Vol 16 No 11, November 2007
1009-1963/2007/16(11)/3382-04

Chinese Physics

© 2007 Chin. Phys. Soc.
and IOP Publishing Ltd

A scheme for the implementation of unconventional geometric phase gates with trapped ions*

Xie Hong(谢 鸿), Li Hong-Cai(李洪才)†, Yang Rong-Can(杨榕灿),
Lin Xiu(林 秀), and Huang Zhi-Ping(黄志平)

School of Physics and Optoelectronics Technology, Fujian Normal University, Fuzhou 350007, *China*

(Received 28 December 2006; revised manuscript received 6 March 2007)

We propose a scheme for the realization of unconventional geometric two-qubit phase gates with two identical two-level ions. In the present scheme, the two ions are simultaneously illuminated by a standing-wave laser pulse with its pulse frequency being tuned to the ionic transition. The gate operation time can be much shorter, making the system robust against decoherence. In addition, we choose the appropriate experimental parameters to construct the geometric phase gate in one step, and thus avoid implementing the pure geometric single qubit operation.

Keywords: unconventional geometric phase gate, trapped ions, two-level ions
PACC: 4250

1. Introduction

It has been shown that the basic building blocks of a quantum computer are quantum logic gates, i.e. any quantum computation can be implemented by a series of universal two-qubit logic gates and one-qubit local operators. This discovery motivates an intensive research into quantum logic gates such as dynamic quantum logic gates and geometric quantum logic gates. So far, most of the quantum information processings and quantum computations are based on dynamic quantum logic gates. However, dynamic quantum phase gates are very sensitive to parameter fluctuation, which is a blockage to large-scale quantum computation. In contrast, the geometric phase gates depends only on the area of the path that the state takes in Hilbert space, and does not depend on the details of the path,[1,2] thus it may be fault-tolerant and possess high fidelity.[3]

There are two kinds of geometric phase gates: conventional geometric quantum phase gates[2,4−6] and unconventional ones.[7−11] The conventional phase gate can be constructed by driving the qubits to undergo appropriate adiabatic cyclic evolutions and they require additional operations to cancel the effects from the corresponding dynamical phases, which may result in additional errors. While the unconventional ones can be obtained by displacing the qubits along a closed path in the phase space and they do not require additional operations to cancel the effects from the dynamical phases. This is because the dynamic phase created in the whole procedure is proportional to the geometric phase, leading the total phase to be dependent only on global geometric features. Thus, the unconventional geometric gate possesses all advantages owned by the conventional geometric gate.

Until now, there are numerous schemes for the implementation of the unconventional geometric gate proposed in cavity QED system[8,10−13] and in the trap-ion system.[3,7,9] In this paper, we propose an alternative scheme for implementing geometric phase gate with two trapped ions. Compared with the schemes depicted in Ref.[9–11], our scheme only use two-level ions as memory, and the laser beam is tuned to the ionic transition. Furthermore, we choose the appropriate experimental parameters to construct the geometric phase gate without implementing the pure geometric single qubit operation, since it is much difficult than the common single qubit operation.[7] Thus in our scheme the experimental procedure is quite simple.

The unconventional geometric phase gate can be constructed by displacing the qubit along a closed loop in the phase space. The displacement operator is expressed as

$$D(\alpha) = \mathrm{e}^{\alpha a^{+} - \alpha^{*} a}, \tag{1}$$

*Project supported by the National Natural Science Foundation of China (Grant No 10574022), and the Funds of the Natural Science of Fujian Province, China (Grant Nos Z0512006 and A0210014).
†Corresponding author. E-mail: hcli45@fjnu.edu.cn

where a^+ and a are the creation and annihilation operators of the harmonic oscillator, respectively. The displacement operator satisfies

$$D(\beta)D(\alpha) = D(\alpha+\beta)\mathrm{e}^{\mathrm{i}\mathrm{Im}(\beta\alpha^*)}. \tag{2}$$

