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The stability of Bose–Einstein condensates in a two-dimensional shallow trap*

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The stability of Bose–Einstein condensates (BECs) loaded into a two-dimensional shallow harmonic potential well is studied. By using the variational method, the ground state properties for interacting BECs in the shallow trap are discussed. It is shown that the possible stable bound state can exist. The depth of the shallow well plays an important role in stabilizing the BECs. The stability of BECs in the shallow trap with the periodic modulating of atom interaction by using the Feshbach resonance is also discussed. The results show that the collapse and diffusion of BECs in a shallow trap can be controlled by the temporal modulation of the scattering length.

Keywords: Bose–Einstein condensates, shallow harmonic potential well, stability
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1. Introduction

The dynamics of trapped dilute Bose–Einstein condensate (BEC) has been successfully modelled by the mean-field theory.[1] In particular, a nonlinear Schrödinger equation for the macroscopic wavefunction of the BEC gas provides an accurate description of the ground state and of the excitation spectrum of the condensate at zero temperature.[2−16] The nonlinear term in the Gross-Pitaevskii (GP) equation which comes from the interaction between the particles, plays an important role in stabilizing the BEC. The term is proportional to the collision scattering length, and may be either positive (attractive) or negative (repulsive). When it is attractive, if the nonlinearity is too strong, the system becomes highly attractive and it eventually collapses. And if the nonlinearity is too weak, the system becomes weakly attractive in the final stage and it expands to infinity. Recently developed experimental technique[17] makes it possible to effectively control the sign of the scattering length using an external magnetic field. Such a modulation of scattering length is possible by manipulating an external magnetic field near the Feshbach resonance[18] and has been employed in different studies of the BEC.[19] A scheme of stabilizing the BEC in two dimensions has been suggested recently by the rapid periodic temporal modulation of scattering length $a_{\rm s}$ of angular frequency ω via $a_{\rm s} \to a_0[1 - c\sin(\omega t)]$, where a_0 and c are constants and t is time.[20,21] It is worth noting that this issue is similar to what was recently considered in several areas, e.g. nonlinear optics[22] and BEC.[20,23−26]

Usually, the condensates are formed in a parabolic trap. But it is interesting to note that in recent years much attention has been increasingly payed to BECs in a shallow well.[27,28] The BEC states in the shallow trap can be created exponentially by reducing the laser intensity in optical trapping and/or by reducing the electric current in magnetic trapping. It is shown that the possible bound states can exist in this shallow trap. Because of the macroscopic quantum tunnelling, the dynamics of the possible bound states in the shallow trap is mostly controlled by the nonlinear attractive interaction. So, the formation and study of such a bound state in a shallow trap could be of utmost interest, especially, this will provide a more stringent test for the mean-field models.

As discussed in Ref.[28], it is possible to have stable BEC bound states in localized exponentially screening shallow harmonic potential wells in 2D and 3D cases separately. But recently, there has been carried out another study of BEC bound states in shallow traps by Moiseyev *et al.*[29] They have concluded that there exist no stable bound states in 3D shallow traps.

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However for both of the cases the BEC can collapse with the nonlinear parameter g increasing. Thus, in the present investigation, we replace the parabolic trap with a localized exponentially-screening shallow harmonic potential.[28] The aim of the present work is to consider the dynamics of BEC in a shallow harmonic well by using the two-dimensional nonlinear Schrödinger model with a periodic modulation of the nonlinearity. The stability of BEC in the shallow trap with the periodic modulating of atom interaction by using the Feshbach resonance is discussed. The results show that the collapse and diffusion of BECs in the shallow trap can be controlled by the temporal modulation of the scattering length. The stability of the ground state is also considered by the variational method and the stability diagram of the ground state is obtained.

