&\quad +4\rho_{12}\rho_{22}\rho_{24} + \rho_{22}^3 + \rho_{11}\rho_{14}\rho_{44}.
\end{aligned}$$

The degree of entanglement between the two atoms $E_{\rm AB}(t)$ may be expressed in terms of the negative eigenvalues $\lambda_{\rm a}^{i}(t)$ $(i = 1,2,3,4)$ for the partial transposition of density matrix.

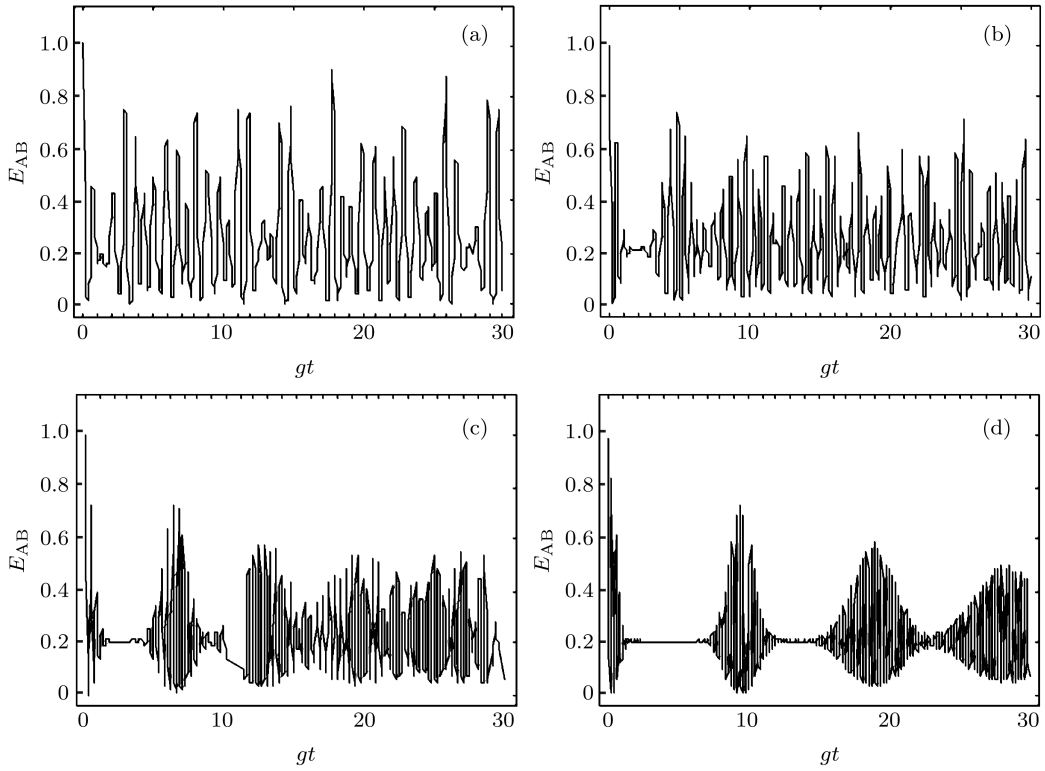


Fig.1. Time evolution of the entanglement between two atoms for $\varphi = \pi/2$. (a) $|\alpha| = 0.5$, (b) $|\alpha| = 2$, (c) $|\alpha| = 4$, (d) $|\alpha| = 6$.