Considering a path consisting of N short straight sections $\Delta\alpha_j$, the total operation is

$$\begin{aligned} D_{\mathrm{total}} &= D(\Delta\alpha_N)\mathrm{L}\cdots D(\Delta\alpha_1) \\ &= D\Big(\sum_{j=1}^{N}\Delta\alpha_j\Big)\exp\Big[\mathrm{i}\mathrm{Im}\Big(\sum_{j=2}^{N}\Delta\alpha_j\sum_{k=1}^{j-1}\Delta\alpha_k^*\Big)\Big]. \end{aligned} \tag{3}$$

Since an arbitrary path γ can be obtained in the limit $N\to\infty$, we have

$$D_{\mathrm{total}} = D\Big(\int_\gamma \mathrm{d}\alpha\Big)\mathrm{e}^{\mathrm{i}\Theta}, \tag{4}$$

where $\Theta = \mathrm{Im}(\int_\gamma \alpha^*\mathrm{d}\alpha)$.
For a closed path we obtain

$$D_{\mathrm{total}} = D(0)\mathrm{e}^{\mathrm{i}\Theta} = \mathrm{e}^{\mathrm{i}\Theta}, \tag{5}$$

where the phase operator Θ is determined by the area of the loop in the phase space.

2. Implementation of unconventional geometric phase gates

We now consider two identical two-level ions with the excited state $|e\rangle$ and the ground state $|g\rangle$ confined in a potential trap. The two ions are driven by a standing-wave laser beam tuned to the ionic transition. In the rotating-wave approximation, the Hamiltonian for this system is given by [14]

$$\begin{aligned} H &= H_0 + H_{\mathrm{int}}, \\ H_0 &= \nu a^\dagger a + \frac{1}{2}\sum_{j=1}^{2}\omega_0\sigma_{z,j}, \\ H_{\mathrm{int}} &= \frac{\Omega}{2}\sum_{j=1}^{2}\{\sigma_j^+\sin[\eta(a+a^\dagger)+\phi] \\ &\quad\times \mathrm{e}^{-\mathrm{i}\omega_0 t} + \mathrm{H.c.}\}, \end{aligned} \tag{6}$$

where $\sigma_{z,j} = (|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)$, $\sigma_j^+ = |e_j\rangle\langle g_j|$ and $\sigma_j^- = |g_j\rangle\langle e_j|$; ω_0 is the transition frequency for the two-level ion, ν is the vibrational frequency of the centre-of-mass mode, Ω and ϕ are the Rabi frequency and the phase of the laser field respectively, and η is Lamb–Dicke parameter. We can rewrite the interaction term as

$$\begin{aligned} H_{\mathrm{int}} = \frac{\Omega}{2}\sum_{j=1}^{2}\{&\sigma_j^+[\sin\eta(a+a^\dagger)\cos\phi \\ &+ \cos\eta(a+a^\dagger)\sin\phi]\mathrm{e}^{-\mathrm{i}\omega_0 t} + \mathrm{H.c.}\}. \end{aligned} \tag{7}$$

In the Lamb–Dicke regime ($\eta\sqrt{\overline{n}+1}\ll 1$), where $\overline{n}$ is the mean phonon number, we can retain terms only to the first order in η.

$$\begin{aligned} H_{\mathrm{int}} = \frac{\Omega}{2}\sum_{j=1}^{2}\{&\sigma_j^+[\eta(a+a^\dagger)\cos\phi \\ &+ \sin\phi]\mathrm{e}^{-\mathrm{i}\omega_0 t} + \mathrm{H.c.}\}. \end{aligned}$$

Then, in the interaction picture the interaction Hamiltonian is given by

$$\begin{aligned} H_{\mathrm{I}} &= \mathrm{e}^{\mathrm{i}H_0 t}H_{\mathrm{int}}\mathrm{e}^{-\mathrm{i}H_0 t} \\ &= \Omega\sum_{j=1}^{2}\sigma_{x,j}[\eta(a\mathrm{e}^{-\mathrm{i}\nu t} + a^\dagger\mathrm{e}^{\mathrm{i}\nu t})\cos\phi + \sin\phi], \end{aligned} \tag{8}$$

where $\sigma_{x,j} = (\sigma_j^+ + \sigma_j^-)/2$.

