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A new hyperchaotic dynamical system

Liu Chong-Xin(刘崇新)†

Institute of Electrical Engineering, Xi'an Jiaotong University,
State Key Laboratory of Electrical Insulation and Power Equipment, Xi'an 710049, *China*

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In this paper a new hyperchaotic system is reported. Some basic dynamical properties, such as continuous spectrum, Lyapunov exponents, fractal dimensions, strange attractor and Poincaré mapping of the new hyperchaotic system are studied. Dynamical behaviours of the new hyperchaotic system are proved by not only numerical simulation and brief theoretical analysis but also an electronic circuit experiment.

Keywords: hyperchaos, dynamical behaviours, circuit experiment
PACC: 0545, 4265

1. Introduction

In the last few years, hyperchaos dynamics have been extensively studied due to its great potential applications in engineering, physics and mathematics, and so on. This paper reports on the finding of a new hyperchaotic system.[1] This new hyperchaotic system is a continuous time four-dimensional autonomous system, which exhibits complex and abundant hyperchaotic dynamics behaviours according to the detailed numerical and theoretical analyses. This paper briefly explores and analyses its some basic hyperchaotic dynamical properties, and shows that simulation results support and verify brief theoretical derivations, and this hyperchaotic dynamical system can be realized by an electronic oscillation circuit. The circuit experiment results show agreement with numerical simulations.

It is believed that the application of the hyperchaotic systems will enhance the security of communication scheme in information processing.

2. Systems description and analysis

First, consider the Liu chaotic system[2]

$$\begin{aligned} \dot{x} &= a(y-x), \\ \dot{y} &= bx - kxz, \\ \dot{z} &= -cz + hx^2, \end{aligned} \tag{1}$$

where a, b, c, k and h are all positive real parameters; when $a = 10$, $b = 40$, $k = 1$, $c = 2.5$, and $h = 4$, the Liu system is chaotic. The chaotic attractor obtained from the Liu system according to the detailed numerical and theoretical analyses is also a butterfly-shape attractor, exhibiting the abundant and complex chaotic dynamics.

Here, the reverse structure form of the Liu system is described by

$$\begin{aligned} \dot{x} &= a(y-x), \\ \dot{y} &= bx + kxz, \\ \dot{z} &= -cz - hx^2. \end{aligned} \tag{2}$$

Apparently, in system (2) the sign of the multiplier in the second equation is positive and the sign of the quadratic term in the third equation is negative, which is the only difference between system (2) and system (1).

Similarly, here a, b, c, k and h are all positive real parameters; when $a = 10$, $b = 40$, $k = 1$, $c = 2.5$, and $h = 4$, system (2) is also chaotic. The Lyapunov exponents of this chaotic system are obtained to be

$$\lambda_{\mathrm{L}_1} = 1.64328, \quad \lambda_{\mathrm{L}_2} = 0, \quad \text{and} \quad \lambda_{\mathrm{L}_3} = -14.42.$$

There is only one positive Lyapunov exponent in chaotic system (2); apparently, system (2) is also a three-dimensional autonomous chaotic system. The strange attractors of system (2) are shown in Fig.1. From the figure we can see that the strange attractor is a reverse butterfly-shape attractor; therefore, system (2) is a reverse structure Liu system.

