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The mass of the isodoublet of the 2^1S_0 meson nonet revisited

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In the $q\bar{q}$ quark model, the states $\pi(1300)$, $\eta(1295)$ and $\eta(1475)$ are assigned as the 2^1S_0 meson nonet. The mass of the isodoublet remains interesting and needs further confirmation in experiments. This paper employs the phenomenological trajectory on the (N, M^2)-plot and meson–meson mixing methods to calculate the mass of this state, the calculated results are 1341 MeV and 1311.58 ± 59 MeV respectively. These predictions can be compared with experiments in the future.

Keywords: meson–meson mixing, isodoublet
PACC: 1440, 3520X

1. Introduction

It is well known that the strong interactions are described by the non-Abelian gauge field theory Quantum Chromodynamics (QCD). Since the mesons are the ideal laboratory for the study of strong interactions in the strongly coupled non-perturbative regime, the meson spectroscopy would provide important clues to the understanding of the strong interactions.

Hadron spectroscopy is a very active field with the recent discovery of new light and heavy quark mesons. Great effort has been made to study meson spectra over the mass region 1000–2400 MeV[1] in the last decade. To be able to interpret the nature of new resonances, it is important that we have a template to compare observed states with theoretical predictions. The constituent quark model offers the most complete description of hadron properties and is probably the most successful phenomenological model of hadron structure. In the constituent quark model, mesons with a given set of quantum numbers $J^{\rm PC}$ appear in nonets.

In the new edition of Particle Data Group (PDG),[2] the state $\pi(1300)$ is assigned as the isovector and the states $\eta(1295)$ and $\eta(1475)$ are assigned as the isoscalar of the 2^1S_0 meson nonet (see Table 1). The assignment of the isodoublet $K(1460)$[3] needs the further confirmation experimentally, because it was observed in the two experiments $K^-p \to K^-2\pi p$[4] and $K^{\pm}p \to K^+2\pi p$[5] two decades ago.

Table 1. The assignments for the observed $J^{\rm PC} = 0^{-+}$ mesons suggested by PDG.[2]

$N^{2S+1}L_J$	$I=1$	$I=0$	$I=1/2$
1^1S_0	π	η η'	K
2^1S_0	$\pi(1300)$	$\eta(1295)$ $\eta(1440)$	$K(1460)*$
3^1S_0	$\pi(1800)$	$\eta(1760)$ $\eta(x)$[6]	$K(1830)$

In the work, based on meson–meson mixing and the phenomenological trajectory on the (N, M^2)-plot, the mass of isodoublet of 2^1S_0 meson nonet are estimated. The results could be useful in comparison with the experimental data in the new experiment.

2. The phenomenological trajectories on the (N, M^2)-plot:

A series of recent papers by Anisovich *et al*[1,7] indicates that meson states fit to the linear trajectories with sufficiently good accuracy. The trajectories on the (N, M^2)-plot can be represented as

$$M^2 = M_0^2 + (N-1)\mu^2, \qquad (1)$$

where M_0 is the mass of basic meson and μ^2 is the trajectory slope parameter, N is the radial quantum number. The trajectory of isodoublet of the 2^1S_0 meson nonet is shown in Fig.1. From the Eq.(1), we can

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obtain the mass of isodoublet of the 2^1S_0 meson nonet and the value is determined to be 1341 MeV.

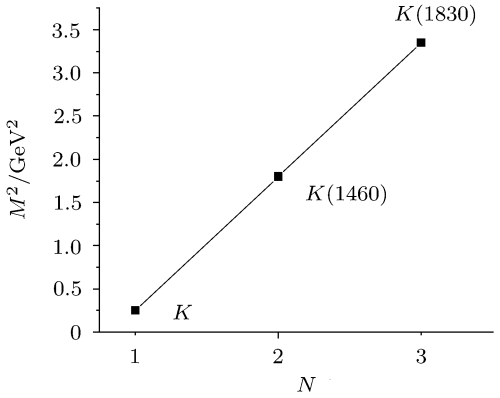


Fig.1. The (N, M^2)-plot for the isodoublet of 1^1S_0, 2^1S_0 and 3^1S_0.

