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Structural polarization properties of vector Gaussian beam in the far field*

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Based on the vector angular spectrum representation of optical beam and the method of stationary phase, the analytical TE and TM terms of vector Gaussian beam have been presented in the far field. By using the local polarization matrix, the polarization properties of the TE and TM terms in the far field are investigated, and it is found that the degree of their polarization is only determined by the spatial location. When the source is completely polarized, the TE and TM terms are both completely polarized in the far field. When the source is completely unpolarized, the TE and TM terms in the far field are partially polarized. The whole beam is also partially polarized except on the propagating axis. Moreover, the degrees of polarization of TE and TM terms are both larger than that of the whole beam.

Keywords: vector Gaussian beam, vectorial structure, local polarization matrix, degree of polarization

PACC: 4225B, 4225J

1. Introduction

Within the paraxial regime, the longitudinal electric field of optical beam vanishes, and that is no restriction within the two transverse electric fields of optical beam. In the non-paraxial regime, the longitudinal electric field cannot be neglected and given by the transversality condition of the electric field. Moreover, non-paraxial optical beam must satisfy the Maxwell's equations. As the Maxwell equations can be separated into the transverse and longitudinal field equations, an arbitrary optical beam as a solution of the Maxwell equations is essentially composed of two terms.[1−4] One is the TE term with the electric field transverse to the propagation axis, and the other is the TM term with the associated magnetic field transverse to the propagation axis. As the transversality condition of the electric field should be obeyed and the polarized direction of every plane wave component must be perpendicular to its own wave vector, the decomposition of optical beam into the TE and TM terms is unique. Therefore, the transversality condition will impact on the polarization properties of non-paraxial optical beam, its TE and TM terms. Although the polarization of partially coherent optical beams has been extensively investigated, it is concentrated on the whole beam.[5−7] As the TE and TM terms are orthogonal to each other in the far field and most practical applications of laser beams are involved in the far field, the analytical TE and TM terms of vector Gaussian beam in the far field are presented by using the vector angular spectrum representation and the method of stationary phase. By means of the local polarization matrix, the polarization properties of vector Gaussian beam, its TE and TM terms, in the far field are studied in this paper.

2. The degrees of polarization

In the Cartesian coordinate system, the z-axis is taken to be the propagation axis. The half space $z \geq 0$ is filled with a homogeneous, isotropic, nonconducting and transparent medium with electric permittivity ε and magnetic permeability μ. A vector Gaussian beam propagates toward the half space $z \geq 0$, and the initial transverse electric fields of vector Gaussian beam in the source plane are stationary, ergodic and quasi-monochromatic. The transverse electric fields of vector Gaussian beam in the source plane $z = 0$ are given by

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$$\begin{pmatrix} E_x(\boldsymbol{\rho},0;t) \\ E_y(\boldsymbol{\rho},0;t) \end{pmatrix} = \exp\left(-\frac{\rho^2}{w_0^2}\right) \begin{pmatrix} \exp\{\mathrm{i}[\phi_x(t)-\omega t]\} \\ \exp\{\mathrm{i}[\phi_y(t)-\omega t]\} \end{pmatrix}, \quad (1)$$

where $\boldsymbol{\rho} = x\boldsymbol{i} + y\boldsymbol{j}$ with $\rho = (x^2+y^2)^{1/2}$; w_0 is the beam waist width; $\phi_x(t)$ and $\phi_y(t)$ are the phases in the source plane; ω is the angular frequency. By using a vector angular spectrum method,[8,9] the propagating electric field of vector Gaussian beam is found to be

$$\boldsymbol{E}(\boldsymbol{\rho},z;t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \boldsymbol{A}(p,q)\exp[\mathrm{i}k(px+qy+\gamma z)]\mathrm{d}p\mathrm{d}q \quad (2)$$

with

$$\boldsymbol{A}(p,q) = A_x\left(\boldsymbol{i} - \frac{p}{\gamma}\boldsymbol{k}\right) + A_y\left(\boldsymbol{j} - \frac{q}{\gamma}\boldsymbol{k}\right), \quad (3)$$

where $\gamma = (1-p^2-q^2)^{1/2}$; $k = 2\pi/\lambda$ is the wave number with $\lambda = 2\pi/\omega(\mu\varepsilon)^{1/2}$ being the light wavelength in the medium; $\boldsymbol{A}(p,q)$ is the vector angular spectrum; A_x and A_y are the transverse components of the vector angular spectrum. The longitudinal electric field in Eq.(2) stems from the transversality condition of electric field. A_x is given by the Fourier transform of the x-component of the electric field in the source plane