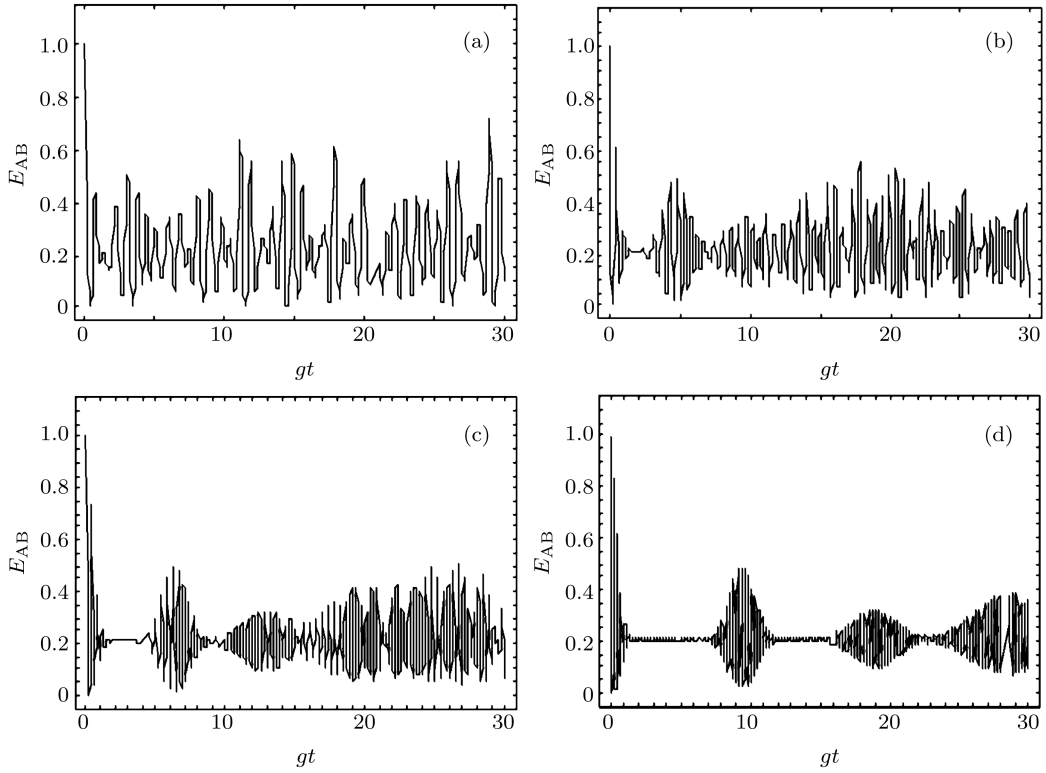


Fig.2. Time evolution of the entanglement between two atoms for $\varphi = \pi$. (a) $|\alpha| = 0.5$, (b) $|\alpha| = 2$, (c) $|\alpha| = 4$, (d) $|\alpha| = 6$.

Figures 1, 2 and 3 present the numerical results of the entanglement time evolution of the two atoms for two parameters φ and $|\alpha|$, and $\varphi = \pi/2$, $\varphi = \pi$, $\varphi = 0$, respectively. It is shown that the entanglement between the two atoms is strongly dependent on φ, the phase angle between the two coherent states, and the strength of the light field $|\alpha|$. In Figs.1–3, the ordinate represents the degree of entanglement between the two atoms $E_{\rm AB}$, and the x-coordinate represents the scaled time gt.

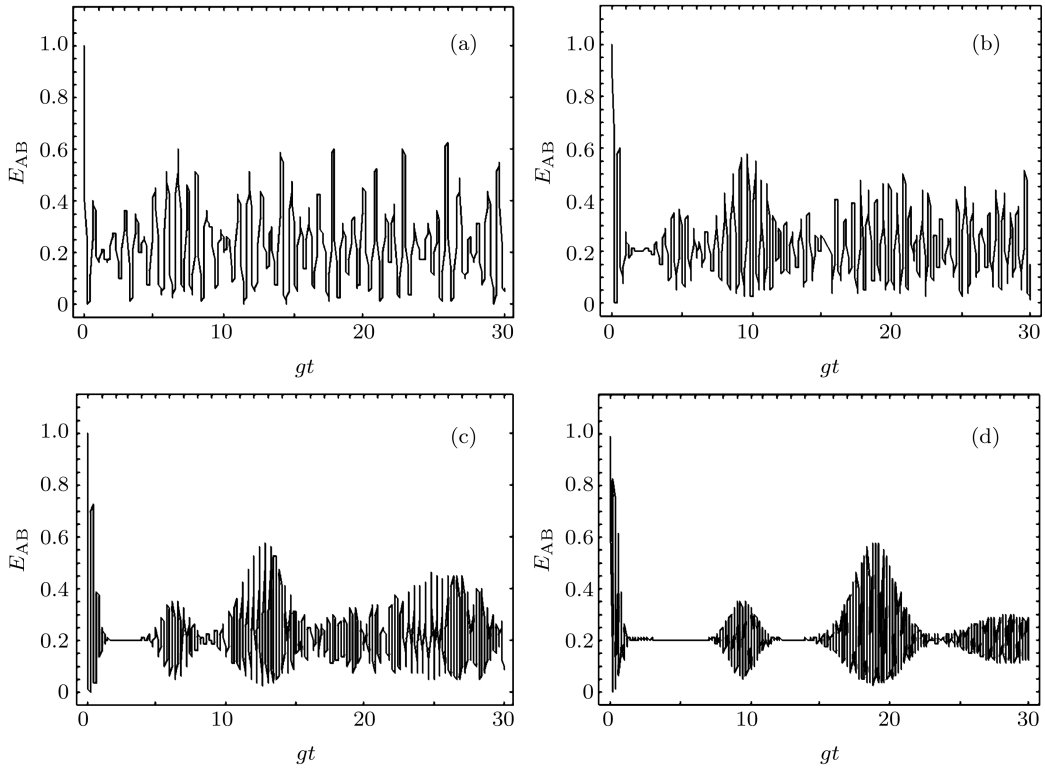


Fig.3. Time evolution of the entanglement between two atoms for $\varphi = 0$. (a) $|\alpha| = 0.5$, (b) $|\alpha| = 2$, (c) $|\alpha| = 4$, (d) $|\alpha| = 6$.

In Figs.1–3, the result tells us that for different $|\alpha|$, the time evolution of the degree of entanglement between the two atoms $E_{\rm AB}$ is quite complicated. When $|\alpha|$ is small, the degree of entanglement changes in disorder, and the two atoms are in the entanglement state. When $|\alpha|$ is large enough, we see that the two atoms are always in the entanglement state, and it exhibits ordinary collapses and revivals at 0.2 degree of the entanglement.

4. Conclusion

The time evolution of the degree of entanglement between two atoms in a system of two entangled atoms interacting with the Schrödinger cat state field has been studied. The influence of the phase angle of the two coherent states and the strength of the light field on the degree of entanglement between two atoms by using numerical calculations is discussed. The result shows that the two atoms are always in the entanglement state, and the degree of entanglement between the two atoms exhibits ordinary collapses and revivals at 0.2 degree of the entanglement when the light field is large enough. On the other hand, the reinforcement of three different light fields on the degree of entanglement between two atoms is not evident. This result will be helpful for the understanding of entanglement by further investigation of the entangled atoms interacting with the entangled fields.

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