Vol 16 No 11, November 2007
1009-1963/2007/16(11)/3249-07

Chinese Physics

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Synchronization between two different noise-perturbed chaotic systems with unknown parameters*

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(Received 2 February 2007; revised manuscript received 10 April 2007)

In this paper, a general method of synchronizing noise-perturbed chaotic systems with unknown parameters is proposed. Based on the LaSalle-type invariance principle for stochastic differential equations and by employing a combination of feedback control and adaptive control, some sufficient conditions of chaos synchronization between these noise-perturbed systems with unknown parameters are established. The model used in the research is the chaotic system, but the method is also applicable to the hyperchaotic systems. Unified system and noise-perturbed Rössler system, hyperchaotic Chen system and noise-perturbed hyperchaotic Rössler system are taken for illustrative examples to demonstrate this technique.

Keywords: synchronization, adaptive control, noise perturbation, unknown parameters
PACC: 0545

1. Introduction

Since the idea of synchronizing two identical chaotic systems under different initial conditions was first introduced by Pecora and Carrol[1] in 1990, chaos synchronization, as an important topic in nonlinear science, has been widely investigated in many fields.[2−4] Hence, a lot of approaches to the synchronization of chaotic systems with linear or nonlinear feedback control, adaptive-feedback control, and active control (see Refs.[5–9] and references therein) have been proposed.

In many practical fields such as biological system and cognitive processes, and many other cases, every component cannot be assumed to be identical, and some system's parameters cannot be exactly known beforehand. In view of these reasons, most of researchers[10−13] have studied the synchronization of different systems with unknown parameters.

It is worth noticing that noise, which is commonly regarded as a random and persistent disturbance reducing the clarity of a signal, is omnipresent in nature and in man-made systems. Synchronization of chaotic systems is unavoidably subjected to noise. Derived in Ref.[14] is a recursion formula of transition probability of the noise-induced synchronization arising in a pair of identical uncouple logistic maps linked by common noisy excitation only. In Ref.[15], authors studied the relation between noise-induced synchronization and the usual chaotic synchronization, and demonstrated that noise-induced synchronization was closely related to generalized synchronization. In Ref.[16], it is found that authors applied the simple adaptive-feedback control scheme to synchronizing chaotic non-autonomous systems, and numerical simulations showed the proposed method was robust against the effect of external noise. Based on the LaSalle-type invariance principle for stochastic differential equations, the authors discussed the complete synchronization problem between unidirectionally coupled Chua's circuits within stochastic perturbation in Ref.[17].

Motivated by the above findings, in this paper we are mainly to further ascertain whether it is possible to synchronize two different noise-perturbed chaotic systems with unknown parameters. We give a general method to study this problem. Based on the so-called LaSalle-type invariance principle for stochastic differential equations, it is proved that the suggested approach can realize the chaos synchronization of noise-perturbed chaotic systems. The model used in the present research is the chaotic system, but the method is also applicable to the hyperchaotic systems. Simulation results validate the effectiveness and universality

*Project supported by the National Natural Science Foundation of China (Grant Nos 10472091 and 10332030) and the Graduate Starting Seed Fund of Northwestern Polytechnical University, China (Grant No Z200655).
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of the proposed synchronization method.

2. Problem description

The drive chaotic system considered in this paper is described as

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{F}(\boldsymbol{x})\hat{\boldsymbol{p}}, \tag{1}$$

where $\boldsymbol{x} \in R^n$ is the state vector, $\boldsymbol{f}(\boldsymbol{x}) \in R^n$, $\boldsymbol{F}(\boldsymbol{x}) \in R^{n\times m}$, and $\hat{\boldsymbol{p}} \in R^m$ is the unknown parameter vector. And let $\Omega \subset R^n$ be a chaotic bounded set of Eq.(1) which is globally attractive.

The response system with stochastic perturbation is given as follows:

$$\dot{\boldsymbol{y}} = \boldsymbol{g}(\boldsymbol{y}) + \boldsymbol{G}(\boldsymbol{y})\hat{\boldsymbol{q}} + \boldsymbol{\sigma}(t, \boldsymbol{y}-\boldsymbol{x})\dot{\boldsymbol{\omega}} + \boldsymbol{u}, \tag{2}$$

where $\boldsymbol{y} \in R^n$ is the state vector, $\boldsymbol{g}(\boldsymbol{y}) \in R^n$, $\boldsymbol{G}(\boldsymbol{y}) \in R^{n\times l}$, and $\hat{\boldsymbol{q}} \in R^l$ is the unknown parameter vector. $\boldsymbol{u} \in R^n$ is the control input. Suppose that $\boldsymbol{y}$ is defined on a set S, and $S \supset \Omega$. Accordingly, $\boldsymbol{\omega}(t) = (\omega_1(t), \omega_2(t), \cdots, \omega_{m_1}(t))^{\mathrm{T}}$ is an m_1-dimensional Brownian motion, $\dot{\boldsymbol{\omega}}(t) = (\eta_1, \cdots, \eta_{m_1})^{\mathrm{T}}$ is the m_1-dimensional white noise vector, in which every two elements are statistically independent, i.e.

$$\begin{aligned} &E[\eta_i(t)] = 0, \\ &E[\eta_i(t)\eta_j(t')] = \delta_{ij}\delta(t-t'), \\ &i, j = 1, \cdots, m_1, \end{aligned} \tag{3}$$

and, $\boldsymbol{\sigma}(t, \boldsymbol{y}-\boldsymbol{x}) = (\sigma_1, \sigma_2, \cdots, \sigma_{m_1}) : R^n \to R^{n\times m_1}$ is a function of noise coupling strength.