$$A_x = \frac{1}{\lambda^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_x(\boldsymbol{\rho},0;t)\exp[-\mathrm{i}k(px+qy)]\mathrm{d}x\mathrm{d}y = \frac{\exp\{\mathrm{i}\phi_x(t)-\omega t]\}}{4\pi f^2}\exp\left(-\frac{p^2+q^2}{4f^2}\right), \quad (4)$$

where $f = 1/kw_0$. Similarly, A_y is

$$A_y = \frac{\exp\{\mathrm{i}[\phi_y(t)-\omega t]\}}{4\pi f^2}\exp\left(-\frac{p^2+q^2}{4f^2}\right). \quad (5)$$

According to the vectorial structure of electromagnetic beam,[1−3] three unit vectors $\boldsymbol{s}$, $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ can be defined as follows:

$$\begin{aligned} \boldsymbol{s} &= p\boldsymbol{i} + q\boldsymbol{j} + \gamma\boldsymbol{k}, \\ \boldsymbol{e}_1 &= \frac{q}{(p^2+q^2)^{1/2}}\boldsymbol{i} - \frac{p}{(p^2+q^2)^{1/2}}\boldsymbol{j}, \\ \boldsymbol{e}_2 &= \frac{p\gamma}{(p^2+q^2)^{1/2}}\boldsymbol{i} + \frac{q\gamma}{(p^2+q^2)^{1/2}}\boldsymbol{j} \\ &\quad -(p^2+q^2)^{1/2}\boldsymbol{k}. \end{aligned} \quad (6)$$

Thus, the three unit vectors $\boldsymbol{s}$, $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ form a mutually perpendicular right–handed system

$$\boldsymbol{s}\times\boldsymbol{e}_1 = \boldsymbol{e}_2, \quad \boldsymbol{e}_1\times\boldsymbol{e}_2 = \boldsymbol{s}, \quad \boldsymbol{e}_2\times\boldsymbol{s} = \boldsymbol{e}_1. \quad (7)$$

In this system, the vector angular spectrum $\boldsymbol{A}(p,q)$ can be decomposed into the two terms

$$\boldsymbol{A}(p,q) = [\boldsymbol{A}(p,q)\cdot\boldsymbol{e}_1]\boldsymbol{e}_1 + [\boldsymbol{A}(p,q)\cdot\boldsymbol{e}_2]\boldsymbol{e}_2 = (qA_x - pA_y)\left(\frac{q\boldsymbol{i}-p\boldsymbol{j}}{p^2+q^2}\right) + (pA_x+qA_y)\left(\frac{p\boldsymbol{i}+q\boldsymbol{j}}{p^2+q^2} - \frac{\boldsymbol{k}}{\gamma}\right), \quad (8)$$

where the dot denotes scalar product. Accordingly, the propagating electric field of vector Gaussian beam can be decomposed into the TE and TM terms

$$\boldsymbol{E}(\boldsymbol{\rho},z;t) = \boldsymbol{E}_{\mathrm{TE}}(\boldsymbol{\rho},z;t) + \boldsymbol{E}_{\mathrm{TM}}(\boldsymbol{\rho},z;t) \quad (9)$$

with

$$\boldsymbol{E}_{\mathrm{TE}}(\boldsymbol{\rho},z;t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(qA_x - pA_y)\left(\frac{q\boldsymbol{i}-p\boldsymbol{j}}{p^2+q^2}\right)\exp[\mathrm{i}k(px+qy+\gamma z)]\mathrm{d}p\mathrm{d}q, \quad (10)$$

$$\boldsymbol{E}_{\mathrm{TM}}(\boldsymbol{\rho},z;t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(pA_x + qA_y)\left(\frac{p\boldsymbol{i}+q\boldsymbol{j}}{p^2+q^2} - \frac{\boldsymbol{k}}{\gamma}\right)\exp[\mathrm{i}k(px+qy+\gamma z)]\mathrm{d}p\mathrm{d}q. \quad (11)$$