†Corresponding author. E-mail: liucx@mail.xjtu.edu.cn

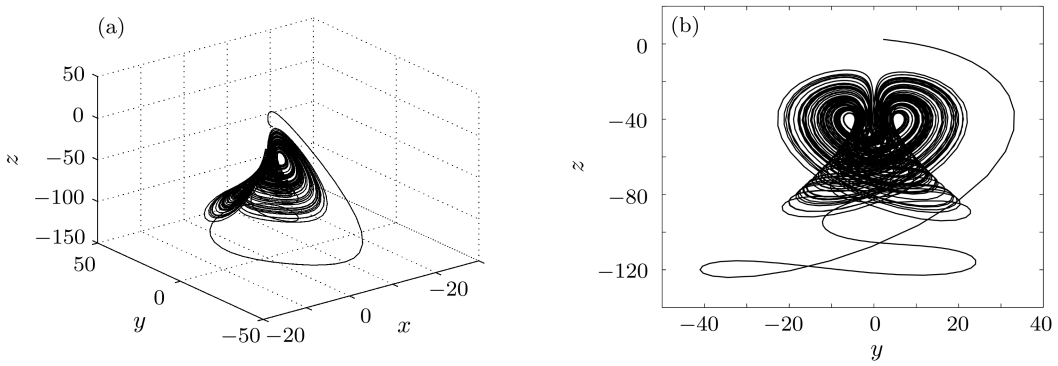


Fig.1. Strange attractors of system (2). (a) Three-dimensional view and (b) $y-z$ phase plane strange attractors.

On the basis of reverse structure Liu system, a new hyperchaotic system can be generated by adding an additional state variable w to system (2). This new hyperchaotic system can be described by the following four-dimensional autonomous differential equation:

$$\begin{aligned} &\dot{x} = a(y-x), \\ &\dot{y} = bx + kxz + ew, \\ &\dot{z} = -cz - hx^2 + mw, \quad \dot{w} = -dy, \end{aligned} \tag{3}$$

where a, b, c, k, h, e, m and d are also all positive real parameters. The values of the parameters in system (3) are $a = 10$, $b = 40$, $c = 2.5$, $k = 1$, $h = 4$, $e = 1$, $m = 1$ and $d = 2.5$, so this new chaotic system is hyperchaotic in this case.

In order to reveal the hyperchaotic dynamical properties of this nonlinear system, first, we discuss equilibria of four-dimensional autonomous system (3). Let

$$\begin{aligned} &a(y-x) = 0, \\ &bx + kxz + ew = 0, \\ &-cz - hx^2 + mw = 0, \quad -dy = 0. \end{aligned} \tag{4}$$

System (3) has only one real equilibrium, which is represented by $O(0,0,0,0)$.

For the equilibrium $O(0,0,0,0)$, system (3) are linearized, and the Jacobian matrix is

$$\begin{aligned} J_0 &= \begin{bmatrix} -a & a & 0 & 0 \\ b+kz & 0 & kx & e \\ -2hx & 0 & -c & m \\ 0 & -d & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} -10 & 10 & 0 & 0 \\ 40 & 0 & 0 & 1 \\ 0 & 0 & -2.5 & 1 \\ 0 & -2.5 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{5}$$

To gain its eigenvalues, we let

$$|\lambda I - J_0| = 0. \tag{6}$$

These eigenvalues that are corresponding to the equilibrium $O(0,0,0,0)$ are respectively obtained as follows:

$$\begin{aligned} &\lambda_1 = -25.5786, \quad \lambda_2 = -2.5, \\ &\lambda_3 = 15.5156, \quad \text{and} \quad \lambda_4 = 0.0630, \end{aligned}$$

where λ_1 and λ_2 are negative real roots, while λ_3 and λ_4 are positive real roots. Therefore, the equilibrium $O(0,0,0,0)$ is an unstable saddle in this nonlinear four-dimensional autonomous system.

The initial values of system (3) are appointed as $(2.4, 2.2, 0.8, 0)$. According to numerical as well as detailed theoretical analyses,[3] it has been confirmed that new system (3) displays sophisticated and abundant hyperchaotic dynamical behaviours.

The hyperchaotic strange attractors are shown in Figs.2 and 3, and they are also new reverse butterfly-shape attractors.

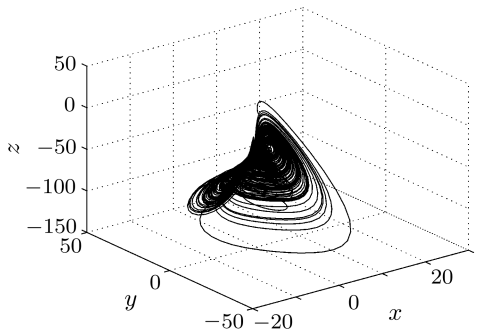


Fig.2. Three-dimensional view of system (3).

