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Gravitational frequency-shift and deflection of light by quintessence*

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In this paper we investigate the gravitational frequency-shift and deflection of light in the Schwarzschild black hole space-time surrounded by quintessence. With the analytical and numerical methods, we find that the gravitational frequency-shift of light in the Schwarzschild black hole space-time surrounded by the quintessence increases as the values of the normalization factor c increases, but the gravitational frequency-shift of light decreases with the quintessential state parameter $\omega_{\rm q}$ increasing. We also calculate the deflection of light by quintessence and find that the deflection rate decreases as the values of the quintessential parameters c and $\omega_{\rm q}$ increase.

Keywords: frequency-shift, deflection, quintessence
PACC: 0420, 0430, 1180

1. Introduction

A large number of astronomical observations, such as type Ia supernovae,[1−5] compressed laryonic matter (CMB) experiment[6−8] and the large scale structure,[9−12] convincingly showed the accelerating expansion of the universe. The expansion implies the valuable contribution of matter with the negative pressure to the evolution of the universe. It is believed that the accelerating expansion is attributed to the fact that the universe is dominated by an exotic component with the negative pressure, called 'dark energy' which accounts for 70 percent of the energy density of the universe. There are several candidates for dark energy: The first one is the cosmological constant,[13,14] and the second one is the so-called dynamic candidates such as: phantom,[15−18] quintessence,[19−21] K-essence[22,23] and quintom.[24−26] The difference in dark energy among these candidates is the size of parameter $\omega_{\rm q}$, namely the ratio between the pressure and energy density of the dark energy. For quintessence, the equation of state is given by the relationship between the pressure $p_{\rm q}$ and the energy density $\rho_{\rm q}$, i.e. $p_{\rm q} = \omega_{\rm q}\rho_{\rm q}$ with $\omega_{\rm q}$ being in the rang of $-1 \leq \omega_{\rm q} \leq -1/3$, which causes the acceleration. The borderline case of $\omega_{\rm q} = -1$ of the extraordinary quintessence covers the cosmological constant term. As has been recognized,[27−29] that the acceleration is a challenge to the traditional theory of the quantum gravity because of the outer horizon, which does not allow one to introduce the observable S-matrix in terms of asymptotic past and future states. The outer horizon of the de Sitter space differs significantly from the inner horizon of a black hole, which has an asymptotically flat space far away from the black hole.

Up to now a lot of gravitational effects in the solar system and binary systems are well described by the Einstein general relativity.[30−42] In general relativity, the repulsion necessary to obtain an accelerating expansion of the universe can be provided by including a vacuum energy. This corresponds to the well-known modification to the Einstein equations including an additional cosmological term. Observations suggested the cosmological constant $\Lambda \sim 10^{-52}\,{\rm m}^{-2}$. Or alternatively, the vacuum energy can be considered as a dynamical field, such as quintessence.

It is well known that a lot of test gravitational effects have supported the general relativity, such as gravitational red-shift, deflection of light, gravitational time delay, perihelion shift and geodetic precession. Kiselev[43] constructed a general solution to the spherically symmetric Einstein equations for the quintessence-surrounded Schwarzschild black hole. In this paper we investigate the gravitational frequency-shift and the deflection of the light in the

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Schwarzschild black hole space-time surrounded by the quintessence. It is a significant case to understanding the accelerating expansion of the universe and properties of the dark matter and dark energy.

The remainder of this paper is organized as follows. In Section 2, we briefly review the space-time structure of the Schwarzschild black hole surrounded by the quintessence. In Section 3, we straightly calculate the gravitational frequency-shift of the light by the quintessence. In Section 4, we mainly investigate the deflection of light in the given space–time. A brief conclusion is given in the last section.

2. Schwarzschild black hole space–time surrounded by quintessence

In this section we briefly review the Schwarzschild black hole space–time surrounded by quintessence.[43] Following the notation in Ref.[44], the metric of the Schwarzschild black hole space–time surrounded by the quintessence takes the form

$$\mathrm{d}s^2 = \mathrm{e}^{\nu}\mathrm{d}t^2 - \mathrm{e}^{\lambda}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2), \quad (1)$$

with the function $\nu = \nu(r)$ and $\lambda = \lambda(r)$. If we define the normalization of gravitational constant G as $4\pi G = 1$, the Einstein equations have the form

$$2T_t^t = -\mathrm{e}^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) + \frac{1}{r^2}, \quad (2)$$

$$2T_r^r = -\mathrm{e}^{-\lambda}\left(\frac{1}{r^2} - \frac{\nu'}{r}\right) + \frac{1}{r^2}, \quad (3)$$

$$2T_\theta^\theta = 2T_\varphi^\varphi = -\frac{1}{2}\mathrm{e}^{-\lambda}\left(\nu'' + \frac{\nu'^2}{2} + \frac{\nu' - \lambda'}{r} - \frac{\nu'\lambda'}{2}\right). \quad (4)$$