As the condition $kr = k(\rho^2+z^2)^{1/2} \to \infty$ is satisfied in the far field, the method of stationary phase is applicable. By using the method of stationary phase,[10,11] the analytical TE term of vector Gaussian beam in the far field is obtained as follows:

$$\boldsymbol{E}_{\mathrm{TE}}(\boldsymbol{\rho},z;t) = -\frac{\mathrm{i}z_r z}{\rho^2 r^2}\exp\left(-\frac{\rho^2}{4f^2r^2} + \mathrm{i}(kr-\omega t)\right)\{y\exp[\mathrm{i}\phi_x(t)] - x\exp[\mathrm{i}\phi_y(t)]\}(y\boldsymbol{i} - x\boldsymbol{j}), \quad (12)$$

where $z_r = kw_0^2/2$ is the confocal parameter. Similarly, the analytical TM term in the far field is acquired as

$$\boldsymbol{E}_{\mathrm{TM}}(\boldsymbol{\rho}, z; t) = -\frac{\mathrm{i}z_r}{\rho^2 r^2} \exp\left(-\frac{\rho^2}{4f^2 r^2} + \mathrm{i}(kr - \omega t)\right) \{x \exp[\mathrm{i}\phi_x(t)] + y \exp[\mathrm{i}\phi_y(t)]\}(xz\boldsymbol{i} + yz\boldsymbol{j} - \rho^2 \boldsymbol{k}). \tag{13}$$

Now, we investigate the structural polarization properties in the far field. The TE term in the far field can be regarded as a sum of two spatial electric vectors

$$\boldsymbol{E}_{\mathrm{TE}}(\boldsymbol{\rho}, z; t) = \boldsymbol{E}_{\mathrm{TE1}}(\boldsymbol{\rho}, z; t) + \boldsymbol{E}_{\mathrm{TE2}}(\boldsymbol{\rho}, z; t) \tag{14}$$

with

$$\boldsymbol{E}_{\mathrm{TE1}}(\boldsymbol{\rho}, z; t) = -\frac{\mathrm{i}z_r zy\{y \exp[\mathrm{i}\phi_x(t)] - x \exp[\mathrm{i}\phi_y(t)]\}}{\rho^2 r^2} \exp\left(-\frac{\rho^2}{4f^2 r^2} + \mathrm{i}(kr - \omega t)\right) \left(\boldsymbol{i} - \frac{x}{z}\boldsymbol{k}\right), \tag{15}$$

$$\boldsymbol{E}_{\mathrm{TE2}}(\boldsymbol{\rho}, z; t) = \frac{\mathrm{i}z_r zx\{y \exp[\mathrm{i}\phi_x(t)] - x \exp[\mathrm{i}\phi_y(t)]\}}{\rho^2 r^2} \exp\left(-\frac{\rho^2}{4f^2 r^2} + \mathrm{i}(kr - \omega t)\right) \left(\boldsymbol{j} - \frac{y}{z}\boldsymbol{k}\right). \tag{16}$$

The longitudinal components of $\boldsymbol{E}_{\mathrm{TE1}}(\boldsymbol{\rho}, z; t)$ and $\boldsymbol{E}_{\mathrm{TE2}}(\boldsymbol{\rho}, z; t)$ are related to their corresponding transverse components. The decomposition of the TE term is shown in Fig.1. The angle between the two spatial electric vectors of $\boldsymbol{E}_{\mathrm{TE1}}(\boldsymbol{\rho}, z; t)$ and $\boldsymbol{E}_{\mathrm{TE2}}(\boldsymbol{\rho}, z; t)$ is

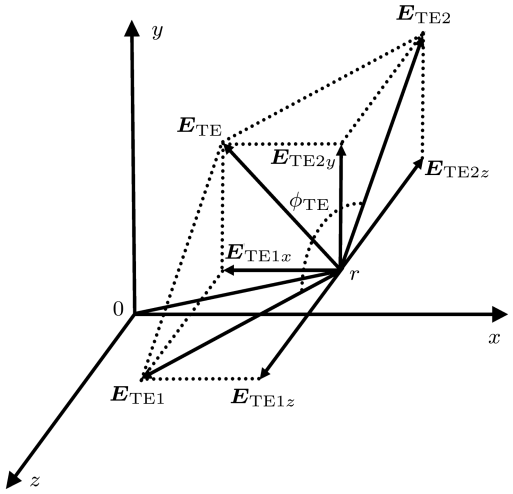


Fig.1. Scheme of the decomposition of the TE term.