2. Governing equations

Usually condensates are studied in a parabolic trap $V(R) = m^2\omega^2\rho^2/2$, where m is the mass of an atom, ρ is the radial distance and ω is the angular frequency of the trap. In the present study, we consider the exponentially-screening well $V(r) = m^2\omega^2\rho^2\exp(-C\rho^2)/2$, where C is a positive parameter. If $C = 0$ the trap will become an infinite trap. A small deviation of C from 0 leads to a shallow potential well of which the depth decreases with c. We reduce the potential well to the form

$$V(r) = r^2\exp(-cr^2) \tag{1}$$

in units of $\hbar\omega$, where $r = \rho/l$, and $c = Cl^2$ with $l = \sqrt{\hbar/m\omega}$. We take the radially-symmetric time-dependent mean-field GP equation with attractive nonlinearity in the present study.[28] The two-dimensional GP equation for the BEC wavefunction $\psi(r;t)$ can be written in dimensionless form as[23]

$$\left[-\mathrm{i}\frac{\partial}{\partial t} - \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + r^2\mathrm{e}^{-cr^2} - g|\psi(r;t)|^2\right] \times\psi(r;t) = 0, \tag{2}$$

where $-g$ is the attractive nonlinearity. In this paper we consider attractive nonlinearity corresponding to positive g value mainly. Here length and time are expressed in units of l and $(\omega/2)^{-1}$, respectively. In the 2D case, in terms of number of atoms N and atomic scattering length a_s, the nonlinearity is given by $g = 2\pi n$. A scaled nonlinearity n is defined as $n = Na_\mathrm{s}/l$. The normalization condition in Eq.(2) is $2\pi\int r\mathrm{d}r|\psi(r;t)|^2 = 1$.

3. The ground state

To study the stability of the ground state in a shallow harmonic potential well, we adopt the following simple trial function for $\psi(r)$[26]

$$\psi(r) = A\mathrm{e}^{-r^2/(2R^2)}, \tag{3}$$

where A and R are the normalization and width respectively. $\pi A^2R^2 = 1$. The ground state energy of BEC gas confined in a 2D shallow well is given by the functional

$$E = 2\pi\int\left[\left|\frac{\partial\psi}{\partial r}\right|^2 + r^2\mathrm{e}^{-cr^2}|\psi(r)|^2 - \frac{g}{2}|\psi(r)|^4\right]r\mathrm{d}r. \tag{4}$$

Then the trial wavefunction (3) is substituted into the ground state energy (4), and the energy is obtained as

$$E = \frac{1}{R^2} + \frac{R^2}{(1+cR^2)^2} - \frac{g}{4\pi R^2}. \tag{5}$$

It is important to note that energy (5) can has a minimum only when $g < 4\pi$. That is, the system can has a possible bound state only when $g < 4\pi$. So, in the following, we just consider the case of $g < 4\pi$. After minimizing the energy functional $\dfrac{\partial E}{\partial R} = 0$, we obtain the width of ground state

$$-\frac{2}{R_0^3} + \frac{2R_0}{(cR_0^2+1)^2} - \frac{4cR_0^3}{(cR_0^2+1)^3} + \frac{g}{2\pi R_0^3} = 0. \tag{6}$$

Because $g < 4\pi$, Eq.(6) can reduce to

$$\left(c^3 - \frac{4c\pi}{g-4\pi}\right)R_0^6 + \left(3c^2 + \frac{4\pi}{g-4\pi}\right)R_0^4 + 3cR_0^2 + 1 = 0. \tag{7}$$

In Fig.1(a), we plot the ground state width R_0 against nonlinearity g for different values of c. It is clear that the width of the BEC decreases with g increasing. For small values of g, R_0 is sensitive to c, but with g increasing, the effect of c becomes weak. We can also find that for large values of c, a possible stable ground state can exist only when $g > g_\mathrm{min}$. For example, when $c = 0.3$, $g > g_\mathrm{min} \approx 8$ should be satisfied. It should be noted from Fig.1(a) that for all values of c, there exists a critical g_crit and as $g > g_\mathrm{crit} = 4\pi$, the BEC will collapse. In Fig.1(b), we plot the ground state energy versus g for different values of c. We can also find that with the value of g increasing for a fixed c, the energy decreases. On the other hand, the width and energy keep decreasing as g increases to the collapse point g_crit.