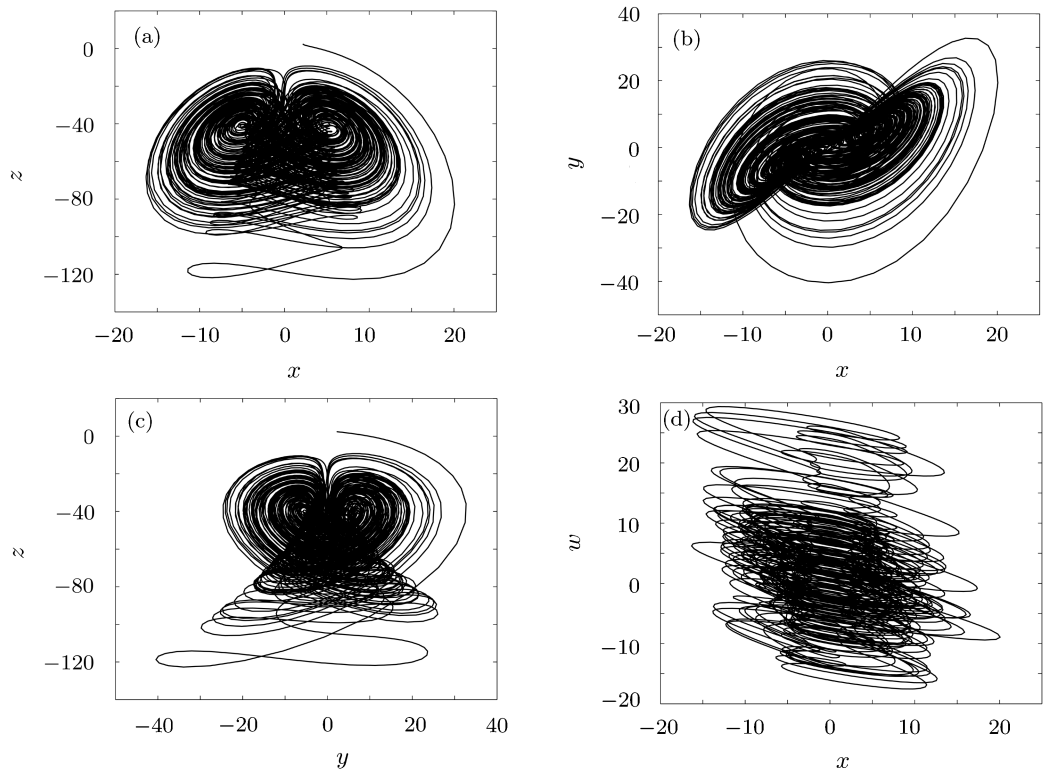


Fig.3. (a) $x-z$ phase plane strange attractors of system (3), (b) $x-y$ phase plane strange attractors of system (3), (c) $y-z$ phase plane strange attractors of system (3), (d) $x-w$ phase plane strange attractors of system (3).

According to the aforementioned analyses, apparently, they are new chaotic attractors, while the sensitive dependence on the initial conditions is a prominent characteristic of chaotic behaviour: when the appointed initial values are changed, the chaotic dynamical behaviour of this system disappears immediately.

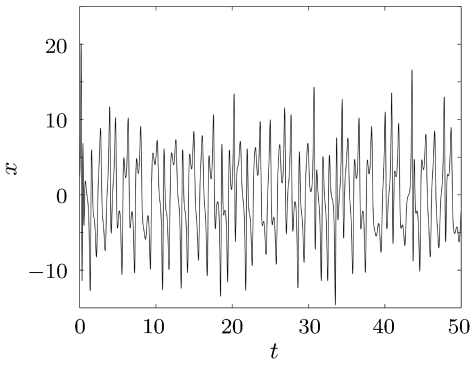


Fig.4. $w(t)$ waveform of system (3).