$$\begin{aligned}\phi_{\mathrm{TE}} &= \pi - \arccos\left(\frac{|\boldsymbol{E}_{\mathrm{TE1}}|^2 + |\boldsymbol{E}_{\mathrm{TE2}}|^2 - |\boldsymbol{E}_{\mathrm{TE}}|^2}{2\,|\boldsymbol{E}_{\mathrm{TE1}}|\,|\boldsymbol{E}_{\mathrm{TE2}}|}\right) \\ &= \pi - \arccos\left(\frac{xy}{\sqrt{bc}}\right),\end{aligned} \tag{17}$$

where

$$b = z^2 + x^2, \quad \text{and} \quad c = z^2 + y^2. \tag{18}$$

In the far field, the degree of polarization is[5,10,12]

$$P_{\mathrm{TE}}(\boldsymbol{\rho}, z) = \left(1 - \frac{4\mathrm{Det}J_{\mathrm{TE}}(\boldsymbol{\rho}, \boldsymbol{\rho}, z)}{[\mathrm{Tr}J_{\mathrm{TE}}(\boldsymbol{\rho}, \boldsymbol{\rho}, z)]^2}\right)^{1/2}, \tag{19}$$

where Det and Tr denote the determinant and the trace of the matrix, respectively. The 2×2 local polarization matrix $J_{\mathrm{TE}}(\boldsymbol{\rho}, \rho, z)$ is given by

$$J_{\mathrm{TE}}(\boldsymbol{\rho}, \boldsymbol{\rho}, z) = \begin{bmatrix} < \boldsymbol{E}^*_{\mathrm{TE1}}(\boldsymbol{\rho}, z; t)\boldsymbol{E}_{\mathrm{TE1}}(\boldsymbol{\rho}, z; t) > & < \boldsymbol{E}^*_{\mathrm{TE1}}(\boldsymbol{\rho}, z; t)\boldsymbol{E}_{\mathrm{TE2}}(\boldsymbol{\rho}, z; t) > \\ < \boldsymbol{E}^*_{\mathrm{TE2}}(\boldsymbol{\rho}, z; t)\boldsymbol{E}_{\mathrm{TE1}}(\boldsymbol{\rho}, z; t) > & < \boldsymbol{E}^*_{\mathrm{TE2}}(\boldsymbol{\rho}, z; t)\boldsymbol{E}_{\mathrm{TE2}}(\boldsymbol{\rho}, z; t) > \end{bmatrix}, \tag{20}$$

where the asterisk means the complex conjugation, and the angle brackets is referred to an average with respect to the time variable t. When the source of vector Gaussian beam is completely polarized, namely the difference between $\phi_x(t)$ and $\phi_y(t)$ is constant, the local polarization matrix of TE term in the far field becomes

$$J_{\mathrm{TE}}(\boldsymbol{\rho}, \boldsymbol{\rho}, z) = H_0 \begin{bmatrix} by^2 - 2\sqrt{bc}xy\cos\phi_{\mathrm{TE}} + cx^2\cos^2\phi_{\mathrm{TE}} & -\sqrt{bc}xy\sin\phi_{\mathrm{TE}} + \dfrac{c}{2}x^2\sin(2\phi_{\mathrm{TE}}) \\ -\sqrt{bc}xy\sin\phi_{\mathrm{TE}} + \dfrac{c}{2}x^2\sin(2\phi_{\mathrm{TE}}) & cx^2\sin^2\phi_{\mathrm{TE}} \end{bmatrix}, \tag{21}$$

where

$$H_0 = \frac{z_r^2\{x^2 + y^2 - 2xy\cos[\phi_x(t) - \phi_y(t)]\}}{\rho^4 r^4} \exp\left(-\frac{\rho^2}{2f^2 r^2}\right). \tag{22}$$

By substituting Eq.(21) into Eq.(19), the degree of polarization of TE term of a completely polarized Gaussian beam is obtained to be 1. Similarly, the degrees of polarization of TM term and the whole beam are both verified to be 1. However, when the source is completely unpolarized, the degree of polarization in the far field no longer maintains a constant value of 0. The completely unpolarized source is featured by that $\phi_x(t)$ and $\phi_y(t)$ are random. In the far field, the local polarization matrix of TE term becomes

$$J_{\mathrm{TE}}(\boldsymbol{\rho},\boldsymbol{\rho},z)=H_1\begin{bmatrix} by^2+cx^2\cos^2\phi_{\mathrm{TE}} & cx^2\sin\phi_{\mathrm{TE}}\cos\phi_{\mathrm{TE}} \\ cx^2\sin\phi_{\mathrm{TE}}\cos\phi_{\mathrm{TE}} & cx^2\sin^2\phi_{\mathrm{TE}} \end{bmatrix}, \tag{23}$$

where

$$H_1=\frac{z_r^2\{x^2+y^2-2xy<\cos[\phi_x(t)-\phi_y(t)]>\}}{\rho^4 r^4}\exp\left(-\frac{\rho^2}{2f^2r^2}\right). \tag{24}$$

