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Faithfully probabilistic teleportation of an unknown atomic state and cavity field state with a single measurement*

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This paper shows that, based on the single-photon JC model depicting the resonant interaction of a two-level atom with a single cavity mode, an unknown atomic state and cavity photon superposition state can be faithfully teleported with only a single measurement. The scheme is probabilistic, its success lies on the event that the sender atom (or the medi-atom, for teleportation of cavity field state) is detected in the higher state. The scheme is in contrast to the previous ones of using a maximally two-particle entangled state as quantum channel.

Keywords: faithful teleportation, a single measurement, cavity QED
PACC: 4250

Quantum teleportation,[1] which describes the disembodied transmission of information, is regarded as a very useful primitive operation in quantum computation and quantum information processing.[2,3] Since its first suggestion,[1] much attentions have been paid to it and a great deal of works have been done, which lead to its experimental demonstration in various systems, including optical systems,[4,5] nuclear magnetic resonance,[6] and trapped ions.[7,8]

Microwave cavity QED, with Rydberg atoms crossing superconducting microwave cavities, provides one of those candidates for quantum computation and quantum information processing.[9] In the context of microwave cavity QED, a number of schemes have been proposed for teleportation of an unknown atomic state,[10−15] and an unknown cavity field state.[16−18] However, none of these schemes have been experimentally demonstrated. The main challenge consists in the so-called joint Bell-state measurement, performed on the Bell operator basis for the particle whose state is to be teleported, plus its partner composing the quantum channel. Recently, a scheme has been proposed for conditional teleportation of an unknown atomic state and cavity field state with a single measurement.[19] The scheme is based on the single-photon JC model that describes the resonant interaction of a two-level atom with a single cavity mode, which has been experimentally achieved and used for preparing some kinds of quantum states.[20] However, the scheme is only approximate, as the fidelity is only about 0.987. We have shown in Ref.[21] that, based on the resonant interaction of a Λ-type atom with a bimodal cavity field,[22,23] an unknown atomic state can be conditionally, but faithfully (the fidelity is 1.0 in principle) teleported with a single measurement. Whereas this type of atom-cavity interaction mechanism has no longer been realized in microwave cavity QED experiments.

In this paper, we will discuss how single-photon JC model can also be used for faithful teleportation of an unknown atomic state and cavity photon superposition state, through the method of a single measurement. In contrast to the previous ones,[19,21] the different feature of the scheme is that a non-maximally two-particle entangled state acts as the quantum channel.

Figure 1(a) displays the atomic level configuration and the setup for teleportation of an unknown atomic state. Atoms $A_j (j = 1, 2)$ employed here have a higher state $|e\rangle$ and a lower state $|g\rangle$. Atom A_1 is

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initially in an unknown state. Atoms A_2 is the one to receive the teleported state. C is an ideal millimetre-wave superconducting microwave single-mode cavity. $D_{e(g)}$ denotes the atomic state-selective field ionization detector.

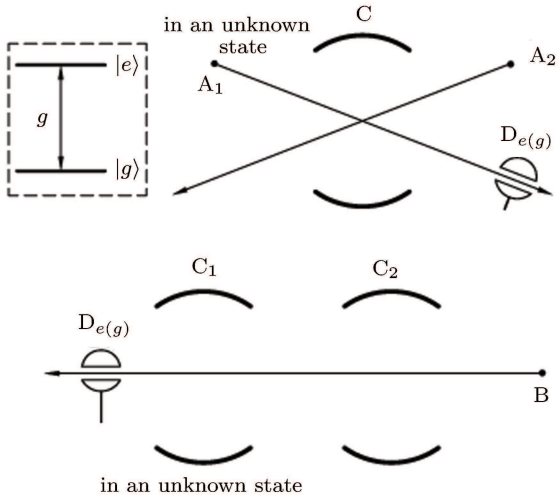


Fig.1. (a) Atomic level configuration and the setup for teleportation of an unknown atomic state. (b) The setup for teleportation of an unknown cavity field state.

We now consider the interaction of the atom with a single-mode cavity field. The relevant atomic levels $|e\rangle$ and $|g\rangle$ are circular Rydberg states with principal quantum numbers 51 and 50 respectively. The frequency of the transition $|e\rangle \leftrightarrow |g\rangle$ is at 51.1 GHz. We assume that this transition is coupled resonantly to the cavity mode with the vacuum Rabi oscillation frequency g. In the interaction picture, the interaction Hamiltonian for the atom–cavity system under the dipole and rotating wave approximation can be written as[20,24−26]

$$H = \mathrm{i}g(a^+|g\rangle\langle e| - a|e\rangle\langle g|), \tag{1}$$

where a^+ and a are creation and annihilation operators for the cavity mode respectively. Under domination of the Hamiltonian (1), the time evolution of the relevant states of the joint atom–cavity field system is given by

$$|g\rangle|n\rangle \rightarrow \cos(g\sqrt{n}\tau)|g\rangle|n\rangle - \sin(g\sqrt{n}\tau)|e\rangle|n-1\rangle, \tag{2}$$

$$|e\rangle|n-1\rangle \rightarrow \cos(g\sqrt{n}\tau)|e\rangle|n-1\rangle + \sin(g\sqrt{n}\tau)|g\rangle|n\rangle, \tag{3}$$

where $|n\rangle$ denotes the state with n photons in the cavity mode, τ is the resonant interaction time.