The appropriate general expression for the energy–momentum tensor of quintessence is given by

$$T_t^t = \rho_{\mathrm{q}}(r), \quad (5)$$

$$T_i^j = \rho_{\mathrm{q}}(r)\alpha\left[-(1+3B)\frac{r_i r^j}{r_n r^n} + B\delta_i^j\right]. \quad (6)$$

We set $< r_i r^j >= \frac{1}{3}\delta_i^j r_n r^n$. Therefore we derive the following relations:

$$p_{\mathrm{q}} = \omega_{\mathrm{q}}\rho_{\mathrm{q}}, \quad (7)$$

$$\omega_{\mathrm{q}} = \frac{1}{3}\alpha. \quad (8)$$

The quintessential state has

$$-1 < \omega_{\mathrm{q}} < 0 \Rightarrow -3 < \alpha < 0. \quad (9)$$

The additivity and linearity condition which fixes the free parameter of the energy–momentum tensor for the matter yields

$$B = -\frac{3\omega_{\mathrm{q}} + 1}{6\omega_{\mathrm{q}}}, \quad (10)$$

which implies

$$T_t^t = T_r^r = \rho_{\mathrm{q}}, \quad (11)$$

$$T_\theta^\theta = T_\varphi^\varphi = -\frac{1}{2}\rho_{\mathrm{q}}(3\omega_{\mathrm{q}} + 1). \quad (12)$$

So the spherically symmetric metric of the Schwarzschild black hole space–time surrounded by quintessence is

$$\mathrm{d}s^2 = \left(1 - \frac{2M}{r} - \frac{c}{r^{3\omega_{\mathrm{q}}+1}}\right)\mathrm{d}t^2 - \left(1 - \frac{2M}{r} - \frac{c}{r^{3\omega_{\mathrm{q}}+1}}\right)^{-1}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta d\varphi^2), \quad (13)$$

where M is the mass of the black hole, ω_{q} is the quintessential state parameter, c is the normalization factor which is related to ρ_{q} by $\rho_{\mathrm{q}} = -\frac{c}{2}\frac{3\omega_{\mathrm{q}}}{r^{3(1+\omega_{\mathrm{q}})}}$, and ρ_{q} is the density of quintessence. The above exact spherically symmetric solution for Einstein equations describes the black hole surrounded the quintessential matter with the energy–momentum tensor, which satisfies the additivity and linearity condition in accordance with Eqs.(11) and (12).

3. Gravitational frequency-shift of light by quintessence

In this section we concentrate on the gravitational frequency-shift in the Schwarzschild black hole space–time surrounded by the quintessence, because this space–time is static, the ratio between the frequencies of the light, measured at different positions, is governed by

$$\frac{\nu}{\nu_{\mathrm{A}}} = \sqrt{\frac{g_{tt}(r)}{g_{tt}(r_{\mathrm{A}})}}. \quad (14)$$

In the vicinity of the gravitational source, we assume $\frac{c}{r^{3\omega_{\mathrm{q}}+1}} \ll \frac{2M}{r}$, so

$$\frac{\nu}{\nu_{\mathrm{A}}} = \sqrt{\frac{g_{tt}^{(0)}(r)}{g_{tt}^{(0)}(r_{\mathrm{A}})}}\left[1 + \frac{c}{2}\left(\frac{1}{g_{tt}^{(0)}(r_{\mathrm{A}})r_{\mathrm{A}}^{(3\omega_{\mathrm{q}}+1)}} - \frac{1}{g_{tt}^{(0)}(r)r^{(3\omega_{\mathrm{q}}+1)}}\right)\right], \quad (15)$$

where $g_{tt}^{(0)}(r) = 1 - \dfrac{2M}{r}$. And here supposing $M \ll r$, we further expand the above square-root-expression to the second-order $\dfrac{M}{r}$ and obtain

$$\frac{\nu}{\nu_{\rm A}} = 1 - \frac{M}{r} + \frac{M}{r_{\rm A}} + \frac{c}{2}\left(\frac{1}{r_{\rm A}^{(3\omega_{\rm q}+1)}} - \frac{1}{r^{(3\omega_{\rm q}+1)}}\right). \quad (16)$$

By analysing Fig.1, we can find that the frequency-shift increases as the values of the normalization factor c increases, but it is contrary to the quintessential state parameter $\omega_{\rm q}$. We can find that the influence of the normalization factor c on the frequency-shift will vanish when the quintessential state parameter $\omega_{\rm q}$ increases enough.

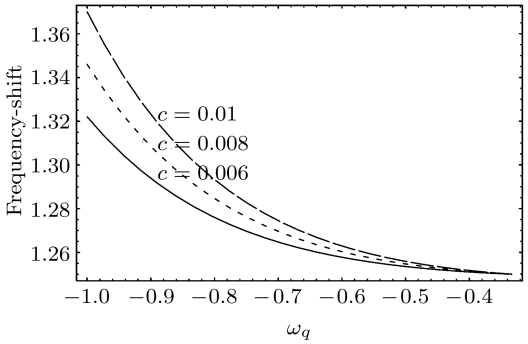


Fig.1. Gravitational frequency-shift of light by quintessence for $r_{\rm A} = 2M$, $r = 2r_{\rm A}$, $M = 1$, the horizon axis is $-1 < \omega_{\rm q} < -\dfrac{1}{3}$.