The waveforms of $x(t)$ in time domain are shown in Fig.4, apparently, they are non-periodic in continuous-time four-dimensional autonomous chaotic system (3), which is one of basic chaotic dynamical properties.

The spectrum of non–linear system (3) is also studied, and it is continuous as shown in Fig.5.

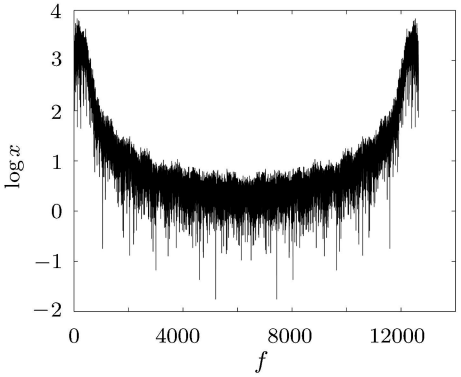


Fig.5. Spectrum of $|x|$ in system (3).

The Poincaré mapping of this non-linear system is also analysed. It can be seen that the Poincaé mappings are these points in confusion as shown in Fig.6.

The bifurcation diagram of x with increasing a is given in Fig.7, and it shows abundant and complex dynamical behaviours.

The divergence of system (1) is given by

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} = -a - c = -\beta.$$

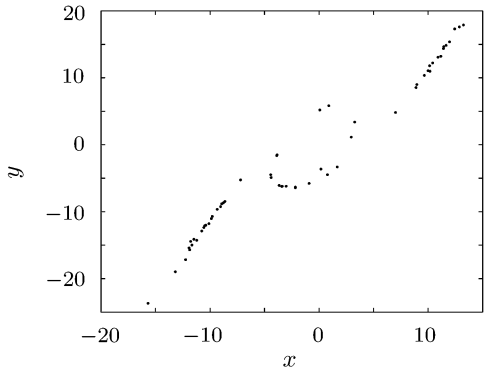


Fig.6. The Poincaré map of $x-y$ plane of system (3).

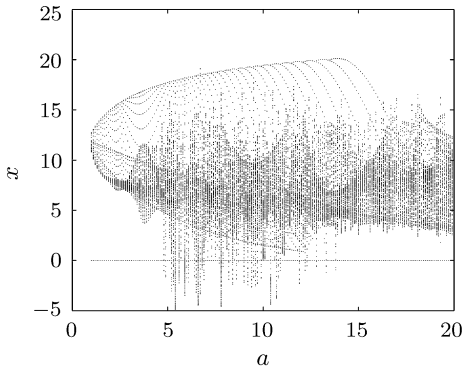


Fig.7. Bifurcation diagram for increasing a with $b = 40$, $h = 4$, $k = 1$, $e = 1$, $m = 1$ and $d = 2.5$ of system (3).

Since $\nabla V = -\beta = -12.5$ and its value is a negative constant, system (3) is dissipative. Therefore, this shows that each volume containing this system trajectory shrinks to zero as $t \to \infty$ at an exponential rate $V(0)\mathrm{e}^{-\beta t}$. Thus, all orbits of system (3) are ultimately confined to a subset of zero volume, and it is apparent that this nonlinear system asymptotic motion settles finally down to an attractor.[4,5]

According to the chaos theory, the Lyapunov exponents measure the exponential rates of divergence and convergence of nearby trajectories in phase space of system (3).

As is well known, there is more than one positive exponent in a hyperchaotic four-dimensional autonomous system.[6,7]

Calculating the two largest values of positive Lyapunov exponents of nonlinear system (3) yields $\lambda_{\mathrm{L}_1} = 1.711$ and $\lambda_{\mathrm{L}_2} = 0.028$.

They show the expanding nature in different directions in phase space.

