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The improved local linear prediction of chaotic time series

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Based on the Bayesian information criterion, this paper proposes the improved local linear prediction method to predict chaotic time series. This method uses spatial correlation and temporal correlation simultaneously. Simulation results show that the improved local linear prediction method can effectively make multi-step and one-step prediction of chaotic time series and the multi-step prediction performance and one-step prediction accuracy of the improved local linear prediction method are superior to those of the traditional local linear prediction method.

Keywords: local linear prediction, Bayesian information criterion, state space reconstruction, chaotic time series

PACC: 0545

1. Introduction

Deterministic chaos, which is determined by nonlinear deterministic dynamical mechanism, is a kind of irregular movement existing widely in nature and society. With the development of chaos theory and research on its application technique, the modelling and prediction of chaotic time series are currently among the most important challenges in the field of chaotic signal processing.[1] Many prediction methods have been proposed in the past few years, and there are three basic methods: global prediction method.[2−4] local prediction method[5−13] and adaptive nonlinear prediction method.[14−18]

Adaptive nonlinear methods can effectively predict chaotic time series, and one-step prediction accuracy is well. However, Refs.[14–18] mainly study the methods' one-step accuracy, and talk a little about the methods' multi-step performance.

Local methods belong the most usually used methods to predict chaotic time series. References [8–11] use adaptive technique to improve local methods, and Ref.[12] uses support vector machine technique to improve local methods. Reference [13] is based on the local quadratic polynomial model and uses the discrete cosine transformation (DCT) to lessen the complexity of solving coefficient matrix.

In this paper the improved local linear prediction method is proposed to predict chaotic time series, where the embedding dimension and the time delay are chosen to be different values in the procedure of finding the nearest neighbour points and in the prediction procedure. This improved method uses spatial correlation and temporal correlation simultaneously. The multi-step prediction performance and one-step prediction accuracy of the improved local linear prediction method are superior to those of the traditional local linear prediction method.

This paper is organized as follows. The improved local linear prediction method is described in Section 2, and the effectiveness of this improved local linear prediction method is illustrated by comparison with the local linear prediction method in Section 3. Section 4 contains a brief summary and discussion.

2. The improved local linear prediction method

There are different sources of predictability in time series. The most common and best-studied one is the existence of linear correlation in time. A linear stochastic prediction model, an autoregressive, a moving average, or a combination of AR and MA, is based on the linear correlation in time and exploit all the information of linear correlation in time. Another source of predictability is nonlinear determinism. Chaos system is a nonlinear deterministic system which is de-

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termined by nonlinear dynamical mechanism. There is inner nonlinear deterministic rule in chaos system. The predictions of chaotic time series are based on nonlinear determinism which implies that equal states have an equal future, and similar states will evolve similarly.

The reconstruction of a state space which is equivalent to the original state space of a system from a scale time series is the basis of chaotic time series predictions. Based on Whitney's topological embedding theorem, Takens[19] has proved the delay embedding theorem which offers the possibility of reconstructing m-dimensional dynamics from a scale time series. Suppose that an observed scalar time series is $\{x(n), n = 1, 2, 3, \cdots, N\}$. According to Takens' delay embedding theorem, an embedding of the strange chaotic attractors can be obtained by constructing delay vectors $X(n) = [x(n), x(n-\tau), \cdots, x(n-(m-1)\tau)]^{\mathrm{T}}, n = (m-1)\tau+1, (m-1)\tau+2, \cdots, N$ from time-delayed coordinates, where m is the embedding dimension and τ is the time delay. In application the key for the state space reconstruction is determining its optimal parameters. When the embedding dimension is larger than the minimum embedding dimension, the trajectory of reconstructed vectors can reflect the true state space evolution of chaotic systems. From the mathematical point of view the time delay τ is arbitrary, since the data items are assumed to have infinite precision. In applications the time delay should not be taken too small that there is little difference between the different elements of the delay vectors, and should not be taken too large that the different coordinates may be almost uncorrelated.

