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Notch filter feedback controlled chaos in buck converter*

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A method of controlling chaos in the voltage-mode buck converter is presented by using an improved notch filter feedback control in this paper. The proposed control part comprises a notch filter and a low-pass filter. The discrepancy between the outputs of the two filters is introduced into the control prototype of the power converter. In this way, the system period-1 solution is kept unchanged. The harmonic balance method is applied to analysing the variation law of the system bifurcation point, and then the stable range of the feedback gain is ascertained. The results of simulation and experiment are also given finally.

Keywords: chaos, control of chaos, buck converter, notch filter
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1. Introduction

Bifurcation and chaos commonly exist in DC–DC converters with Feedback control,[1−6] which deteriorate the performance of the power converters, for instance, output ripple, conversion efficiency.[7] Furthermore there are few effective approaches to precisely forecasting and controlling the state variable or the output in chaos. Therefore the effective way of suppressing chaotic behaviour in DC–DC converter is of great significance in engineering.

The voltage-mode buck converter is the earliest power converter for the investigation of chaotic behaviour. The output voltage changes into a chaotic state from stable period-1 with the variation of its input voltage. The output behaves in a quasirandom way, which deteriorates the system performance. Some papers devoted to the controlling chaos in this kind of system have been published. In Refs.[7–9], chaotic state in a DC–DC converter was controlled into period-1 via an external periodic signal perturbing the system parameters. Thus the perturbing signal circuits are necessary, for the quantitative analysis of the control parameters is quite difficult. Another way to control chaos is to adopt the output or state variable feedback. In Ref.[10], time-delay feedback was first applied to the control of chaos in the voltage-mode buck converter. The control parameters were ascertained analytically by using the Floquet theory. However, the control parameter is not only an analytic solution from the viewpoint of stability. In Refs.[11,12], the voltage derivative feedback and state variable feedback were applied to the control of chaos in the voltage-mode buck converter. Both of them were hardly realized in engineering because the feedback signals are introduced into the power circuits.

Another advanced chaos control method is based on filters. In Ref.[13], the control of chaos in the buck converter was achieved by using a washout filter which is a kind of high-pass filter. The band-stop notch filter can also effectively control the chaotic system,[14] but its application to a switching power converter has not been reported to date.

From the above analyses, this paper aims to apply the output notch filter feedback to the control of chaos in the voltage-mode buck converter. The notch filter frequency is set to be the switching frequency of the power converter because the period-1 is our control target. Thus, only the feedback gain needs to be ascertained. The stable range of the feedback gain is obtained by applying the harmonic balance method to the analysis of the bifurcation point variation law. In this way, the system stable range can be widened, and the output in steady state or period-1 is kept unchanged.

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2. Notch filter feedback controlled buck converter

In the voltage-mode buck converter, the variation of its input voltage can cause the occurrence of period-doubling bifurcation, and then the output changes into a chaotic state.[6,10,11] Here the sub-harmonic, whose frequency is less than the switching frequency, will exist in this system. Notch filter can suppress the occurrence of sub-harmonic.[13] Therefore it can stabilize the system in period-1 by adding a notch filter to the feedback control of the buck converter. The corresponding transfer function of the feedback notch filter is

$$G_{\rm f}(s) = p\frac{s^2+\omega_{\rm n}^2}{s^2+2\xi\omega_{\rm n}s+\omega_{\rm n}^2}, \tag{1}$$

where ξ is the damping factor, p is the feedback gain, and $\omega_{\rm n}$ is the notch frequency.

As we expect the system to be controlled in period-1, $\omega_{\rm n}$ should be set to be the switching frequency of the power converter. While the system is in period-1, s is equal to $\mathrm{j}2\pi/T$, where T is the switching period of the power converter. So here the relation $s^2+\omega_{\rm n}^2=0$ is satisfied, and we have $G_{\rm f}(\mathrm{j}2\pi T)=0$. Therefore, the notch filter output in period-1 is a constant steady-state component. The control signal in a steady-state should approach zero so as to keep the period-1 solution of the system to be controlled unchanged. This expectation can promise well, if we choose the feedback control signal as the difference signal between the outputs of the notch filter and a low-pass filter. And here the low-pass filter is used to filter out the average steady-state component from the output voltage.