Another Lyapunov exponent is $\lambda_{\mathrm{L}_3} = 0$.

It shows the critical nature between the expanding and the contracting natures in different directions in phase space.

Negative Lyapunov exponent of nonlinear system (3) is $\lambda_{\mathrm{L}_4} = -14.2233$.

It shows the contracting nature in different directions in phase space.

Therefore, by theoretical analysis and numerical simulation, the Lyapunov dimension of this hyperchaotic system is

$$D_{\mathrm{L}} = j + \frac{1}{|\lambda_{\mathrm{L}j+1}|}\sum_{i=1}^{j}\lambda_{\mathrm{L}i} = 3 + \frac{(\lambda_{\mathrm{L}_1} + \lambda_{\mathrm{L}_2} + \lambda_{\mathrm{L}_3})}{|\lambda_{\mathrm{L}_4}|}$$
$$= 3 + \frac{1.711 + 0.028 + 0}{|-14.2233|} = 3.12. \quad (7)$$

The Lyapunov dimension of this new hyperchaotic system is also fractional dimension.

The above theoretical analysis and numerical simulation both show that system (3) is really a new hyperchaotic system and more sophisticated topological structure and abundant hyperchaotic dynamical properties.[8−10]

3. Circuitry experiment confirmation of the hyperchaotic system

In this section, an electronic oscillator circuit is designed to realize new hyperchaotic system (3) based on Ref.[11]. The designed hyperchaotic oscillator circuit is shown in Fig.8 and it comprises linear resistors, linear capacitors, operational amplifiers and analogue multipliers. The four state variables x, y, z and w are respectively obtained from the terminal outputs of u_{C_1}, u_{C_2}, u_{C_3} and u_{C_4} in this electronic circuit. The experimental phase portraits of the new reverse butterfly-shape attractor (3) are shown in Fig.9. The circuit experiment results show agreement with numerical simulations. The designed chaos oscillator shown in Fig.8 is very useful in electronic technique and communication engineering.[11−14]

In general, a new hyperchaotic system is established by adding an additional state variable to a three-dimensional autonomous chaotic system or two-dimensional non-autonomous chaotic system, and this approach is effective.[15−23]