Assumption 1 For any $\boldsymbol{y} = (y_1, y_2, \cdots, y_n)^{\mathrm{T}}$ and $\boldsymbol{z} = (z_1, z_2, \cdots, z_n)^{\mathrm{T}} \in S$, there exists a positive constant K such that

$$|g_i(\boldsymbol{y}) - g_i(\boldsymbol{z})| \le K\left\|\boldsymbol{y}-\boldsymbol{z}\right\|, \quad i = 1, 2, \cdots, n. \tag{4}$$

We call the above condition the uniform Lipschitz condition, and K is referred to as the uniform Lipschitz constant. Note that condition (4) holds as long as $\partial g_i/\partial y_j (i, j = 1, 2, \cdots, n)$ are bounded.

Assumption 2 For any $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)^{\mathrm{T}}$ and $\boldsymbol{y} = (y_1, y_2, \cdots, y_n)^{\mathrm{T}} \in \Omega \subset R^n$, $\sigma_i(t, \boldsymbol{y}-\boldsymbol{x}) (i = 1, 2, \cdots, n)$ satisfies the Lipschitz condition. Moreover, there exists a non-negative constant k_1 such that

$$\begin{aligned} &\sigma_i^{\mathrm{T}}(t, \boldsymbol{y}-\boldsymbol{x})\sigma_i(t, \boldsymbol{y}-\boldsymbol{x}) \le k_1(\boldsymbol{y}-\boldsymbol{x})^{\mathrm{T}}(\boldsymbol{y}-\boldsymbol{x}), \\ &i = 1, 2, \cdots, n \end{aligned} \tag{5}$$

and $\sigma_i(t, 0, 0) \equiv 0$.

The implementation of chaos synchronization is converted into how to design the control input $\boldsymbol{u}$, which is able to synchronize the states of both the drive and the response systems with stochastic perturbations. Let $\boldsymbol{e} = \boldsymbol{y} - \boldsymbol{x}$, then the dynamic equation of synchronization error can be expressed as

$$\begin{aligned} \dot{\boldsymbol{e}} = {} & \boldsymbol{g}(\boldsymbol{y}) - \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{y})\hat{\boldsymbol{q}} - \boldsymbol{F}(\boldsymbol{x})\hat{\boldsymbol{p}} \\ & + \boldsymbol{\sigma}(t, \boldsymbol{e})\dot{\boldsymbol{\omega}} + \boldsymbol{u}. \end{aligned} \tag{6}$$

Hence, the objective of synchronization is to make $\lim\limits_{t\to\infty} \|\boldsymbol{e}(t; t_0, \boldsymbol{e}_0)\| = 0$, in light of the theory of stochastic differential equations, for any initial condition $\boldsymbol{e}_0 \in R^n$, where $\boldsymbol{e}(t; t_0, \boldsymbol{e}_0)$ denotes a unique global solution $(t \ge t_0)$ of error dynamics (6). Clearly, $\boldsymbol{e}(t; 0) = 0$ is a trivial solution of the error dynamics (6).

Inspired by the work in Ref.[18], we choose a control input as

$$\boldsymbol{u} = \boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{g}(\boldsymbol{x}) + \boldsymbol{\varepsilon}\boldsymbol{e} + \boldsymbol{F}(\boldsymbol{x})\boldsymbol{p} - \boldsymbol{G}(\boldsymbol{y})\boldsymbol{q} \tag{7}$$

with the feedback strength $\boldsymbol{\varepsilon} = \mathrm{diag}(\varepsilon_i) \in R^{n\times n}$ updated by the following law

$$\dot{\varepsilon}_i = -\beta_i e_i^2, \quad i = 1, 2, \cdots, n \tag{8}$$

and the parameters $\boldsymbol{p}$ and $\boldsymbol{q}$ updated according to the following noise-perturbed update laws

$$\dot{\boldsymbol{p}} = -(\boldsymbol{F}(\boldsymbol{x}))^{\mathrm{T}}\boldsymbol{e} + \boldsymbol{\varsigma}(t, \boldsymbol{e})\dot{\boldsymbol{\mu}}, \tag{9}$$

$$\dot{\boldsymbol{q}} = (\boldsymbol{G}(\boldsymbol{y}))^{\mathrm{T}}\boldsymbol{e} + \boldsymbol{\xi}(t, \boldsymbol{e})\dot{\boldsymbol{\upsilon}}, \tag{10}$$