The first step of local linear prediction is to find the neighbour points of the current state $X(n)$ in the reconstructed state space. According to the reconstructed trajectory, we compute the distances between the current delay vectors $X(n)$ and its $n-1$ preceding delay vectors $X(i)(i = 1, 2, \cdots, n-1)$

$$d(i) = \|X(i) - X(n)\|_2, \tag{1}$$

and find the k nearest neighbour points $X(n_i)(i = 1, 2, \cdots, k)$, where the embedding dimension and the time delay are chosen as the proper value m_{opt} and τ_{opt} which are optimal for the state space construction, and $k > m_{\mathrm{opt}}$.[5] The local linear model in a m_{opt}-dimensional delay embedding space is a moving average model of order m_{opt}, such that the prediction is a linear superposition of the m_{opt} elements of delay vector, that is

$$\begin{aligned}\hat{x}(n+1) &= a_0 + \sum_{i=1}^{m_{\mathrm{opt}}} a_i x(n-(i-1)\tau_{\mathrm{opt}}) \\ &= AX'(n),\end{aligned} \tag{2}$$

where $A = [a_0, a_1, a_2, \cdots, a_{m_{\mathrm{opt}}}]$, $X'(n) = [1, x(n), x(n-\tau_{\mathrm{opt}}), \cdots, x(n-(m_{\mathrm{opt}}-1)\tau_{\mathrm{opt}})]^{\mathrm{T}}$. Deterministic predictions assume that the reconstructed trajectory is governed by a deterministically continuously mapping F between the current state and the future state. If the state at time j was similar to the current state $X(n)$, continuity of F guarantees that the state $X(j+1)$ will also be close to the future state $X(n+1)$.

In the prediction procedure of Eq.(2), it is not necessary that $m = m_{\mathrm{opt}}$ and $\tau = \tau_{\mathrm{opt}}$. The parameters m and τ should take values which is more suitable for making prediction. In this paper the parameters' values are chosen as $\tau = 1$ and $m = p$, where p is the order of moving average model in Eq.(2), then Eq.(2) can be written as

$$\hat{x}(n+1) = a_0 + \sum_{i=1}^{p} a_i x(n-i+1) = A_p X'_p(n), \tag{3}$$

where $A_p = [a_0, a_1, a_2, \cdots, a_p]$, $X'_p(n) = [1,\ x(n),\ x(n-1),\ \cdots,\ x(n-p+1)]^{\mathrm{T}}$.

The entries of the coefficient vector A_p depend on the location of current states $X(n)$ and its topological neighbourhood $X(n_i)(i = 1, 2, \cdots, k)$. The coefficient vector A_p can be obtained from the following equation:

$$A_p B = D, \tag{4}$$

where B is a $(p+1) \times k$ matrix whose ith column is made of $X'_p(n_i)$ and D is a $1 \times k$ vector whose ith column is made of $x(n_i+1)$ and

$$X'_p(n_i) = [1, x(n_i), x(n_i-1), \cdots, x(n_i-p+1)]^{\mathrm{T}},$$

$$D = [x(n_1+1), x(n_2+1), x(n_3+1), \cdots, x(n_k+1)].$$

The same scheme may be followed to calculate $\hat{x}(n+2)$, which follows the last point, now it is $X(n+1)$.

We search for the optimal order of moving average model of Eq.(3) that minimizes the following Bayesian information criterion (BIC):[20]

$$C(p) = \ln \varepsilon^2(p) + p\frac{\ln L}{L}, \tag{5}$$

where $\varepsilon^2(p)$ is the prediction error of a p-order moving average model, and L is the length of the prediction data. In this paper we use the multi-step prediction error

$$\varepsilon^2(p) = \frac{1}{L}\sum_{i=1}^{L}[x(n+i) - \hat{x}(n+i)]^2. \tag{6}$$

3. Simulation and results

To illustrate the performance of the improved local linear prediction method, we use the Lorenz model as an example. This model describes the Rayleigh–Benard convection arising from the two dimensional Navier–Stokes equation, which is formulated for a fluid slab of finite thickness subjected to gravity loading, heated from below and between the top cold and the bottom hot surface the temperature is held constant. The partial differential equations were transformed to a set of three ordinary differential equations. Later Lorenz described the way he derived the model both physically and mathematically. The Lorenz equation can be written in the following forms:

$$\begin{aligned} \dot{x} &= \sigma(y - x), \\ \dot{y} &= x(r - z) - y, \\ \dot{z} &= xy - bz, \end{aligned} \tag{7}$$

where x is the amplitude of the convection motion, y is the temperature between the ascending and descending currents, z is the distortion of the vertical temperature profile from linearity, and σ, r, and b are dimensionless parameters. The parameters are most commonly selected to be $\sigma = 10, r = 28$, and $b = 8/3$ for a rich dynamical behaviour.