Assume that atom A_1 is initially in the unknown state

$$|\psi\rangle_{A_1} = c_e|e\rangle_{A_1} + c_g|g\rangle_{A_1} \tag{4}$$

with $|c_e|^2 + |c_g|^2 = 1$. Now we want to teleport such a state to atom A_2. In order to do so, we first excite atom A_2 to its higher state $|e\rangle_{A_2}$ and send it through a vacuum single-mode cavity. After an interaction time τ, the system which is composed of atom A_2 and the cavity evolves into the state

$$|\psi\rangle_{A_2-C} = \cos(g\tau)|e\rangle_{A_2}|0\rangle_C + \sin(g\tau)|g\rangle_{A_2}|1\rangle_C, \tag{5}$$

which constitutes the quantum channel for the teleportation.

Thus the whole system is in the state

$$|\psi\rangle_{A_1-A_2-C} = |\psi\rangle_{A_1} \otimes |\psi\rangle_{A_2-C} = (c_e|e\rangle_{A_1} + c_g|g\rangle_{A_1})[\cos(g\tau)|e\rangle_{A_2}|0\rangle_C + \sin(g\tau)|g\rangle_{A_2}|1\rangle_C]. \tag{6}$$

Then atom A_1 is sent through the cavity. After another interaction time τ', the whole system evolves into the state

$$\begin{aligned}|\psi'\rangle_{A_1-A_2-C} = {} & c_e\cos(g\tau)|e\rangle_{A_2}[\cos(g\tau')|e\rangle_{A_1}|0\rangle_C + \sin(g\tau')|g\rangle_{A_1}|1\rangle_C] \\ & + c_e\sin(g\tau)|g\rangle_{A_2}[\cos(\sqrt{2}g\tau')|e\rangle_{A_1}|1\rangle_C + \sin(\sqrt{2}g\tau')|g\rangle_{A_1}|2\rangle_C] + c_g\cos(g\tau)|e\rangle_{A_2}|g\rangle_{A_1}|0\rangle_C \\ & + c_g\sin(g\tau)|g\rangle_{A_2}[\cos(g\tau')|g\rangle_{A_1}|1\rangle_C - \sin(g\tau')|e\rangle_{A_1}|0\rangle_C].\end{aligned} \tag{7}$$

Now a measurement is performed on atom A_1. If this atom is found in the state $|e\rangle_{A_1}$, the system which is composed of atom A_2 and the cavity collapses onto the state

$$|\psi'\rangle_{A_2-C} = [c_e\cos(g\tau)\cos(g\tau')|e\rangle_{A_2} - c_g\sin(g\tau)\sin(g\tau')|g\rangle_{A_2}]|0\rangle_C + c_e\sin(g\tau)\cos(\sqrt{2}g\tau')|g\rangle_{A_2}|1\rangle_C. \tag{8}$$

In order to make atom A_2 get the perfect teleported state, the following two conditions i.e.

$$\cos(g\tau)\cos(g\tau') = -\sin(g\tau)\sin(g\tau'),$$
$$\cos(\sqrt{2}g\tau') = 0 \tag{9}$$

should be fulfilled. The probability of success is

$$P_{\rm a} = \cos^2(g\tau)\cos^2(g\tau') = \sin^2(g\tau')\cos^2(g\tau'), \tag{10}$$

which reaches its optimal value $P_{\rm a}^{\rm opt} = 0.2469$ when $g\tau' = 5\pi/(2\sqrt{2})$ (Here we only consider the common resonant atom–cavity interaction time). In other words, if we set $\tau = 0.8412/g$ and $\tau' = 5\pi/(2\sqrt{2}g)$, the unknown state carried by atom A_1 is faithfully teleported to atom A_2, with the probability of success being 0.2469. The cavity mode comes back to its initial vacuum state.

Now we use the idea for teleportation of an unknown cavity photon superposition state. The setup for its implementation is shown in Fig.1(b). C_1 and C_2 are two ideal millimetre-wave superconducting microwave single-mode cavities. Cavity C_1 is initially in an unknown state. Cavity C_2 is the one to receive the teleported state. Atom B (with two states $|e\rangle_{\rm B}$ and $|g\rangle_{\rm B}$) acting as the medi-atom completes the teleportation between two cavities. $D_{e(g)}$ denotes the atomic state-selective field ionization detector.