where $\beta_i (i = 1, 2, \cdots, n)$ are arbitrary positive constant, $\dot{\boldsymbol{\mu}}(t)$ and $\dot{\boldsymbol{\upsilon}}(t)$ are m_2-dimensional and m_3-dimensional white noise vectors, respectively, in which every two elements are statistically independent. $\boldsymbol{\varsigma}(t, \boldsymbol{e}) = (\varsigma_1, \varsigma_2, \cdots, \varsigma_n) : R^n \to R^{n\times m_2}$, and $\boldsymbol{\xi}(t, \boldsymbol{e}) = (\xi_1, \xi_2, \cdots, \xi_n) : R^n \to R^{n\times m_3}$ are functions of noise coupling strength, and there exist non-negative constants k_2 and k_3 such that $\boldsymbol{\varsigma}(t, \boldsymbol{e})$ and $\boldsymbol{\xi}(t, \boldsymbol{e})$ satisfy assumption 2.

Then the combination of the control input and noise-perturbed update laws can make a contribution to achieving the required synchronization by the Lasalle invariance principle for stochastic differential equations,[19] which will be described in the following theorem.

Theorem 1 Under control input (7), noise-perturbed update laws (9) and (10), the trivial solution of the error system (6) is asymptotically stable, i.e. noise-perturbed response system (2) and drive system (1) can be completely synchronized.

Proof Construct a non-negative function in the form of

$$V(\boldsymbol{e},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{q}},\boldsymbol{\varepsilon}) = \frac{1}{2}(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e}+\tilde{\boldsymbol{p}}^{\mathrm{T}}\tilde{\boldsymbol{p}}+\tilde{\boldsymbol{q}}^{\mathrm{T}}\tilde{\boldsymbol{q}}) + \frac{1}{2}\sum_{i=1}^{n}\frac{1}{\beta_i}(\varepsilon_i+L)\varepsilon_i^2, \tag{11}$$

where $\tilde{\boldsymbol{p}}=\boldsymbol{p}-\hat{\boldsymbol{p}}$, $\tilde{\boldsymbol{q}}=\boldsymbol{q}-\hat{\boldsymbol{q}}$, and L is a constant bigger than $n(K+k_1+k_2+k_3)$, i.e. $L>n(K+k_1+k_2+k_3)$. By the Itô-differential rule, the stochastic derivative function along the trajectories of Eq.(6) can be obtained as follows:

$$\dot{V}(\boldsymbol{e},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{q}},\boldsymbol{\varepsilon}) = \ell V(\boldsymbol{e},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{q}},\boldsymbol{\varepsilon}) + V_{\varepsilon}(\boldsymbol{e},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{q}},\boldsymbol{\varepsilon})\dot{\boldsymbol{\omega}} + V_{\tilde{p}}(\boldsymbol{e},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{q}},\boldsymbol{\varepsilon})\dot{\boldsymbol{\mu}} + V_{\tilde{q}}(\boldsymbol{e},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{q}},\boldsymbol{\varepsilon})\dot{\boldsymbol{\upsilon}}. \tag{12}$$

The generator for ℓV the evolution of V is given by

$$\begin{aligned}
\ell V(\boldsymbol{e},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{q}},\boldsymbol{\varepsilon}) &= \dot{\boldsymbol{e}}^{\mathrm{T}}\boldsymbol{e}+\tilde{\boldsymbol{p}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{p}}}+\tilde{\boldsymbol{q}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{q}}}+\sum_{i=1}^{n}\frac{1}{\beta_i}(\varepsilon_i+L)\dot{\varepsilon}_i+\sum_{i=1}^{n}\sigma_i^{\mathrm{T}}(t,\boldsymbol{e}(t))\sigma_i(t,\boldsymbol{e}(t))\\
&\quad+\sum_{i=1}^{n}\varsigma_i^{\mathrm{T}}(t,\boldsymbol{e}(t))\varsigma_i(t,\boldsymbol{e}(t))+\sum_{i=1}^{n}\xi_i^{\mathrm{T}}(t,\boldsymbol{e}(t))\xi_i(t,\boldsymbol{e}(t))\\
&= (\boldsymbol{g}(\boldsymbol{y})-\boldsymbol{g}(\boldsymbol{x})+\boldsymbol{F}(\boldsymbol{x})\hat{\boldsymbol{p}}-\boldsymbol{G}(\boldsymbol{y})\hat{\boldsymbol{q}}+\boldsymbol{\varepsilon}\boldsymbol{e})^{\mathrm{T}}\boldsymbol{e}-\tilde{\boldsymbol{p}}^{\mathrm{T}}(\boldsymbol{F}(\boldsymbol{x}))^{\mathrm{T}}\boldsymbol{e}+\tilde{\boldsymbol{q}}^{\mathrm{T}}(\boldsymbol{G}(\boldsymbol{x}))^{\mathrm{T}}\boldsymbol{e}+\sum_{i=1}^{n}\frac{1}{\beta_i}(\varepsilon_i+L)\dot{\varepsilon}_i\\
&\quad+\sum_{i=1}^{n}\sigma_i^{\mathrm{T}}(t,\boldsymbol{e}(t))\sigma_i(t,\boldsymbol{e}(t))+\sum_{i=1}^{n}\varsigma_i^{\mathrm{T}}(t,\boldsymbol{e}(t))\varsigma_i(t,e(t))+\sum_{i=1}^{n}\xi_i^{\mathrm{T}}(t,\boldsymbol{e}(t))\xi_i(t,\boldsymbol{e}(t))\\
&= (\boldsymbol{g}(\boldsymbol{y})-\boldsymbol{g}(\boldsymbol{x}))^{\mathrm{T}}\boldsymbol{e}-L\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e}+\sum_{i=1}^{n}\sigma_i^{\mathrm{T}}(t,\boldsymbol{e}(t))\sigma_i(t,\boldsymbol{e}(t))\\
&\quad+\sum_{i=1}^{n}\varsigma_i^{\mathrm{T}}(t,\boldsymbol{e}(t))\varsigma_i(t,\boldsymbol{e}(t))+\sum_{i=1}^{n}\xi_i^{\mathrm{T}}(t,\boldsymbol{e}(t))\xi_i(t,\boldsymbol{e}(t))\\
&= \sum_{i=1}^{n}(g_i(y)-g_i(x))e_i-L\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e}+\sum_{i=1}^{n}\sigma_i^{\mathrm{T}}(t,\boldsymbol{e}(t))\sigma_i(t,\boldsymbol{e}(t))+\sum_{i=1}^{n}\varsigma_i^{\mathrm{T}}(t,\boldsymbol{e}(t))\varsigma_i(t,\boldsymbol{e}(t))\\
&\quad+\sum_{i=1}^{n}\xi_i^{\mathrm{T}}(t,\boldsymbol{e}(t))\xi_i(t,\boldsymbol{e}(t)).
\end{aligned} \tag{13}$$