The schematic diagram and the block diagram of the voltage-mode buck converter with notch filter (NF) feedback control are shown in Figs.1(a) and 1(b) respectively, where the additional control is framed by the dot line, which comprises a notch filter and a low-pass filter (LPF), with $V_{\rm in}$ being the input voltage, $V_{\rm ref}$ the reference voltage, $v_{\rm ramp}$ the saw-tooth voltage, $v_{\rm d}$ the diode voltage, $v_{\rm con}$ the original feedback control voltage, $v'_{\rm con}$ the new feedback control voltage, $\Delta v_{\rm con}$ the output voltage of additional control part, and k_1 and k_2, respectively, the feedback gains for the original control and for notch filter control. When $\Delta v_{\rm con}$ is equal to zero, the system turns into a conventional voltage-mode buck converter.

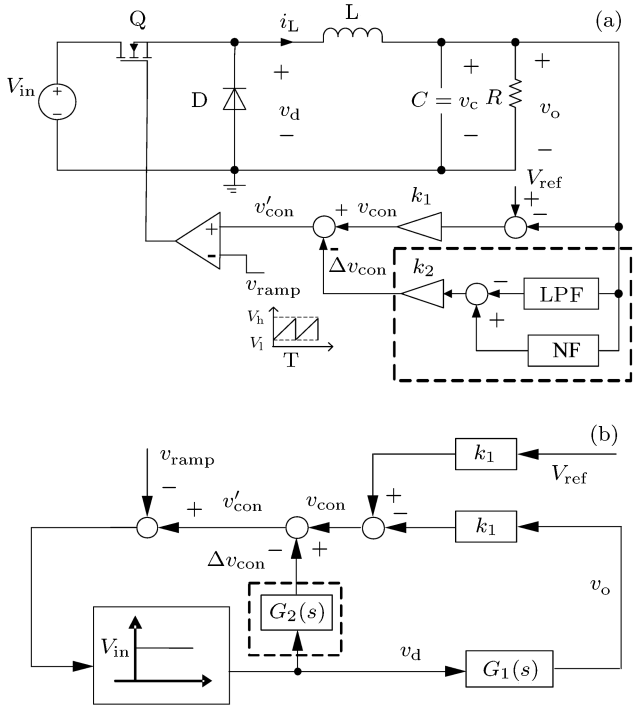


Fig.1. The voltage-mode buck converter with notch filter feedback control. Subfigure (a) shows the schematic diagram and (b) the block diagram.

From Figs.1(a) and 1(b), we have

$$v_{\rm con} = k(V_{\rm ref} - v_{\rm o}), \tag{2}$$

$$v'_{\rm con} = v_{\rm con} - \Delta v_{\rm con}. \tag{3}$$

We define the frequency domain variable corresponding to $v_{\rm o}(t)$ as $v_{\rm o}(s)$ and do the same to the other time domain variables. $G_1(s)$ is the transfer function of the filter which is composed of L, R and C as shown in Fig.1(a), so we have

$$G_1(s) = \frac{v_{\rm o}(s)}{v_{\rm d}(s)} = \frac{1}{LCs^2 + (L/R)s + 1}. \tag{4}$$

The transfer function of the notch filter feedback control is $G_2(s)$ as shown in Fig.1(b), which can be obtained as fellows:

$$\begin{aligned}\Delta v_{\rm con}(s) &= k_2 v_{\rm o}(s)(G_{\rm f}(s) - G_3(s)) \\ &= k_2 G_1(s) V_{\rm d}(s)(G_{\rm f}(s) - G_3(s)),\end{aligned} \tag{5}$$

$$G_2(s) = \frac{\Delta v_{\rm con}(s)}{V_{\rm d}(s)} = k_2 G_1(s)(G_{\rm f}(s) - G_3(s)), \tag{6}$$

where $G_3(s)$ is the transfer function of the low-pass filter, which is written as

$$G_3(s) = \frac{2\pi/(k_3 T)}{s + 2\pi/(k_3 T)}. \tag{7}$$

As is known, the larger the value of k_3, the better the performance of the low-pass fitter, but the worse the dynamic property of the system. So the value of k_3 should be appropriately chosen. And we define the hybrid transfer function as

$$G(s) = k_1 G_1(s) - G_2(s). \tag{8}$$

In the following, we will apply the harmonic balance method[15] to analysing the bifurcation point variation law in fore and aft the notch filter feedback control. Figures 2(a) and 2(b) show the waveforms of the feed voltage $v'_{\rm con}$, saw-tooth voltage $v_{\rm ramp}$ and pulse drive signal in period-1 and period-2 respectively, where $v_{\rm ramp} = V_{\rm l} + (V_{\rm h} - V_{\rm l}) \bmod (t/T, 1)$.