We choose the superposition states $|\pm\rangle_j$ of the ions as qubits, where $|\pm\rangle_j = (1/\sqrt{2})(|e\rangle_j \pm |g\rangle_j)$. It is easy to check that the states $|\pm\rangle_j$ satisfies $\sigma_{x,j}|\pm\rangle_j = \pm(1/2)|\pm\rangle_j$.

During the infinitesimal time interval $[t, t+\mathrm{d}t]$, the evolution operator is

$$\begin{aligned} \mathrm{d}U &= \mathrm{e}^{-\mathrm{i}H_{\mathrm{i}}\mathrm{d}t} \\ &= \mathrm{e}^{-\mathrm{i}\Omega}\sum_{j=1}^{2}[\eta(a\mathrm{e}^{-\mathrm{i}\nu t} + a^\dagger\mathrm{e}^{\mathrm{i}\nu t})\cos\phi + \sin\phi]\sigma_{x,j}\mathrm{d}t. \end{aligned} \tag{9}$$

This leads to

$$\begin{aligned} \mathrm{d}U|+\rangle_j &= \mathrm{e}^{\mathrm{i}\mathrm{d}\theta_1}\mathrm{e}^{a^\dagger\mathrm{d}\alpha_1 - a\mathrm{d}\alpha_1^*}|+\rangle_j; \\ \mathrm{d}U|-\rangle_j &= \mathrm{e}^{\mathrm{i}\mathrm{d}\theta_2}\mathrm{e}^{a^\dagger\mathrm{d}\alpha_2 - a\mathrm{d}\alpha_2^*}|-\rangle_j, \end{aligned} \tag{10}$$

where

$$\begin{aligned} \mathrm{d}\theta_1 &= -\frac{\Omega}{2}\sin\phi\mathrm{d}t, \\ \mathrm{d}\alpha_1 &= -\frac{\mathrm{i}}{2}\Omega\eta\cos\phi\mathrm{e}^{\mathrm{i}\nu t}\mathrm{d}t; \\ \mathrm{d}\theta_2 &= \frac{\Omega}{2}\sin\phi\mathrm{d}t, \\ \mathrm{d}\alpha_2 &= \frac{\mathrm{i}}{2}\Omega\eta\cos\phi\mathrm{e}^{\mathrm{i}\nu t}\mathrm{d}t. \end{aligned} \tag{11}$$

Thus, the different initial system states will evolve as

$$\begin{aligned} |+_1\rangle|+_2\rangle|\phi_{\mathrm{v}}(0)\rangle \to{}& \mathrm{e}^{2\mathrm{i}\mathrm{d}\theta_1}D(2\mathrm{d}\alpha_1) \\ &\times|+_1\rangle|+_2\rangle|\phi_{\mathrm{v}}(0)\rangle, \end{aligned} \tag{12a}$$

$$|+_1\rangle|-_2\rangle|\phi_{\mathrm{v}}(0)\rangle \to |+_1\rangle|-_2\rangle|\phi_{\mathrm{v}}(0)\rangle, \tag{12b}$$

$$|-_1\rangle|+_2\rangle|\phi_{\mathrm{v}}(0)\rangle \to |-_1\rangle|+_2\rangle|\phi_{\mathrm{v}}(0)\rangle, \tag{12c}$$

$$|-_1\rangle|-_2\rangle|\phi_{\rm v}(0)\rangle \to {\rm e}^{2{\rm i}{\rm d}\theta_2}D(2{\rm d}\alpha_2) \times |-_1\rangle|-_2\rangle|\phi_{\rm v}(0)\rangle, \tag{12d}$$

where $|\phi_{\rm v}(0)\rangle$ is the initial state of the vibrational mode. After an interaction time t, the system states becomes

$$|+_1\rangle|+_2\rangle|\phi_{\rm v}(0)\rangle \to {\rm e}^{{\rm i}\theta_1(t)}{\rm e}^{{\rm i}\Theta_1}D(2\alpha_1) \times |+_1\rangle|+_2\rangle|\phi_{\rm v}(0)\rangle, \tag{13a}$$

$$|+_1\rangle|-_2\rangle|\phi_{\rm v}(0)\rangle \to |+_1\rangle|-_2\rangle|\phi_{\rm v}(0)\rangle, \tag{13b}$$