By assumptions 1 and 2, we can obtain

$$\ell V \le \sum_{i=1}^{n}\|\boldsymbol{e}\|\,|e_i|-L\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e}+\sum_{i=1}^{n}k_1+\sum_{i=1}^{n}k_2+\sum_{i=1}^{n}k_3 \le (n(K+k_1+k_2+k_3)-L)\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e}\le 0. \tag{14}$$

Hence, following from the LaSalle-type invariance principle for the stochastic differential equations proposed in Ref.[19], it can be concluded that the trivial solution of error system (6) is asymptotically stable, i.e. noise-perturbed response system (2) can synchronize with the drive system (1) asymptotically. *QED.*

Remark 1 Systems (1) and (2) describe a lot of classic chaotic systems, such as Lorenz system,[20] Chen system,[21] Rössler system, unified system[22] and so on.

Remark 2 The method proposed in this paper is the chaotic system, but it is also applicable to the hyperchaotic systems and multi-scroll chaotic systems.[23]

3. Illustrative examples

In this section, two examples and corresponding numerical simulations are given to illustrate the validity of the proposed method. In example 1, we consider the synchronization between a drive system with unknown parameters and a noise-perturbed response

with known parameters. In example 2, the synchronization between a drive system and a noise-perturbed response system all with unknown parameters is considered. In the two examples, example 1 is discussed in detail, while example 2 briefly.

Example 1 To observe the synchronization between a unified system and a noise-perturbed Rössler system, take the unified system[22] as the drive system

$$\begin{aligned}\dot{x}_1 &= (25\hat{p}+10)(x_2-x_1),\\ \dot{x}_2 &= (28-35\hat{p})x_1 - x_1x_3 + (29\hat{p}-1)x_2,\\ \dot{x}_3 &= x_1x_2 - (8+\hat{p})x_3/3,\end{aligned} \tag{15}$$

where unknown parameter $\hat{p}\in[0,1]$. Obviously, system (15) becomes the generalized Lorenz system[20] for $\hat{p}\in[0,0.8)$, while system (15) becomes the original Chen system[21] for $\hat{p}\in(0.8,1]$. When $\hat{p}=0.8$, system (15) becomes the Lü system.[24] In particular, system (15) contains the Lorenz and Chen systems as two extremes and the Lü system bridges the gap between the Lorenz and Chen systems. Moreover, system (15) is always chaotic in the whole interval $\hat{p}\in[0,1]$.

And take the noise-perturbed Rössler system as the response system

$$\begin{aligned}\dot{y}_1 &= -y_2 - y_1 + \sigma_1(t,e(t)) + u_1,\\ \dot{y}_2 &= y_1 + \hat{q}_1y_2 + \sigma_2(t,e(t)) + u_2,\\ \dot{y}_3 &= y_1y_3 - \hat{q}_2y_2 + \hat{q}_3 + \sigma_3(t,e(t)) + u_3.\end{aligned} \tag{16}$$