From Fig.2, the following relationship can be obtained, wherein d is the duty ratio of the power converter:

in period-1

$$v'_{\rm con}(dT) = v_{\rm ramp}(dT), \tag{9}$$

in period-2

$$v'_{\rm con}(dT + \delta) = v_{\rm ramp}(dT + \delta),$$

$$v'_{\rm con}(T + dT - \delta) = v_{\rm ramp}(T + dT - \delta). \tag{10}$$

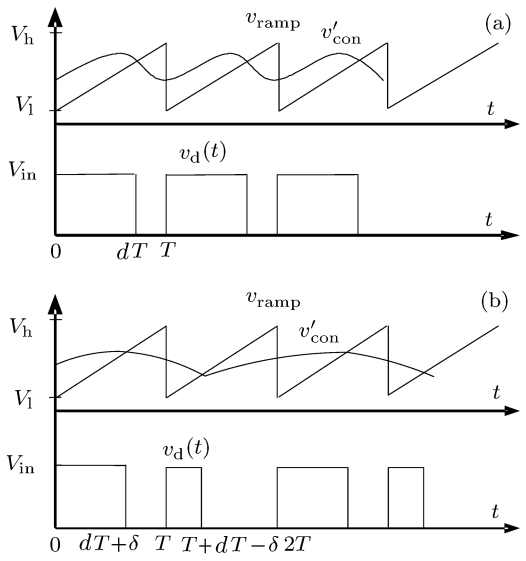


Fig.2. The waveforms of the feed voltage, saw-tooth voltage and pulse drive signal in period-1 (a) and period-2 (b).

Fourier series decomposition is applied to $v_{\rm d}(t)$ in either in period-1 or period-2, and we have in period-1

$$v_{\rm d}(t) = \sum_{n_1=-\infty}^{+\infty} c_{n1} {\rm e}^{{\rm j}n_1\omega_{\rm s}t}, \tag{11}$$

where

$$c_{n1} = \frac{V_{\rm in}}{{\rm j}2n_1\pi}(1 - {\rm e}^{-{\rm j}n_1\omega_{\rm s}dT}),$$

$\omega_{\rm s} = 2\pi/T$, in period-2

$$v_{\rm d}(t) = \sum_{n_2=-\infty}^{+\infty} c_{n2} {\rm e}^{(1/2){\rm j}n_2\omega_{\rm s}t}, \tag{12}$$

where while n_2 is odd number,

$$c_{n2} = \frac{V_{\rm in}}{{\rm j}2n_2\pi}({\rm e}^{-(1/2){\rm j}n_2\omega_{\rm s}(dT-\delta)} - {\rm e}^{-(1/2){\rm j}n_2\omega_{\rm s}(dT+\delta)}),$$

while n_2 is even number

$$\begin{aligned}c_{n2} = \frac{V_{\rm in}}{{\rm j}2n_2\pi}(&2 - {\rm e}^{-(1/2){\rm j}n_2\omega_{\rm s}(dT+\delta)} \\ &- {\rm e}^{-(1/2){\rm j}n_2\omega_{\rm s}(dT-\delta)}).\end{aligned}$$

Thus, in periodic-1 there exists

$$\begin{aligned}v'_{\rm con}(dT) &= k_1 V_{\rm ref} - \sum_{n_1=-\infty}^{+\infty} c_{n1} {\rm e}^{{\rm j}n_1\omega_{\rm s}dT} G({\rm j}n_1\omega_{\rm s}) \\ &= k_1 V_{\rm ref} - c_{10} - \Big(\frac{V_{\rm in}}{\pi}\Big) \\ &\quad \times {\rm Im}\Big(\sum_{n_1=1}^{+\infty} \frac{({\rm e}^{{\rm j}n_1\omega_{\rm s}dT} - 1)}{n_1} G({\rm j}n_1\omega_{\rm s})\Big),\end{aligned} \tag{13}$$

where $c_{10} = [v_{\rm d}(t)]_{\rm ave} = dV_{\rm in}G(0)$.