4. Deflection of light by quintessence

Deflection of the light in the gravitational field is a famous test of the Einstein general relativity theory. In order to calculate the deflection of the light in the Schwarzschild black hole space–time surrounded by the quintessence, we construct the Lagrangian of the equation of motion for the light ray

$$L = \left(1 - \frac{2M}{r} - \frac{c}{r^{3\omega_{\rm q}+1}}\right)\dot{t}^2 - \left(1 - \frac{2M}{r} - \frac{c}{r^{3\omega_{\rm q}+1}}\right)^{-1}\dot{r}^2 - r^2\dot{\varphi}^2, \quad (17)$$

where the overhead dot denotes differentiation with respect to the affine parameter γ, here we restrict the motion within the equatorial plane. Because the metric (13) is independent of t and φ, we have

$$E \equiv \left(1 - \frac{2M}{r} - \frac{c}{r^{3\omega_{\rm q}+1}}\right)\frac{{\rm d}t}{{\rm d}\gamma},$$

$$\text{and} \quad J \equiv r^2\frac{{\rm d}\varphi}{{\rm d}\gamma}. \quad (18)$$

The energy and angular momentum of the particle along the orbit both are conserved. For the light Lagrangian (17) vanishes, so we have

$$\frac{{\rm d}\varphi}{{\rm d}r} = \pm\frac{1}{r^2}\left[\frac{1}{k^2} - \frac{\left(1 - \dfrac{2M}{r} - \dfrac{c}{r^{3\omega_{\rm q}+1}}\right)}{r^2}\right]^{-\frac{1}{2}}, \quad (19)$$

where $k = J/E$. The sign $\pm$ refers to the increasing/decreasing of r. The distance between the closest approaches $r_{\rm A}$ is determined by

$$\frac{{\rm d}r}{{\rm d}\varphi}|_{r=r_{\rm A}} = \frac{1}{k^2} - \frac{1 - \dfrac{2M}{r_{\rm A}} - \dfrac{c}{r_{\rm A}^{3\omega_{\rm q}+1}}}{r_{\rm A}^2} = 0. \quad (20)$$

Substituting Eq.(20) into Eq.(19) yields

$$\frac{{\rm d}\varphi}{{\rm d}r} = \pm\frac{1}{r^2}\left[\frac{1 - \dfrac{2M}{r_{\rm A}} - \dfrac{c}{r_{\rm A}^{3\omega_{\rm q}+1}}}{r_{\rm A}^2} - \frac{(1 - \dfrac{2M}{r} - \dfrac{c}{r^{3\omega_{\rm q}+1}})}{r^2}\right]^{-\frac{1}{2}}. \quad (21)$$

In order to investigate the deflection rate of the light by the quintessence, we numerically simulate Eq.(21), and the obtained results are shown in Fig.2. From this figure we find that the deflection rate decreases as the values of the quintessential parameters c and $\omega_{\rm q}$ increase, and that the deflection rate is more sensitive to parameter $\omega_{\rm q}$ at $\omega_{\rm q} \to -1$.

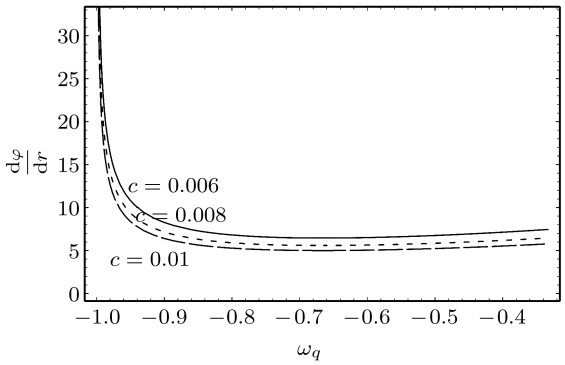


Fig.2. Deflection rate $\dfrac{{\rm d}\varphi}{{\rm d}r}$ of light by quintessence for $r_{\rm A} = 2M$, $r = 2r_{\rm A}$, $M = 1$, the horizon axis is $-1 < \omega_{\rm q} < -\dfrac{1}{3}$.

5. Conclusion

In this paper we have investigated the gravitational frequency-shift and the deflection of the light in the Schwarzschild black hole space–time surrounded by the quintessence. Under the condition of $\frac{c}{r^{3\omega_q+1}} \ll \frac{2M}{r}$, we find that the gravitational frequency-shift of light by quintessence is $\frac{c}{2}\left(\frac{1}{r_A^{(3\omega_q+1)}} - \frac{1}{r^{(3\omega_q+1)}}\right)$ and that the frequency-shift increases as the values of the normalization factor c increases, but it is contrary to the quintessential state parameter ω_q. we can find that the influence of the normalization factor c on the frequency-shift vanishes as the quintessential state parameter ω_q increases enough. We also calculate the deflection of light by quintessence, we find that the deflection rate decreases as both the quintessential parameters c and ω_q increase, and that the deflection rate is more sensitive to parameter ω_q at $\omega_q \to -1$.

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