Convert systems (15) and (16) into the forms of Eqs.(1) and (2) as follows:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{F}(\boldsymbol{x})\hat{\boldsymbol{p}}, \tag{17}$$

where $\boldsymbol{x}=(x_1,\ x_2,\ x_3)^{\mathrm{T}}$ is the state vector, $\hat{\boldsymbol{p}}$ is the unknown parameter, $\boldsymbol{f}(\boldsymbol{x})=(10(x_2-x_1),\ 28x_1-x_1x_3-x_2,\ x_1x_2-8x_3/3)^{\mathrm{T}}$, and $\boldsymbol{F}(\boldsymbol{x})=\begin{pmatrix}25(x_2-x_1)\\ -35x_1+29x_2\\ -x_3/3\end{pmatrix}$, and

$$\dot{\boldsymbol{y}} = \boldsymbol{g}(\boldsymbol{y}) + \boldsymbol{G}(\boldsymbol{y})\hat{\boldsymbol{q}} + \boldsymbol{\sigma}(t,\boldsymbol{y}-\boldsymbol{x})\dot{\boldsymbol{\omega}} + \boldsymbol{u}, \tag{18}$$

where $\boldsymbol{y}=(y_1,y_2,y_3)^{\mathrm{T}}$ is the state vector, $\hat{\boldsymbol{q}}=0$, $\boldsymbol{g}(\boldsymbol{y})=\begin{pmatrix}-y_2-y_1\\ y_1\\ y_1y_3\end{pmatrix}$, $\boldsymbol{G}(\boldsymbol{y})=\begin{pmatrix}0&0&0\\ y_2&0&0\\ 0&1&-y_3\end{pmatrix}$, and $\boldsymbol{u}=(u_1,u_2,u_3)^{\mathrm{T}}$ is the control input. The elements of noise coupling strength function for noise-coupled system (18) can be written as

$$\begin{aligned}\boldsymbol{\sigma}_1(t,\boldsymbol{e}(t)) &= (a_1e_1(t),0,0),\\ \boldsymbol{\sigma}_2(t,\boldsymbol{e}(t)) &= (0,a_2e_2(t),0),\\ \boldsymbol{\sigma}_3(t,\boldsymbol{e}(t)) &= (0,0,a_3e_3(t)).\end{aligned} \tag{19}$$

According to Theorem 1, the control input is taken as

$$\begin{aligned}\boldsymbol{u} &= \boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{g}(\boldsymbol{x}) + \boldsymbol{F}(\boldsymbol{x})\boldsymbol{p} + \varepsilon\boldsymbol{e}\\ &= \begin{pmatrix}10(x_2-x_1)\\ 28x_1-x_1x_3-x_2\\ x_1x_2-8x_3/3\end{pmatrix} - \begin{pmatrix}-x_2-x_1\\ x_1\\ x_1x_3\end{pmatrix}\\ &\quad + \begin{pmatrix}(x_2-x_1)p\\ x_1p\\ -x_3p\end{pmatrix} + \begin{pmatrix}\varepsilon_1e_1\\ \varepsilon_2e_2\\ \varepsilon_3e_3\end{pmatrix},\end{aligned} \tag{20}$$

and noise-perturbed parameters update law is taken as

$$\dot{\boldsymbol{p}} = -(\boldsymbol{F}(\boldsymbol{x}))^{\mathrm{T}}\boldsymbol{e} + \boldsymbol{\varsigma}(t,\boldsymbol{e})\dot{\boldsymbol{\mu}}, \tag{21}$$

where $\boldsymbol{\varsigma}(t,\boldsymbol{e}(t))=(be_1,0,0)$, and $\dot{\boldsymbol{\mu}}(t)$ is a three-dimensional white noise vector, in which every two elements are statistically independent. $\boldsymbol{\varsigma}(t,\boldsymbol{e}(t))$ satisfies assumption 2.

In order to check the appropriateness of the above theoretical analysis, we carry out numerical simulations using the well-known Euler–Maruyama numerical scheme with a time step-size of 0.002. According to Theorem 1, we take the updated feedback strength as $\dot{\varepsilon}_i=-\beta_ie_i^2$ with $\beta_i=1.2$, $i=1,2,3$. In this paper, we select unknown parameters $\hat{p}=0.8$. Without loss of the generality, we assign the initial conditions to $p(0)=-0.2$, $\boldsymbol{x}(0)=(0.0,0.0,30.0)^{\mathrm{T}}$, $\boldsymbol{y}(0)=(1.5,-4.4,1.5)^{\mathrm{T}}$, $\varepsilon_1(0)=1.0$, $\varepsilon_2(0)=4.0$ and $\varepsilon_3(0)=3.0$. The parameters in the two systems (19) and (21) are taken as $a_1=0.2$, $a_2=0.4$, $a_3=-0.3$ and $b=0.2$.

The numerical results are displayed in Figs.1–3, and they indicate that all the state variables of synchronization error between systems (17) and (18) quickly approach to zero. Figure 2 shows that the unknown parameter evolution of drive system (17) rapidly converges to a constant. At the same time, the updated feedback strengths go to some constants, separately, which is depicted in Fig.3.