Combining expression (9) with expression (13), the relationship between $V_{\rm in}$ and d can be obtained as follows:

$$V_{\rm in}=\frac{k_1V_{\rm ref}-v_{\rm ramp}(dT)}{dG(0)+(\frac{1}{\pi})\mathrm{Im}[\sum_{n_1=1}^{+\infty}\frac{(\mathrm{e}^{\mathrm{j}n_1\omega_{\rm s}dT}-1)}{n_1}G(\mathrm{j}n_1\omega_{\rm s})]}. \quad (14)$$

In a similar way, combining expression (10) with expression (12), the relationship between $V_{\rm in}$ and d in period-2 can be obtained as follows:

$$V_{\rm in}=\frac{\pi\delta(V_{\rm h}-V_{\rm l})/T}{-\mathrm{Re}[\sum_{n_2=1}^{+\infty}(N+M)]}, \quad (15)$$

where

$$N=\frac{1}{2n_2-1}G\Big(\mathrm{j}\Big(n_2-\frac{1}{2}\Big)\omega_{\rm s}\Big)\sin((2n_2-1)\omega_{\rm s}\delta), \quad \text{and} \quad M=\frac{1}{2n_2}G(\mathrm{j}n_2\omega_{\rm s})(2\mathrm{e}^{\mathrm{j}n_2\omega_{\rm s}d}\sin(n_2\omega_{\rm s}\delta)-\sin(2n_2\omega_{\rm s}\delta)).$$

When δ in expression (15) approaches zero, the input voltage bifurcation point (the critical value between period-1 and period-2) is

$$V_{\rm in}^*=\frac{(V_{\rm h}-V_{\rm l})}{-2\mathrm{Re}[\sum_{n_2=1}^{+\infty}(G(\mathrm{j}(n_2-\frac{1}{2})\omega_{\rm s})+G(\mathrm{j}n_2\omega_{\rm s})(\mathrm{e}^{\mathrm{j}n_2\omega_{\rm s}dT}-1)]}. \quad (16)$$

Considering $G(s)$ as a low-pass filter, the denominator term to the right of expression (16) can be approximated as a first-order term, and then we have

$$V_{\rm in}^*\propto\frac{V_{\rm h}-V_{\rm l}}{2\mathrm{Re}[(G(\mathrm{j}\omega_{\rm s}/2)]}. \quad (17)$$

The conclusions can been drawn from expression (17). As $v_{\rm ramp}$ changes, the bifurcation point will change. The change of the parameters in $G(s)$ will also affect the bifurcation point. So the notch filter feedback control, corresponding to $G_2(s)$ included in $G(s)$, can also affect the bifurcation point. The bifurcation point is enlarged, and the chaotic behaviour is suppressed by choosing an appropriate value of k_2. Combining expression (14) with expression (16), the corresponding bifurcation points $V_{\rm inb}$ for different values of k_2 can be obtained by numerical computation. But it is necessary to choose values large enough for the Fourier series numbers n_1 and n_2. The value of k_2, which ensures the system in period-1, is ascertained according to the bifurcation point variation law.

The influence on the output voltage in steady-state is discussed as below. When an additional control stabilizes the system in period-1, we have

$$G_2(0)=k_2(G_{\rm f}(0)-G_3(0))=0.$$

And when k_3 is large enough, we have

$$|G_3(\mathrm{j}2\pi/T)|=1/k_3\approx 0,$$

$$G_{\rm f}(\mathrm{j}2\pi/T)=0,$$

$$|G_2(\mathrm{j}2\pi/T)|=|k_2(G_{\rm f}(\mathrm{j}2\pi/T)-G_3(\mathrm{j}2\pi/T))|\approx 0.$$

Based on the above analyses, the notch filter feedback control is considered not to change the steady-state solution and the period-1 solution, which is vital to the control of chaos in power converters.

3. Simulation results

In this section, the simulation work will be carried out according to the buck converter schematic diagram shown in Fig.1(a), and the parameters in simulation are in agreement with those in Ref.[3], specifically chosen as follows: $V_{\rm in}=20\mathrm{V}-35$ V, $L=20\,\mathrm{mH}$, $R=22\Omega$, $C=47\,\mu\mathrm{F}$, $v_{\rm ramp}=3.8+(8.2-3.8)\mathrm{mod}\,(t/T,1)$ V, $V_{\rm ref}=11.3$ V, $k_1=8.4$, and $T=0.4$ ms.

Furthermore, the parameters of the notch filter and the low-pass filter are chosen as follows: $\xi=0.707$, $p=1$, $\omega_{\rm n}=2\pi/T$, and $q=2\pi/(25\,T)$.