$$|-_1\rangle|+_2\rangle|\phi_{\rm v}(0)\rangle \to |-_1\rangle|+_2\rangle|\phi_{\rm v}(0)\rangle, \tag{13c}$$

$$|-_1\rangle|-_2\rangle|\phi_{\rm v}(0)\rangle \to {\rm e}^{{\rm i}\theta_2(t)}{\rm e}^{{\rm i}\Theta_2}D(2\alpha_2) \times |-_1\rangle|-_2\rangle|\phi_{\rm v}(0)\rangle, \tag{13d}$$

where

$$\theta_1(t) = -\theta_2(t) = 2\int_0^t {\rm d}\theta_1 = -\Omega\sin(\phi)t,$$

$$\alpha_1 = -\alpha_2 = \int {\rm d}\alpha_1 = \int_0^t \Big(-\frac{\rm i}{2}\Omega\eta\cos\phi{\rm e}^{{\rm i}\nu t}\Big){\rm d}t = -\frac{\Omega\eta}{2\nu}({\rm e}^{{\rm i}\nu t}-1)\cos\phi,$$

$$\Theta_1 = \Theta_2 = {\rm Im}\Big(\int_l 2\alpha_1^*{\rm d}(2\alpha_1)\Big) = 4{\rm Im}\Big[\int_0^t \frac{\Omega\eta}{2\nu}({\rm e}^{-{\rm i}\nu t}-1)\cos\phi\Big(-\frac{\rm i}{2}\Omega\eta\cos\phi{\rm e}^{{\rm i}\nu t}\Big){\rm d}t\Big]$$
$$= \frac{(\Omega\eta)^2}{\nu}\Big[t-\frac{1}{\nu}\sin(\nu t)\Big]\cos^2\phi.$$

In the case of $\nu t = 2\pi$, $\alpha_1 = \alpha_2 = 0$, which means that the displacement is along a closed path and the vibrational mode returns to its initial state. Then we obtain the evolution of the internal states

$$|+_1\rangle|+_2\rangle \to {\rm e}^{-{\rm i}\Omega\sin(\phi)t}{\rm e}^{{\rm i}\Theta_1}|+_1\rangle|+_2\rangle, \tag{14a}$$

$$|+_1\rangle|-_2\rangle \to |+_1\rangle|-_2\rangle, \tag{14b}$$

$$|-_1\rangle|+_2\rangle \to |-_1\rangle|+_2\rangle, \tag{14c}$$

$$|-_1\rangle|-_2\rangle \to {\rm e}^{{\rm i}\Omega\sin(\phi)t}{\rm e}^{{\rm i}\Theta_1}|-_1\rangle|-_2\rangle, \tag{14d}$$

where we have eliminated the state of the vibrational mode.

With the choice of $\Omega\sin(\phi)t = \Theta_1$, the evolution becomes

$$|+_1\rangle|+_2\rangle \to |+_1\rangle|+_2\rangle, \tag{15a}$$

$$|+_1\rangle|-_2\rangle \to |+_1\rangle|-_2\rangle, \tag{15b}$$

$$|-_1\rangle|+_2\rangle \to |-_1\rangle|+_2\rangle, \tag{15c}$$

$$|-_1\rangle|-_2\rangle \to {\rm e}^{2{\rm i}\Theta_1}|-_1\rangle|-_2\rangle, \tag{15d}$$

which stands for a controlled phase operation. If and only if both ions are in the state $|-_1\rangle|-_2\rangle$, the system obtains a phase $2\Theta_1$. When $2\Theta_1 = \pi$, we obtain a π-phase gate.[15,16] This condition can be satisfied with the choice of $\Omega = \nu\tan\phi/(\eta^2\cos\phi)$ and $\eta = 2\tan\phi$. It is noted that the geometric π-phase gate can be applied to implement quantum computation such as Deutsch–Jozsa algorithm[17] and Grover search algorithm,[18] or to generate the cluster states.[19]