Therefore, the degree of polarization of TE term of a completely unpolarized Gaussian beam is found to be

$$P_{\mathrm{TE}}(\boldsymbol{\rho},z)=\left(1-\frac{4x^2y^2z^2r^2}{(z^2\rho^2+2x^2y^2)^2}\right)^{1/2}. \tag{25}$$

Similarly, the degree of polarization of TM term of a completely unpolarized Gaussian beam can be obtained as follows:

$$P_{\mathrm{TM}}(\boldsymbol{\rho},z)=\left(1-\frac{4x^2y^2z^2r^2}{(z^2\rho^2+x^4+y^4)^2}\right)^{1/2}. \tag{26}$$

As a whole beam, the degree of polarization of a completely unpolarized Gaussian beam is given by

$$P(\boldsymbol{\rho},z)=\frac{\rho^2}{z^2+r^2}. \tag{27}$$

As $x^4+y^4\geq 2x^2y^2$, $P_{\mathrm{TM}}(\boldsymbol{\rho},z)\geq P_{\mathrm{TE}}(\boldsymbol{\rho},z)$. In the paraxial case, the electric field is confined to a paraxial realm. Therefore, Eqs.(25)–(27) can be further reduced to

$$P_{\mathrm{TE}}(\boldsymbol{\rho},z)=P_{\mathrm{TM}}(\boldsymbol{\rho},z)=|\cos(2\theta)|, \tag{28}$$

$$P(\boldsymbol{\rho},z)=\frac{\rho^2}{2z^2}, \tag{29}$$

where

$$\theta=\arctan(y/x). \tag{30}$$

As a result, the degree of polarization of the whole beam is very small and approximately equal to zero. However, those of the TE and TM terms are still between 0 and 1 in the paraxial case. In the non-paraxial case, the high divergent electric field results in high degree of polarization. The degrees of polarization of the TE term, the TM term and the whole beam are depicted in a reference plane $z=1000\lambda$ in Fig.2. Although the source of vector Gaussian beam is completely unpolarized, its TE and TM terms in the far field are partially polarized. The whole beam is also partially polarized except on the propagating axis. Moreover, the degrees of polarization of TE and TM terms are both larger than that of the whole beam.

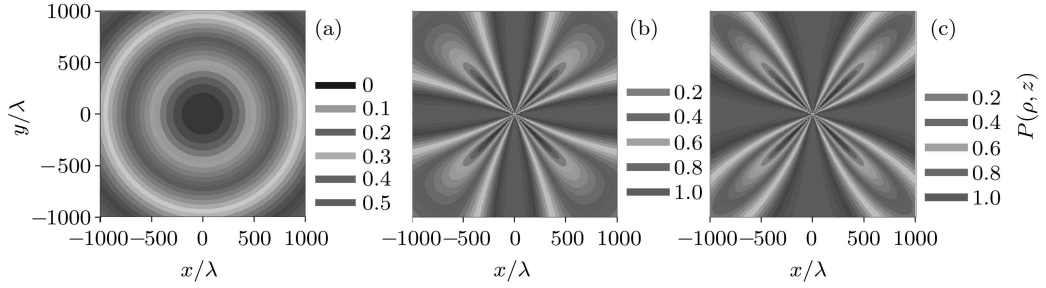


Fig.2. The degrees of polarization of a completely unpolarized Gaussian beam (a) and its TE (b) and TM (c) terms in a reference plane $z=1000\lambda$.

3. Conclusions

In summary, the analytical TE and TM terms of vector Gaussian beam in the far field is presented by using the vector angular spectrum method and the method of stationary phase. The degrees of polarization of vector Gaussian beam and its TE and TM terms in the far field are investigated by using the local polarization matrix, and the degree of their polarization is only determined by the spatial location. When the source is completely polarized, the TE and TM terms are both completely polarized in the far field. When the source is completely unpolarized, its TE and TM terms in the far field are partially polarized. The whole beam is also partially polarized except on the propagating axis. Moreover, the degrees of polarization of the TE and TM terms are both larger than that of vector Gaussian beam. The method used here is also applicable to the elegant Hermite Gaussian beam and Hermite–Laguerre–Gaussian beam.[13−15]

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