Figure 3(a) shows the output bifurcation diagram with an input voltage used as a bifurcation parameter while the notch filter control is invalid, i.e. k_2 is equal to zero. Figure 3(b) shows the Lyapunov exponential spectrum for $V_{\rm in}=24\mathrm{V}-34$ V and $k_2=0$. It is observed that the system changes into period-2 while the input voltage $V_{\rm in}$ is about 24.5 V. And the system is in chaos and the corresponding Lyapunov exponent (LE) is greater than zero as shown in Fig.3(b), while $V_{\rm in}$ is greater than 31.5 V. The input voltage bifurcation point value should be greater than 35 V to ensure the system in period-1 for $V_{\rm in}=20\mathrm{V}-35$ V. Thus, the range of k_2 is ascertained according to the expectant bifurcation point.

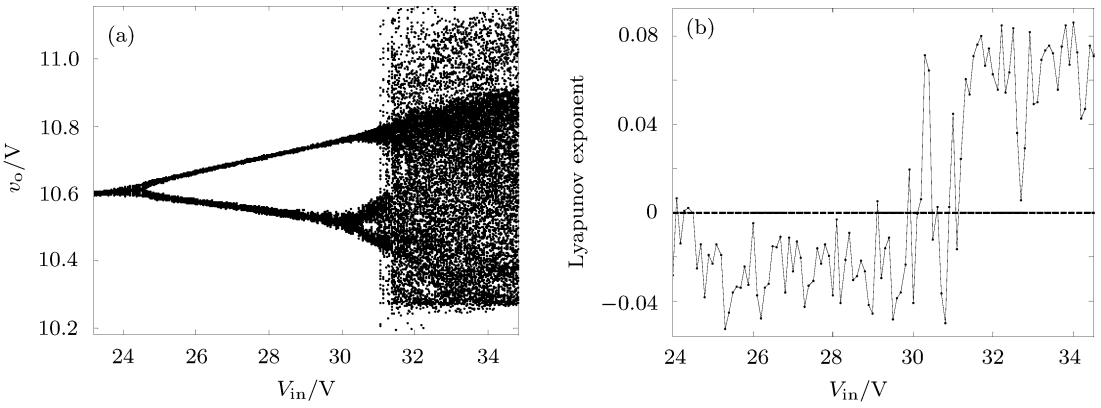


Fig.3. The output bifurcation diagram (a) and the Lyapunov exponent (LE) for $k_2 = 0$ (b).

According to Section 2, combining expression (14) with expression (16), we set n_1 and n_2 to be twenty and use numerical method to obtain the bifurcation point value. Figure 4 shows the bifurcation point diagram for $k_2 = 0 - 5$. The bifurcation point value is about 24.5 V for $k_2 = 0$, which is in agreement with the result in the bifurcation diagram shown in Fig.3(a). From Fig.4, we know that it can stabilize the system in period-1 while V_{in} is equal to 34 V and k_2 is greater than 3.5. Figures 5(a)–5(c) show the capacitive voltage, the inductive current and the feed control voltage of the buck converter while the additional control takes effect at $t = 0.03$ s for $V_{in} = 34$ V and $k_2 = 5$.

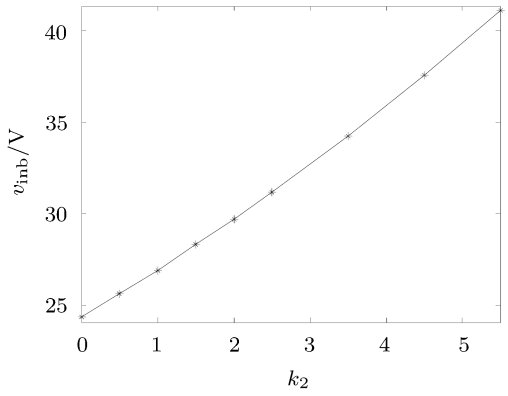


Fig.4. The bifurcation point value V_{in} versus feedback gain k_2.

It can be seen from Fig.5 that the proposed method can change the chaotic state of the system to be controlled into period-1 quickly. Δv_{con} in steady-state is closer to zero, so the period-1 solution is kept unchanged.