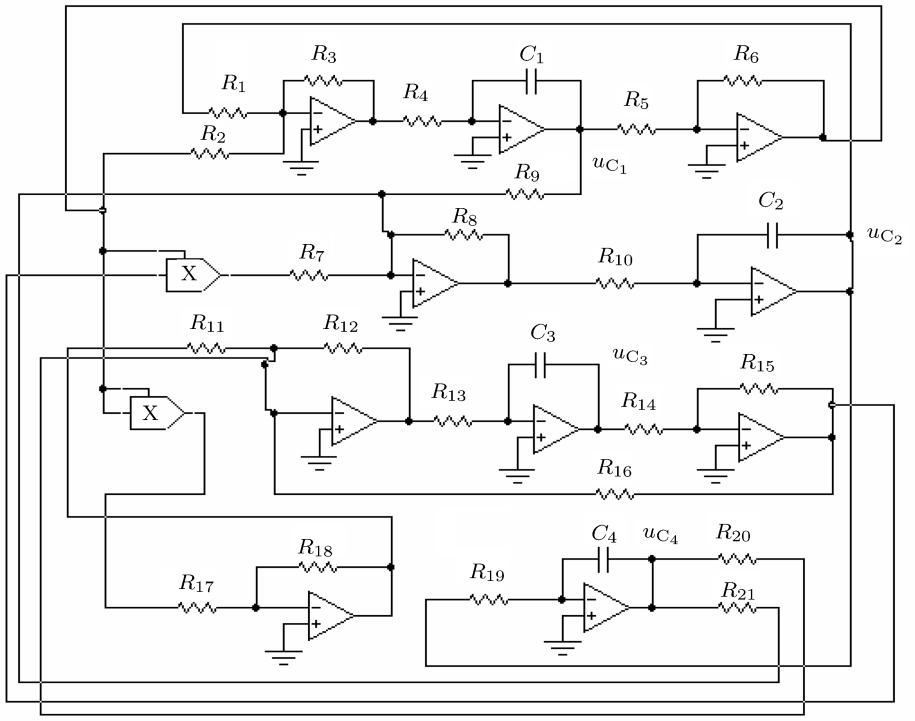


Fig.8. Circuitry realization of the hyperchaotic system (3). resistance/kΩ: R_1, R_3, R_5, R_9, R_{12}, R_{14}, R_{15}, R_{17}, $R_{18} = 10$; R_2, $R_6 = 20$, $R_4 = 100$; $R_7 = 8$; $R_8 = 40$; R_{10}, R_{13}, R_{20}, $R_{21} = 100$; $R_{11} = 1$; $R_{19} = 400$. capacitance/μF: C_1, C_2, C_3, $C_4 = 1$.

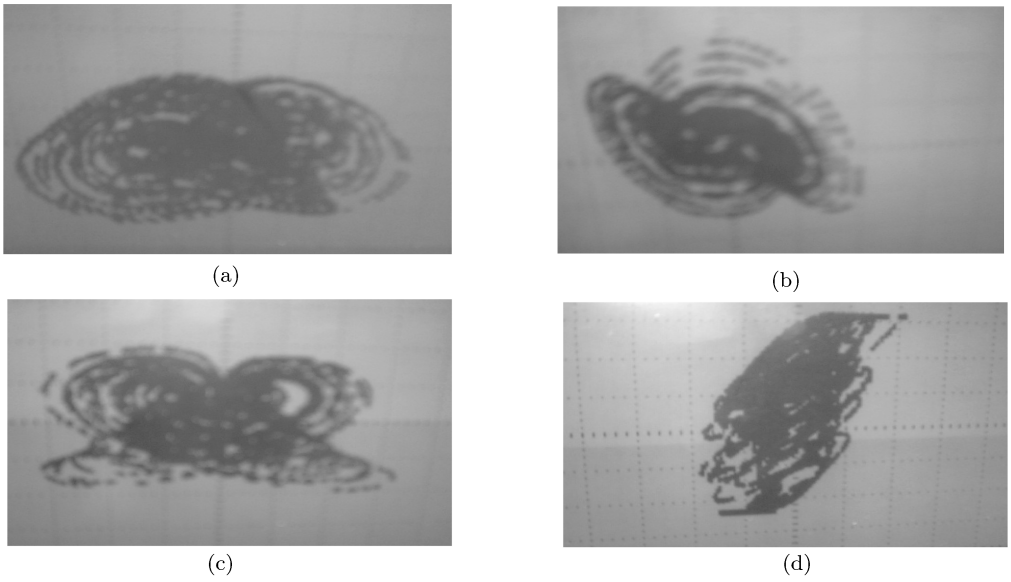


Fig.9. Experimental phase portraits of system (3): (a) $x - z$ plane (1V/div, 1V/div); (b) $x - y$ plane (1V/div, 1V/div); (c) $y - z$ plane (1V/div, 1V/div); (d) $x - w$ plane (1V/div, 5V/div).

4. Conclusion

There are abundant and complex dynamical behaviours in new hyperchaotic system (3). This new hyperchaotic attractor is different from the hyperchaotic Lorenz attractor, hyperchaotic Chen attractor as well as hyperchaotic system proposed by Zengqiang Chen *et al*; but it is also a new reverse butterfly-

shape hyperchaotic attractor of Lorenz–like system. These new hyperchaotic attractors evolved from the Liu chaotic system can be realized with an electronic oscillator circuit and have great potential applications in electronic technique and communication engineering. These new hyperchaotic attractors and their forming mechanism need further studying and exploring. Their topological structures should be completely and thoroughly investigated. It is expected that more detailed theoretical analyses and simulation investigations will appear elsewhere in the near future.

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