3. Meson–meson mixing and the mass of the isodoublet nonet

In the $N = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $S = s\bar{s}$ basis, the form of the mass-squared matrix describing the isoscalar states mixing of a nonet can be written as follows[8]

$$M^2 = \begin{pmatrix} M_N^2 + 2A & \sqrt{2}XA \\ \sqrt{2}XA & M_S^2 + AX^2 \end{pmatrix}, \tag{2}$$

where M_N and M_S are the masses of bare states N and S respectively, A is the total annihilation strength of $q\bar{q}$ pair for the light flavour u and d, X describes the SU(3) breaking ratio of the nonstrange and strange quark propagators via the constituent quark mass ratio $m_u/m_s = 0.6298$.

If we assume physical states $|\psi\rangle$ and $|\psi'\rangle$ to be the eigenvectors of mass-squared matrix, the physical isoscalar states can be related to the bare states N and S by

$$\begin{pmatrix} |\psi\rangle \\ |\psi'\rangle \end{pmatrix} = U \begin{pmatrix} N \\ S \end{pmatrix}, \tag{3}$$

and the unitary matrix U can be described as

$$UM^2U^+ = \begin{pmatrix} M_{\psi'}^2 & 0 \\ 0 & M_\psi^2 \end{pmatrix}, \tag{4}$$

where M_ω^2 and M_ϕ^2 are the square of mass of $|\psi\rangle$ and $|\psi'\rangle$ respectively. If N is the orthogonal partner of the isovector state of a meson nonet, we can expect the N degenerates with isovector state, therefore can get $M_N^2 = M_{I=1}^2$, which is a widely adapted assumption.[9−11] The Gell-Mann–Okubo mass formula[12] implies $M_S^2 = 2M_{I=1/2}^2 - M_{I=1}^2$.

The octet and singlet mix because of SU(3) breaking. The physical states $|\psi\rangle$ and $|\psi'\rangle$ are given in terms of a mixing angle θ by

$$\begin{pmatrix} |\psi\rangle \\ |\psi'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}, \tag{5}$$

where

$$\eta_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3},$$
$$\eta_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}.$$

From Eqs.(3) and (5) we can get

$$U = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} \\ \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \end{pmatrix}. \tag{6}$$

Based on the Eqs.(2), (4) and (6) one can have the following relations:

$$M_{I=1/2}^2 + 2A = \left(\sqrt{\frac{1}{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta\right)^2 M_{\psi'}^2 + \left(\sqrt{\frac{2}{3}}\cos\theta + \sqrt{\frac{1}{3}}\sin\theta\right)^2 M_\psi^2, \tag{7}$$

$$2M_{I=1/2}^2 - M_{I=1}^2 + AX^2 = \left(\sqrt{\frac{1}{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta\right)^2 M_{\psi'}^2 + \left(\sqrt{\frac{2}{3}}\cos\theta + \sqrt{\frac{1}{3}}\sin\theta\right)^2 M_\psi^2. \tag{8}$$

With the help of the singlet-octet mixing angle of meson nonet, we calculate the mass of isodoublet of meson nonet. The results are shown in Table 2.

Table 2. Predicted results from Eq.(8) for different meson nonet.

Nonet	1^1S_0	2^1S_0	1^3S_1	1^3P_2	1^3D_3
$M_{I=1/2}$ [Eq.(8)]	493.45 ± 0.05	1311.58 ± 59	904.8 ± 0.29	1427.7 ± 0.35	1775 ± 3.85
$M_{I=1/2}$ [PDG]	497.648 ± 0.022	~ 1460	891.66 ± 0.26	1425.6 ± 1.5	1776 ± 7
	493.667 ± 0.013				
$M_{I=1/2}$ [Ref.[3]]		1391.9 ± 67.7	901.219 ± 0.604	1432.92 ± 2.73	1779.11 ± 5
$\theta_{\mathrm{quad}}/(^\circ)$	-11.5°	-22.4°	38.7°	29.6°	32°

4. Conclusion

In the scheme of phenomenological trajectory on the (N, M^2)-plot, the mass of the isodoublet of 2^1S_0 is predicted. The result is in good agreement with the value which is calculated from the meson–meson mixing scheme. In Ref.[3], the author described the well-established almost ideally mixing meson nonets. Therefore the description is invalid for non-ideally mixing nonet. In our work, with the help of the singlet-octet mixing angles, we calculate the mass of isodoublet of meson nonet states. This work is valid both for ideally mixing nonet and non-ideally mixing nonet. Our work will be useful for comparison of the experimental data.

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