Assume that cavity C_1 is initially in the unknown state

$$|\psi\rangle_{C_1} = c_0|0\rangle_{C_1} + c_1|1\rangle_{C_1} \tag{11}$$

with $|c_0|^2 + |c_1|^2 = 1$. In order to teleport such a state to cavity C_2, we first send atom B with state $|e\rangle_{\rm B}$ through cavity C_2, which is in the vacuum state $|0\rangle_{C_2}$. After an interaction time t, the system which is composed of atom B and cavity C_2 evolves into the state

$$|\psi\rangle_{\rm B-C_2} = \cos(gt)|e\rangle_{\rm B}|0\rangle_{C_2} + \sin(gt)|g\rangle_{\rm B}|1\rangle_{C_2}. \tag{12}$$

Thus the whole system is in the state

$$|\psi\rangle_{\rm B-C_1-C_2} = |\psi\rangle_{C_1}\otimes|\psi\rangle_{\rm B-C_2} = (c_0|0\rangle_{C_1} + c_1|1\rangle_{C_1})[\cos(gt)|e\rangle_{\rm B}|0\rangle_{C_2} + \sin(gt)|g\rangle_{\rm B}|1\rangle_{C_2}]. \tag{13}$$

Then atom B is sent through cavity C_1. After another interaction time t', the whole system evolves into the state

$$\begin{aligned}|\psi'\rangle_{\rm B-C_1-C_2} = {} & c_0\cos(gt)|0\rangle_{C_2}[\cos(gt')|e\rangle_{\rm B}|0\rangle_{C_1} + \sin(gt')|g\rangle_{\rm B}|1\rangle_{C_1}] + c_0\sin(gt)|1\rangle_{C_2}|g\rangle_{\rm B}|0\rangle_{C_1} \\ & + c_1\cos(gt)|0\rangle_{C_2}[\cos(\sqrt{2}gt')|e\rangle_{\rm B}|1\rangle_{C_1} + \sin(\sqrt{2}gt')|g\rangle_{\rm B}|2\rangle_{C_1}] \\ & + c_1\sin(gt)|1\rangle_{C_2}[\cos(gt')|g\rangle_{\rm B}|1\rangle_{C_1} - \sin(gt')|e\rangle_{\rm B}|0\rangle_{C_1}].\end{aligned} \tag{14}$$

Now atom B is detected. If this atom is found in the state $|e\rangle_{\rm B}$, the system which is composed of two cavities C_1 and C_2 collapses onto the state

$$|\psi\rangle_{C_1-C_2} = [c_0\cos(gt)\cos(gt')|0\rangle_{C_2} - c_1\sin(gt)\sin(gt')|1\rangle_{C_2}]|0\rangle_{C_1} + c_1\cos(gt)\cos(\sqrt{2}gt')|0\rangle_{C_2}|1\rangle_{C_1}. \tag{15}$$

Similarly, the following two conditions

$$\cos(gt)\cos(gt') = -\sin(gt)\sin(gt'),$$
$$\cos(\sqrt{2}gt') = 0 \tag{16}$$

should be satisfied to make sure that the unknown state is faithfully teleported. The probability of success is

$$P_{\rm c} = \cos^2(gt)\cos^2(gt') = \sin^2(gt')\cos^2(gt'). \tag{17}$$

Therefore, by setting $t = 0.8412/g$ and $t' = 5\pi/(2\sqrt{2}g)$, we can faithfully teleport the unknown state from cavity C_1 to C_2, with the probability of success being 0.2469. Cavity C_1 finally remains in the vacuum state.

As we show above, with the chosen atom–cavity interaction time, the state used for the quantum channel in Eq.(5) (or Eq.(12)) is a non-maximally entangled state of the type

$$|\psi\rangle = 0.6666|10\rangle + 0.7454|01\rangle. \tag{18}$$

This is very different from the previous ones, of which

the quantum channel is a maximally entangled state. The price is that the probability of success is reduced to slightly smaller value than 1/4. In Fig.2, we show a graph of complete dependence {[$P_{\rm a}$ vs $g\tau'$], [$F_{\rm a}$ vs $g\tau'$], [$|a|$ vs $g\tau'$], [$|b|$ vs $g\tau'$]}. Where $a = \cos(g\tau) = -\sin(g\tau')$, $b = \sin(g\tau) = \cos(g\tau')$; $F_{\rm a}$ is the fidelity given by

$$\frac{P_{\rm a}}{P_{\rm a} + |c_e|^2 \cos^2(g\tau')\cos^2(\sqrt{2}g\tau')} > \frac{P_{\rm a}}{P_{\rm a} + \cos^2(g\tau')\cos^2(\sqrt{2}g\tau')}$$