3. Discussion and conclusion

Now we give a brief discussion of the experimental data. According to Ref.[20], the $S_{1/2}$ ground state and the metastable $D_{5/2}$ state of the $^{40}Ca^+$ ions can represent the qubits. Since the lifetime of the metastable state is very long, we can neglect the ionic spontaneous emission. Let $\eta = 0.01$, $\overline{n} = 1$, $\phi = 0.1\pi$, $\Omega = \nu\tan\phi/(\eta^2\cos\phi)$. Then the Lamb–Dicke limit $\eta\sqrt{\overline{n}+1} \ll 1$ and the condition $\eta = 2\tan\phi$ can be satisfied. In addition, the decoherence rate for a one-phonon state of a trapped ion is about $\gamma_0 = 10^{-3}\nu/(2\pi)$,[21] and the corresponding decoherence time for a one-phonon state is $T_0 = 1/\gamma_0 = 2\pi\times10^3/\nu$, much longer than the interaction time $t = 2\pi/v$. Thus, our scheme may be feasible based on current technologies.

In conclusion, we have proposed an alternative scheme for the implementation of unconventional geometric gate with trapped ions. In the present protocol, we use two-level ions as memory to remove the possible existence of spectator internal atomic levels, which can act as potential 'leaks' from the two levels spanned by the qubits.[22] The two ions are simultaneously il-

luminated by a standing-wave laser pulse. Because the laser beam is tuned to ionic transition frequency, the gate speed is much faster and the system is robust against decoherence. Moreover, we can realize the geometric phase gate without implementing any pure geometric single qubit operation. Although the intensity of the laser in our scheme must be large, it can be obtained using current technologies. Therefore, our scheme of implementing the unconventional geometric phase gates may be applicable in practice.

References

[1] Berry M V 1984 *Proc. R. Soc.* London A **392** 45

[2] Aharonoy Y and Anandan J 1987 *Phys. Rev. Lett.* **58** 1593

[3] Jones J A, Vedral V, Ekert A and Castagnoli G 2000 *Nature* (London) **403** 869

[4] Zanardi P and Rasetti M 1999 *Phys. Lett.* A **264** 94

[5] Leibfried D, DeMarco B, Meyer V, Lucas D, Barrett M, Britton J, Itano W M, Jelenkovic B, Langer C, Rosenband T and Wineland D J 2003 *Nature* **422** 412

[6] Falci G, Fazio R, Palma G M, Siewert J and Vedral V 2002 *Nature* (London) **407** 355

[7] Duan L M, Cirac J I and Zoller P 2001 *Science* **292** 1695

[8] Wang X B and Keiji M 2001 *Phys. Rev. Lett.* **87** 097901

[9] Zhu S L and Wand Z D 2003 *Phys. Rev. Lett.* **91** 187902

[10] Zheng S B 2004 *Phys. Rev.* A **70** 052320

[11] Zheng S B 2006 *Phys. Rev.* A **74** 032322

[12] Chen C Y, Feng M, Zhang X L and Gao K L 2006 *Phys. Rev.* A **73** 032344

[13] Feng X L, Wang Z S, Wu C F and Oh C H 2006 quant-ph/0606098

[14] Zheng S B 2003 *Phys. Rev. Lett.* **90** 217901

[15] Chen M L and Wang S J 2006 *Acta Phys. Sin.* **55** 4638 (in Chinese)

[16] Cai J W, Mao F, Liao X P and Zheng X J 2006 *Chin. Phys.* **15** 2518

[17] Yang R C, Li H C, Lin X and Chen M X 2006 *Chin. Phys.* **15** 2320

[18] Zheng S B 2005 *Chin. Phys.* **14** 2222

[19] Yang R C, Li H C, Chen M X and Lin X 2006 *Chin. Phys.* **15** 2315

[20] Meejhof D M, Monroe C, King B E, Itano W M and Wineland D J 1996 *Phys. Rev. Lett.* **76** 1796

[21] Schmidt-Kaler F, Häffner H, Riebe M, Gulde S, Lancaster G P T, Deuschle T, Becher C, Roos C F, Eschner J and Blatt R 2003 *Nature* (London) **422** 408

[22] Monroe C, Leibfried D, King B E, Meekhof D F, Itano W M and Wineland D J 1997 *Phys. Rev.* A **55** 2489