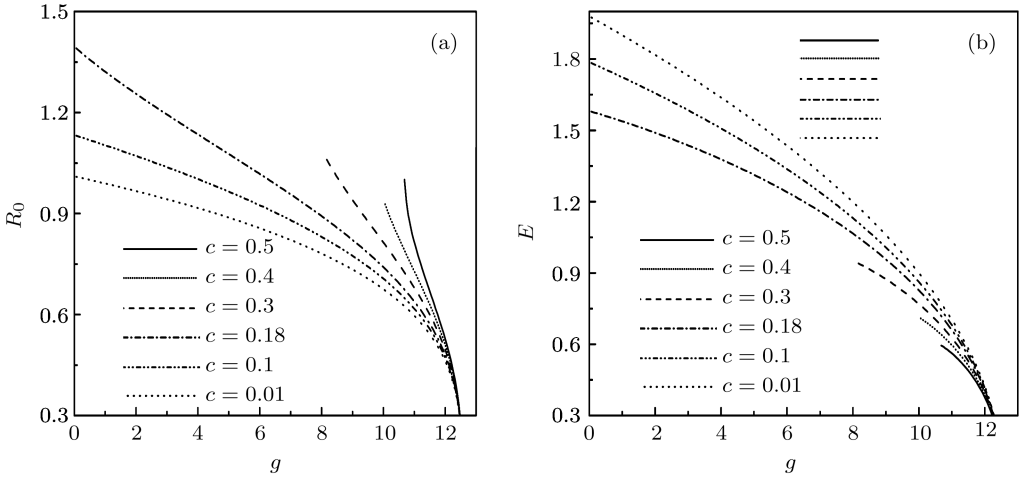


Fig.1. The ground state width R_0 (a) and energy E (b) versus g for different values of c.

The above results indicate that the stability of BEC in the shallow well should be related to the nonlinear strength g and parameter c. According to Eq.(6), the possible stable regime in $c-g$ plane is given in Fig.2(a). It is clear from Fig.2(a) that given a value of g, there exists a maximal $c_{\max}$. If the value of c exceeds the $c_{\max}$, Eq.(7) has no real-valued roots and the potential becomes too shallow to trap the BEC. For the attractive case, a critical $c_{\rm crit} \approx 0.192$ exists. When $c < c_{\rm crit}$, for all $0 < g < g_{\rm crit}$ a possible stable state can exist. When $c > c_{\rm crit}$, there exists a minimum of $g_{\min}$ and the value of $g_{\min}$ increases with c. It is clear from Fig.2(a) that the well depth expressed by c has a significant effect on the stability of BEC. To assure a possible stable bounded state, the value of g should be in the range: $g_{\min} < g < g_{\rm crit}$, for a fixed c.

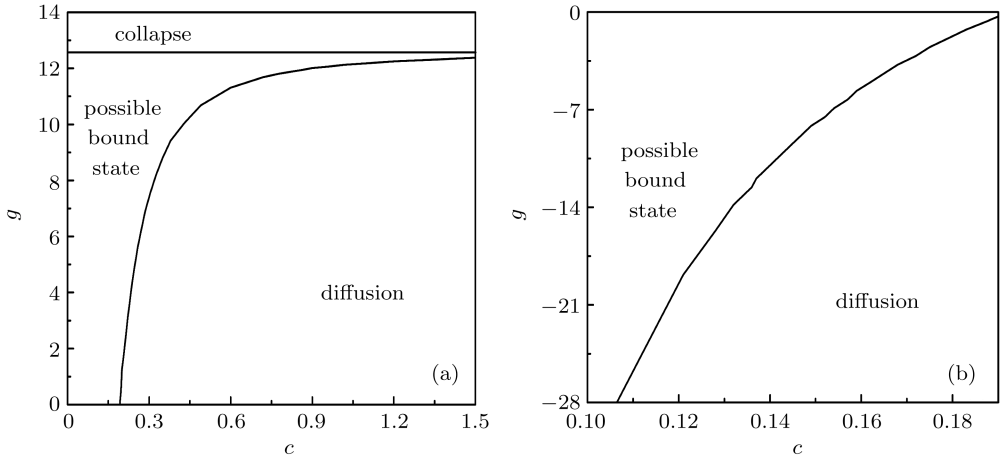


Fig.2. The stability diagram in $g - c$ parameter space for attractive case (a) and repulsive case (b).

In Fig.2(b) we also give the possible stable state for the repulsive case of $g < 0$. It is clear from Fig.2(b) that for a fixed c the possible stable state of BEC can exist when $g_{\min} < g < 0$. The value of $g_{\min}$ decreases with c increasing. For a sufficiently large value of c, i.e. $c = 0.2$, the well is too shallow to trap the repulsive BEC, and BEC will completely diffuse.