The standard fourth-order Runge–Kutta method is used to solve the equations (discarding the transients). The time step is set to be $\delta = 0.01$, and the x values are used to reconstruct the state space with the time delay $\tau_{\rm opt} = 14$ and the embedding dimension $m_{\rm opt} = 3$. Randomly selected two different time series with the length of $N = 10000$ are used as training set. In the numerical simulation of this paper the parameter k take the value of $m_{\rm opt} + 1$.

The multi-step prediction is made as far as 350 and 1000 time steps into the future using the local linear prediction method and the improved local linear prediction method for the two different time series respectively, and the results are shown in Figs.2 and 3. In Figs.2 and 3, we list the actual value for comparison, which isn't included in the multi-step prediction procedure.

The one-step prediction is made as far as 5000 time steps into the future using the local linear prediction method and the improved local linear prediction method, and the results are shown in Fig.4. The mean square error $e_{\rm m}$ is used to evaluate the methods' one-step prediction accuracy, where

$$e_{\rm m} = \frac{1}{n}\sum_{i=1}^{n}[x(i) - \hat{x}(i)]^2. \tag{8}$$

In the one-step prediction procedure, after predicting a future value, the actual value is added to the training set. In Figs.2, 3 and 4, ordinate $x(n)$ and $y(n)$ are the actual value (solid lines o) and the prediction value (dashed lines *) respectively.

The curve $C(p)$–p is shown in Fig.1, where $L = 700$, from which we can see that $p = 10$ minimizes $C(p)$. So the optimal order of MA model in the prediction procedure is chosen as 10.

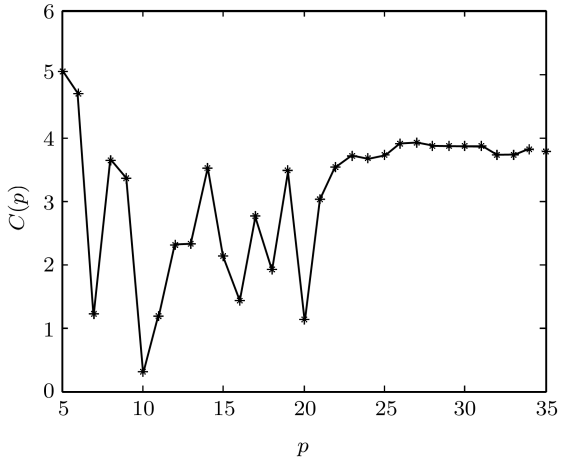


Fig.1. $C(p)$–p for time series from Lorenz system.

From Figs.2(a) and 2(b), it can be seen that, for the first time series, the prediction errors of the local linear method within 248 steps are low, and grow rapidly from the 249th step, where its maximum is 0.0148 within 117 steps, the prediction errors fluctuate in −2.2402–4.0997 from the 118th step to the 248th step. From Figs.2(c) and 2(d), it can be seen that, for the second time series, the prediction errors of the local linear method within 133 steps are low, and grow rapidly from the 134th step, where its maximum is 0.3443 within 133 steps.