(without loss of generality, only the later one is considered in Fig.2). Thus the variation of the probability of success, fidelity, and the coefficients of the quantum channel are legible, corresponding to the variation of $g\tau'$ ($0 \le g\tau' \le 2\pi$). Two real lines A and B correspond to two values $g\tau' = 7\pi/4$ and $5\pi/(2\sqrt{2}\,)$, which are chosen in Ref.[19] and the present scheme respectively. Thus small difference of $g\tau'$ makes the dissimilarity of the two schemes. That is, a maximally entangled state acts as the quantum channel for the approximate teleportation in Ref.[19], while a non-maximally entangled state acts as the quantum channel for the faithful teleportation in the present scheme.

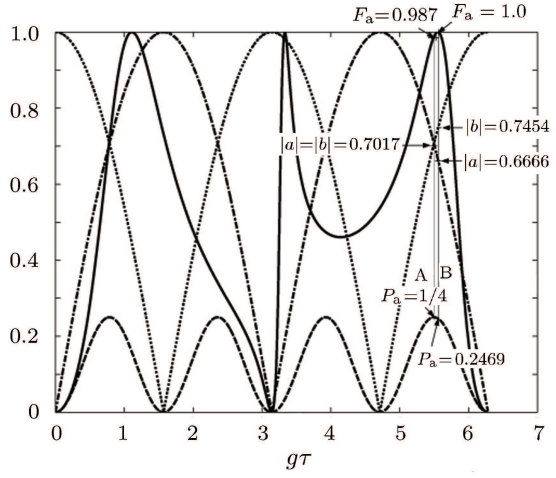


Fig.2. The variation of the probability of success, fidelity, two coefficients of the quantum channel with $g\tau'$. [$F_{\rm a}$ vs $g\tau'$]: solid line; [$P_{\rm a}$ vs $g\tau'$]: dashed line; [$|a|$ vs $g\tau'$]: dash-dot line; [$|b|$ vs $g\tau'$]: dotted line.

We now discuss the possible implementation of the proposed scheme in microwave cavity QED. Our scheme proposed here requires (i) controlled interaction time between the atom and cavity mode, (ii) no atomic spontaneous decay and negligible cavity loss during the whole teleportation process, and (iii) detection of the atom in required state. In our teleportation scheme, the interaction time for two atoms (A_1 and A_2) with the cavity or atom B with two cavities (C_1 and C_2) is different. This can be achieved by choosing a fixed velocity for the atoms and applying Stark effect to make atomic transition close to cavity resonance at preset times. The atomic velocity may be fixed by a combination of pulsed Doppler-selective optical pumping and time-of-flight selection.[20] Setting atom–cavity coupling constant $g = 2\pi \times 25$ kHz,[27] we have $\tau = t = (0.8412/g) \simeq 5.4$ μs, $\tau' = t' = (5\pi/(2\sqrt{2}g)) \simeq 35.4$ μs. If we set the velocity of the atoms to be $v = \sqrt{\pi}w/\tau'$ (the length for the interaction effective zone is $l \sim \sqrt{\pi}w$, waist $w = 6$ mm), thus Stark effect must be applied, to switch the atomic transition away from the cavity resonance for a distance of $0.8475l$ when atom A_2 passes through the cavity or atom B passes through cavity C_2. In the above, we have ignored atomic spontaneous emission and cavity decay. This assumption is valid under the condition $T_{\rm at}$, $T_{\rm cav} \gg T$, where $T_{\rm at}$ and $T_{\rm cav}$ are the atomic lifetime, and cavity photon mean lifetime, respectively, T is the total time required in the teleportation process. For Rydberg atomic levels and superconducting microwave cavities,[20,27] the required condition can be satisfied, as $T_{\rm at} = 30$ ms and $T_{\rm cav} = 130$ ms (corresponding to quality factor $Q = 4.2 \times 10^{10}$ and finesse 4.6×10^9, if the cavity is cooling to the temperature 0.8 K), which are much greater than $T \simeq 20\tau' = 708$ μs (considering the travelling time of the atoms during the whole teleportation process).

To summarize, based on the single-photon JC model, an unknown atomic state and cavity photon superposition state can be faithfully teleported with only a single measurement. The scheme is very different from the previous ones of using a maximally entangled state as quantum channel. The scheme is probabilistic, its success lies on the event that the atom carrying the unknown state (or the medi-atom, for teleportation of cavity field state) is detected in the higher state. The brief discussion on the experimental matter shows that the present scheme is within the scope of what the scheme can be realized in the recent microwave cavity QED technique.

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