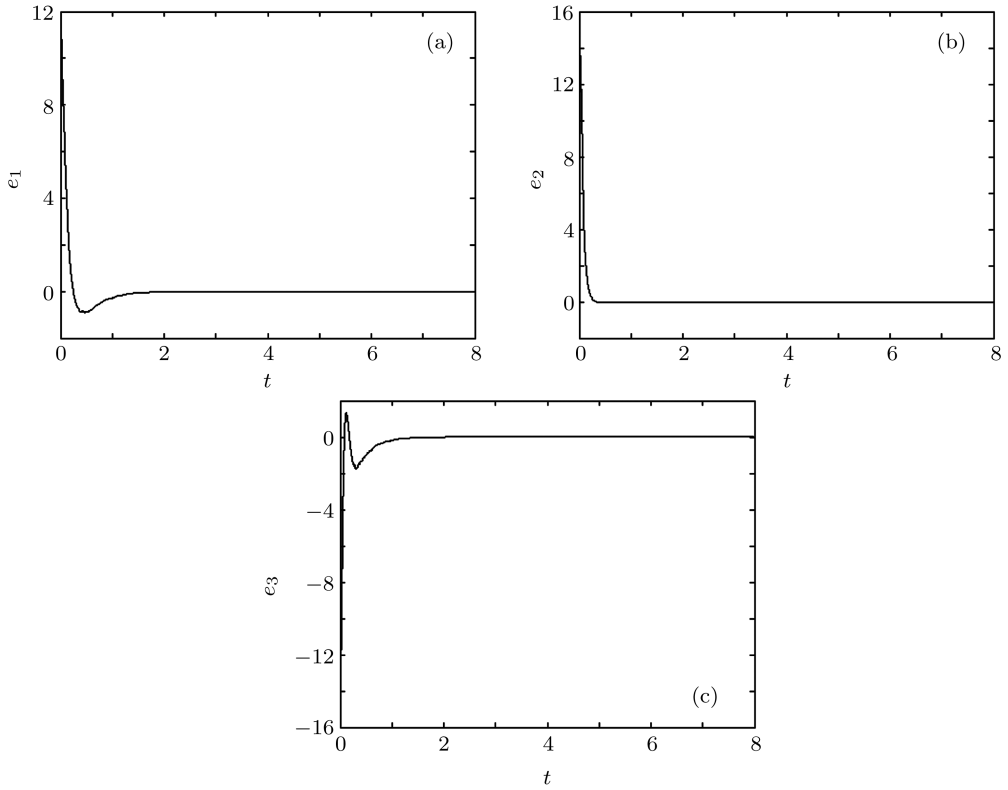


Fig.1. Time evolution of synchronization errors.

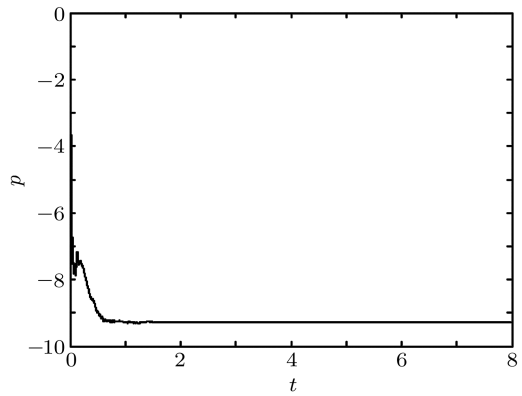


Fig.2. Time evolution of unknown parameter of drive system (17).

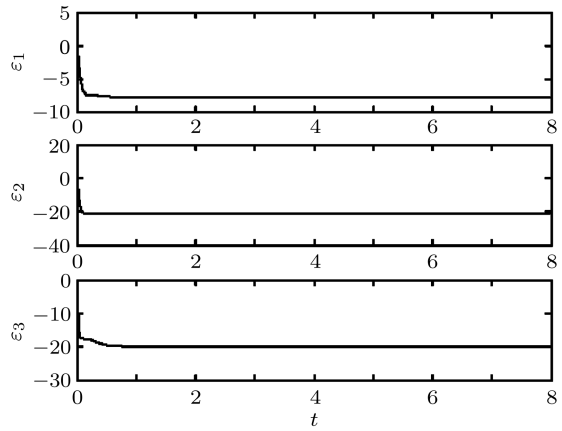


Fig.3. Time evolution of updated feedback strength.

Example 2 Consider a hyperchaotic Chen system as the drive system, which is described as

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{F}(\boldsymbol{x})\hat{\boldsymbol{p}}, \tag{22}$$

where $\boldsymbol{x} = (x_1,\ x_2,\ x_3,\ x_4)^{\mathrm{T}}$ is the state vector, $\hat{\boldsymbol{p}} = (\hat{p}_1,\ \hat{p}_2,\ \hat{p}_3,\ \hat{p}_4,\ \hat{p}_5)^{\mathrm{T}}$ is the unknown parameter, $\boldsymbol{f}(\boldsymbol{x}) = (x_4,\ -x_1x_3,\ x_1x_2,\ x_2x_3)^{\mathrm{T}}$, $\boldsymbol{F}(\boldsymbol{x}) = \begin{pmatrix} x_2 - x_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x_2 & x_1 & 0 \\ 0 & -x_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x_4 \end{pmatrix}$, the noise-perturbed hyperchaotic Rössler system is taken as the response system as follows:

$$\dot{\boldsymbol{y}} = \boldsymbol{g}(\boldsymbol{y}) + \boldsymbol{G}(\boldsymbol{y})\hat{\boldsymbol{q}} + \boldsymbol{\sigma}(t, \boldsymbol{y} - \boldsymbol{x})\dot{\boldsymbol{\omega}} + \boldsymbol{u}, \tag{23}$$

where $\boldsymbol{y} = (y_1,\ y_2,\ y_3,\ y_4)^{\mathrm{T}}$ is the state vector, $\hat{\boldsymbol{q}} = (\hat{q}_1,\ \hat{q}_2,\ \hat{q}_3,\ \hat{q}_4)^{\mathrm{T}}$ is the unknown parameter, $\boldsymbol{u} = (u_1,\ u_2,\ u_3,\ u_4)^{\mathrm{T}}$ is the control input, $\boldsymbol{g}(\boldsymbol{y}) = (-y_2 - y_3,\ y_1 + y_4,\ y_1y_3,\ 0)^{\mathrm{T}}$, and $\boldsymbol{G}(\boldsymbol{y}) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ y_2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -y_3 & y_4 \end{pmatrix}$. For convenience, the elements of noise coupling strength function for noise-coupled system (23) can be written as

$$\sigma_i(t, \boldsymbol{e}(t)) = \mathrm{diag}(a_i e_i(t)) \quad (i = 1, 2, 3, 4). \tag{24}$$

According to Theorem 1, the control input u and the noise-perturbed parameter update laws can be easily set up, separately. For simplicity, we take noise coupling strength functions of Eqs.(9) and (10) as

$$\boldsymbol{\varsigma}(t, \boldsymbol{e}(t)) = (\boldsymbol{0}, \mathrm{diag}(c_i e_i(t)))^{\mathrm{T}} \quad (i = 1, 2, 3, 4), \tag{25}$$

$$\boldsymbol{\xi}(t, \boldsymbol{e}(t)) = \mathrm{diag}(b_i e_i(t)) \quad (i = 1, 2, 3, 4), \tag{26}$$

where $\boldsymbol{0} = (0, 0, 0, 0)^{\mathrm{T}}$ is the zero vector. $\boldsymbol{\mu}$ and $\boldsymbol{\upsilon}$ are five-dimensional and four-dimensional Brownian motions, respectively. $\boldsymbol{\varsigma}(t, \boldsymbol{e}(t))$ and $\boldsymbol{\xi}(t, \boldsymbol{e}(t))$ satisfy assumption 2.

In order to check the appropriateness of the above theoretical analysis, we carry out numerical simulations using the well-known Euler–Maruyama numerical scheme with a time step-size of 0.002. According to Theorem 1, we take the updated feedback strength as $\dot{\varepsilon}_i = -\beta_i e_i^2$, with $\beta_i = 2.1$, $i = 1, 2, 3, 4$. At the same time, we choose the unknown parameter vectors as $\hat{\boldsymbol{p}} = (35, 3, 12, 7, 0.5)^{\mathrm{T}}$ and $\hat{\boldsymbol{q}} = (0.25, 3, 0.5, 0.05)^{\mathrm{T}}$. Without loss of the generality, we assign the initial conditions to $\boldsymbol{p}(0) = (27, 4.13, 7.3, 0.4)^{\mathrm{T}}$, $\boldsymbol{q}(0) = (0.7, 3.5, 1.0, 0.2)^{\mathrm{T}}$, $\boldsymbol{x}(0) = (-0.2,\ 1.5,\ 2.1,\ 0.5)^{\mathrm{T}}$, $\boldsymbol{y}(0) = (4.3, 0.2, 5.0, 4.0)^{\mathrm{T}}$, $\varepsilon_1(0) = 3.0$, $\varepsilon_2(0) = 1.3$, $\varepsilon_3(0) = 0.4$, and $\varepsilon_4(0) = 2.7$. The parameters in the three systems (24), (25) and (26) are taken as $a_1 = 0.2$, $a_2 = 0.4$, $a_3 = -0.3$, $a_4 = 1.3$, $b_1 = 0.1$, $b_2 = 0.2$, $b_3 = -0.4$, $b_4 = 2.1$, $c_1 = 1.2$, $c_2 = 0.2$, $c_3 = 2.1$, and $c_4 = 0.8$.