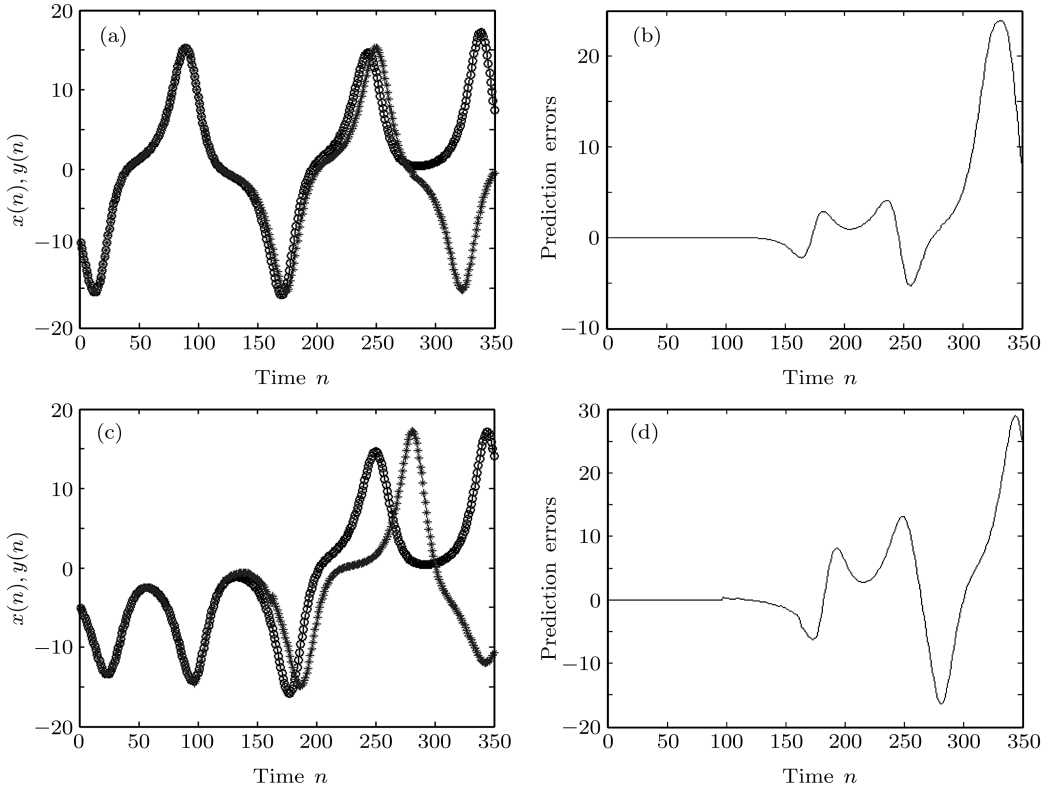


Fig.2. Multi-step prediction results of the local linear prediction method: (a) actual values and prediction values of the first time series; (b) prediction errors of the first time series; (c) actual values and prediction values of the second time series; (d) prediction errors of the second time series.

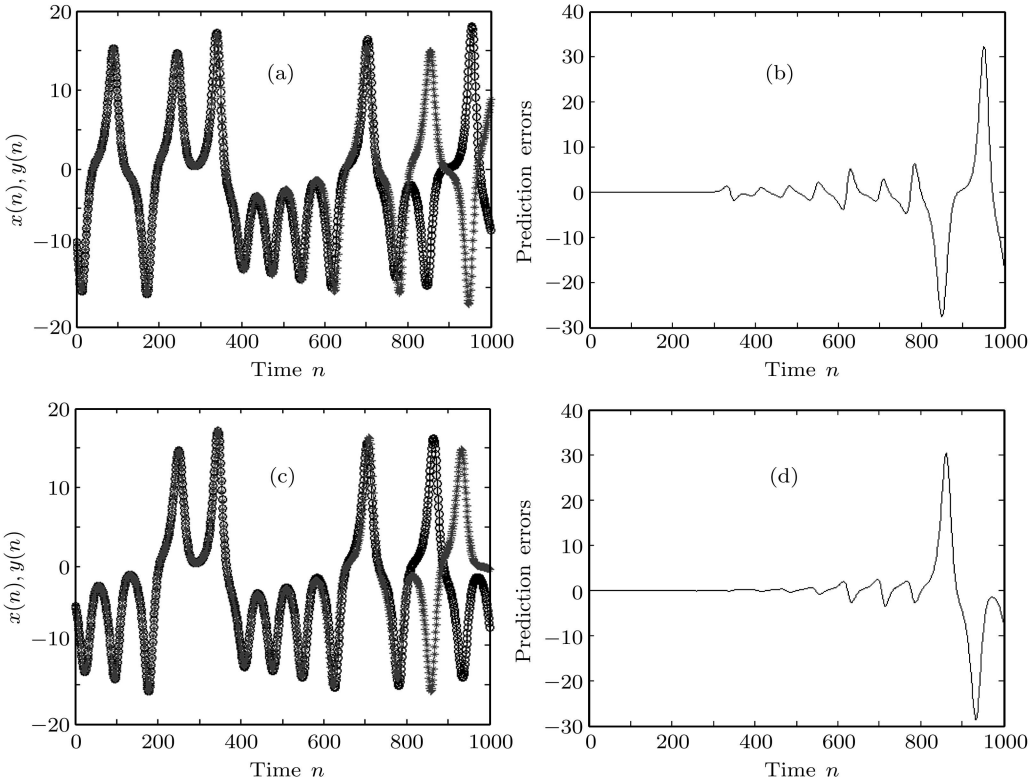


Fig.3. Multi-step prediction results of the improved local linear prediction method: (a) actual values and prediction values of the first time series; (b) prediction errors of the first time series; (c) actual values and prediction values of the second time series; (d) prediction errors of the second time series.