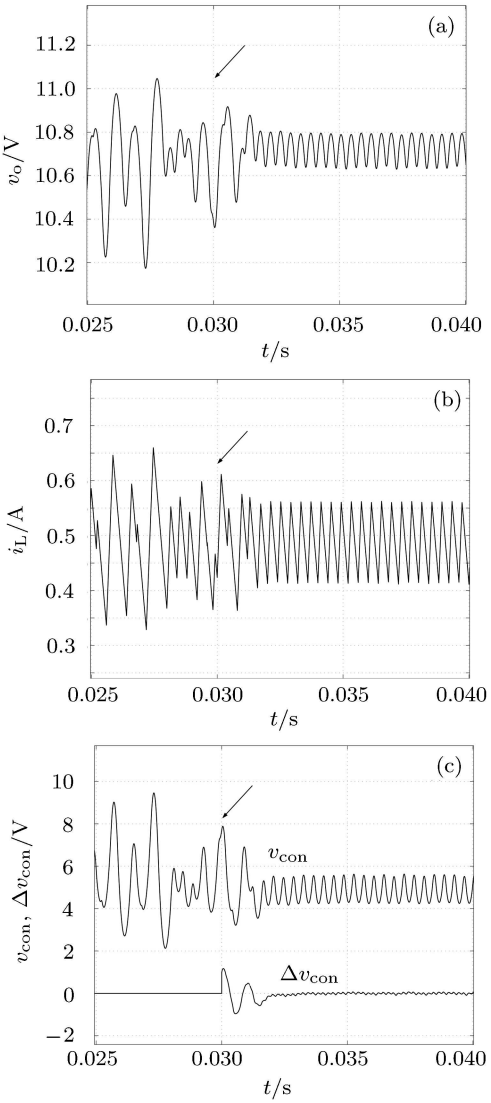


Fig.5. The simulating waveforms for $k_2 = 5$ of the capacitive voltage (a), the inductive current (b), and the feedback control voltage (c).

4. Experimental results

The experimental circuits are built up according to the schematic diagram shown in Fig.1(a), where the notch filter circuit is given by the Wien-bridge and the low-pass filter circuit is constructed by a resistance and a capacitance in series. The experimental parameters are in agreement with those in simulation. Figure 6(a) shows the experimental waveforms of the capacitive voltage (CH1:2V/div) and the inductive current (CH2:500mA/div), and Figure 6(b) shows the experimental waveforms of the original feedback control voltage (CH1:10V/div) and the output of the notch filter feedback control (CH2:10V/div). Without the notch filter feedback control, the voltage and the current are in chaos. On the contrary, the system changes into period-1 from the chaotic state, the voltage and current ripple decrease and the controlled period-1 behaviour acts as that in unstable periodic orbit (UPO) of the original system.

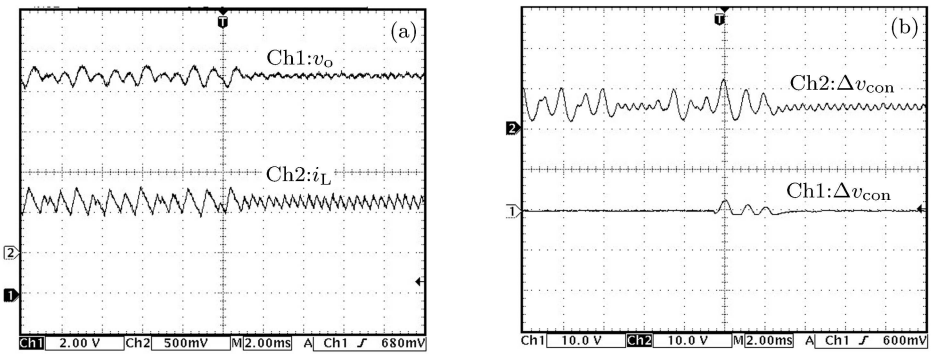


Fig.6. The experimental waveforms for $V_{in} = 34$ V and $k_2 = 5$ of the inductive current and the capacitive voltage (a), and the feedback control voltage and the output of the notch feedback control (b).

5. Conclusion

In this paper we apply the notch filter feedback to the control of chaos in a voltage-mode buck converter, and the control effect is equivalent to that of the time-delay feedback control. It can not only control the chaotic buck converter into period-1 quickly, but also keep the period-1 solution unchanged. And either the notch filter or the low-pass filter adopted in this method is realized by analogue circuits easily. Meanwhile, this method can be deployed to control the chaos in other power converters.

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