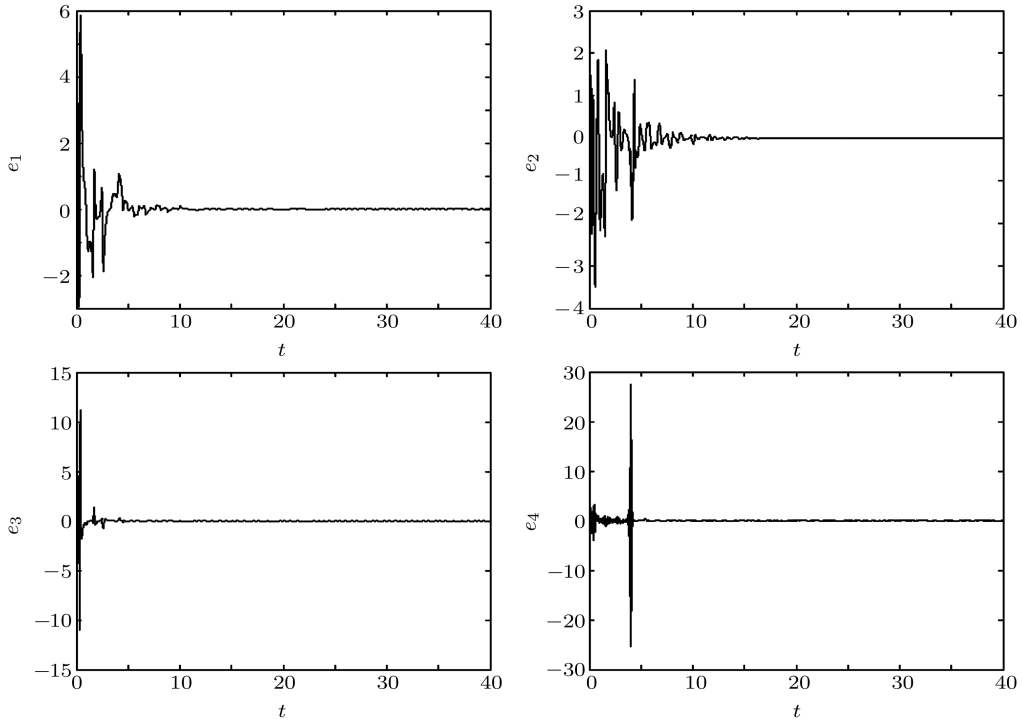


Fig.4. Time evolution of synchronization errors.

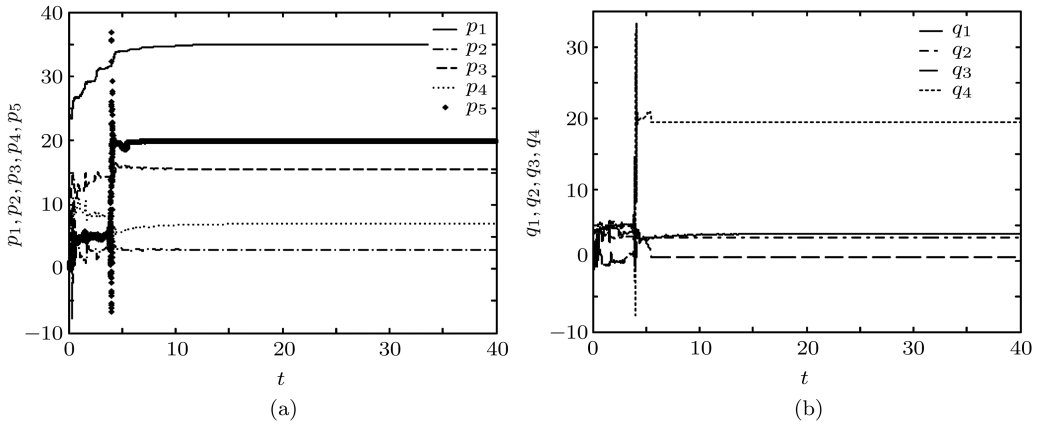


Fig.5. Time evolutions of unknown parameters of drive system (22) (a) and noise-perturbed response system (23) (b).

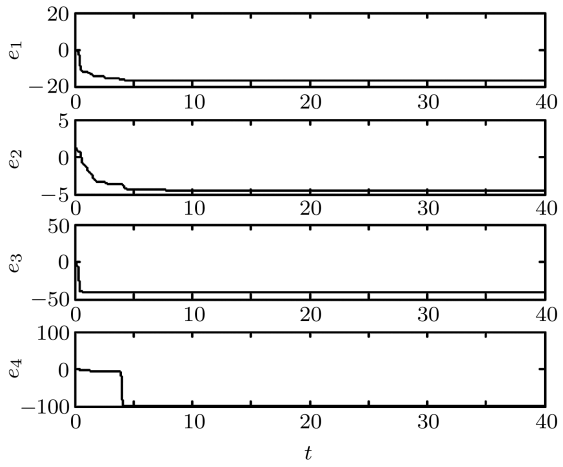


Fig.6. Time evolution of updated feedback strength.

The numerical results are displayed in Figs.4–6, and they indicates that all the state variables of synchronization error between systems (22) and (23) quickly approach to zero. Figure 5 shows that the unknown parameter evolutions of drive system (22) and noise-perturbed response system (23) rapidly converge to some constants. At the same time, the updated feedback strengths go to some constants, separately, which is depicted in Fig.6.

4. Conclusions

In this paper, we present a general method of synchronizing different noise-perturbed chaotic systems with unknown parameters. Based on the LaSalle-type invariance principle for stochastic differential equations, and by employing the combination of feedback control and adaptive control, some sufficient conditions of chaos synchronization between these noise-perturbed systems with unknown parameters are established. The model used in the research is the chaotic system, but the method is also applicable to the hyperchaotic systems. Two illustrative examples are given to demonstrate the validity of this technique, and numerical simulations are also given to show the effectiveness of the method.

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