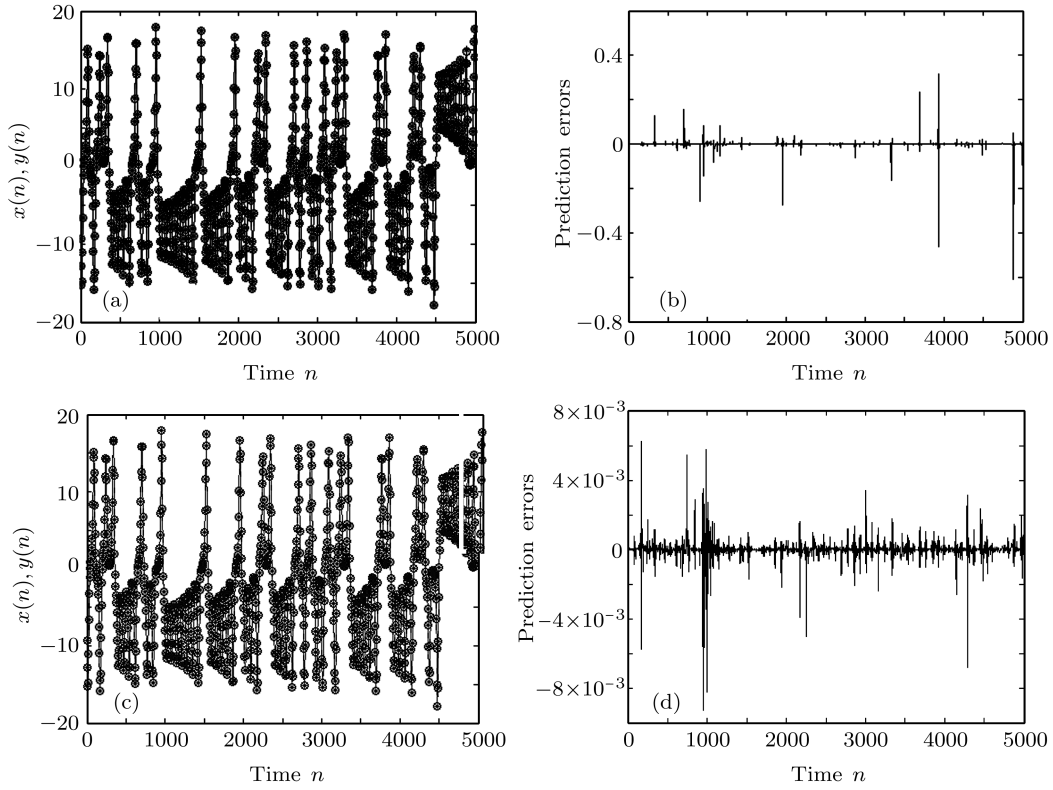


Fig.4. One-step prediction results: (a) actual values and prediction values of the local linear prediction method; (b) prediction errors of the local linear prediction method; (c) actual values and prediction values of the improved local linear prediction method; (d) prediction errors of the improved local linear prediction method.

From Figs.3(a) and 3(b), it can be seen that, for the first time series, the prediction values of the improved local linear prediction method follow the actual trajectory very well within 780 steps, and start diverging significantly from the actual value from the 781st step, where the prediction errors' maximum is only 0.0510 within 297 steps, the prediction errors fluctuate in −1.8161–1.4126 from the 298th step to the 546th step and fluctuate in −4.7248–5.1016 from the 547th step to the 780th step. From Figs.3(c) and 3(d), it can be seen that, for the second time series, the prediction values of the improved local linear prediction method follow the actual trajectory very well within 820 steps, and start diverging significantly from the actual value from the 821st step, where the prediction errors' maximum is only 0.3863 within 523 steps, the prediction errors fluctuate in −3.5009–2.5341 from the 524th step to the 820th step.

So the multi-step prediction performance of the improved local linear prediction method is obviously superior to that of the local linear prediction method.

From Fig.4, it can be seen clearly that the local linear method and the improved local linear method can make one-step prediction to be effective, one-step prediction accuracy is well, and the mean square error e_{m} of the local linear method is 2.4607×10^{-4}, and that of the improved local linear method is 2.7697×10^{-7}, the order of which is lower 3 than that of the local linear method. So the one-step prediction accuracy of the improved local linear method is obviously superior to that of the local linear method.

4. Conclusion

In this paper the improved local linear prediction method is proposed to predict chaotic time series, where in the procedure of finding the nearest neighbour points the embedding dimension and the time delay are chosen as the values which are optimal for the state space construction and in the prediction procedure the embedding dimension and the time delay are chosen as the values based on BIC. This improved method uses spatial correlation and temporal correlation simultaneously. Simulation results show that the multi-step prediction performance and one-step prediction accuracy of the improved local linear prediction method are all superior to those of the traditional local linear prediction method.

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