4. The excited state

Now we discuss the excitation of the BEC in the shallow trap. We replace g with $g(t)$ and employ a variational method with the following Gaussian wavefunction:

$$\psi(r,t) = A(t)\exp\left[-\frac{r^2}{2R^2(t)} + \frac{\mathrm{i}}{2}\beta(t)r^2 + \mathrm{i}\alpha(t)\right], \tag{8}$$

where $A(t)$, $R(t)$, $\beta(t)$ and $\alpha(t)$ are the normalization, width, chirp and phase, respectively. In the 2D case, $A(t) = [\pi^{1/2}R(t)]^{-1}$. The Lagrangian density for generating Eq.(2) is[30]

$$\mathrm{L}(\psi) = \frac{\mathrm{i}}{2}(\dot{\psi}\psi^* - \dot{\psi}^*\psi) - \left|\frac{\partial\psi}{\partial r}\right|^2 - r^2\mathrm{e}^{-cr^2}|\psi|^2 + \frac{1}{2}g(t)|\psi|^4, \tag{9}$$

where the overhead dot represents time derivative, and $g(t) = g_0[1 + \varepsilon\sin(\omega t)]$. The trial wavefunction (8) is substituted into the Lagrangian density and the effective Lagrangian L_{eff} is calculated by integrating the Lagrangian density

$$L_{\mathrm{eff}} = \pi A^2(t)R^2(t)\left[-\frac{1}{2}\dot{\beta}(t)R^2(t) + \frac{1}{4}g(t)A^2(t) - \dot{\alpha}(t) - \frac{1}{R^2(t)} - \beta^2(t)R^2(t) - \frac{R^2(t)}{[1 + cR^2(t)]^2}\right]. \tag{10}$$

The Euler–Lagrange equations for this effective lagrangian are given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L_{\mathrm{eff}}}{\partial\dot{\gamma}(t)} = \frac{\partial L_{\mathrm{eff}}}{\partial\gamma(t)}, \tag{11}$$

where $\gamma(t)$ represents $\beta(t)$, $\alpha(t)$, $A(t)$ and $g(t)$. After some algebra calculation one can eliminate the variables $\beta(t)$, $\alpha(t)$, $A(t)$ and $g(t)$ from these equations and obtain the following second-order differential equation for the evolution of the width R:

$$\frac{\mathrm{d}^2R}{\mathrm{d}t^2} = \frac{4}{R^3} - \frac{g_0[1 + \varepsilon\sin(\omega t)]}{\pi}\frac{1}{R^3} - \frac{4(R - cR^3)}{(1 + cR^2)^3}. \tag{12}$$

4.1. BEC with constant atom interaction ($\varepsilon = 0$)

If $\varepsilon = 0$, Eq.(12) will reduce to

$$\frac{\mathrm{d}^2R}{\mathrm{d}t^2} = -\frac{\mathrm{d}}{\mathrm{d}R}\left[\frac{2(1 - g_0/(4\pi))}{R^2} + \frac{2R^2}{(1 + cR^2)^2}\right]. \tag{13}$$

So in this case the effective potential $U(R)$ for the evolution of the width is given by

$$U(R) = \frac{2(1 - g_0/(4\pi))}{R^2} + \frac{2R^2}{(1 + cR^2)^2}. \tag{14}$$

Now we discuss the effective potential structures of the attractive BEC. Figure 3(a) shows one of those results at different values of nonlinear couple parameter g_0, with the value of c fixed. For $c = 0.25$, the effective potential can possess a minimum when $g_{\min} \approx 5.63 < g < g_{\mathrm{crit}}$. As $g > g_{\mathrm{crit}}$, the wall of the well at the centre is absent and the condensate collapses into an infinitely deep well. This happens for critical nonlinearity $g_0 = 4\pi$. As $g < g_{\min} \approx 5.63$, the attractive interaction between atoms is too weak and the condensate spills out over the top of the shallow well. Figure 3(b) shows the effective potential for different values of c with $g = 9.42$. We can find in Fig.3(b) that with the increase of c the outer wall of the well gradually reduces and for a large enough value of $c(\approx 0.38)$ this wall completely disappears and no bound state is possible. The results given in Figs.3(a) and 3(b) are in good agrement with those in Fig.2(a).

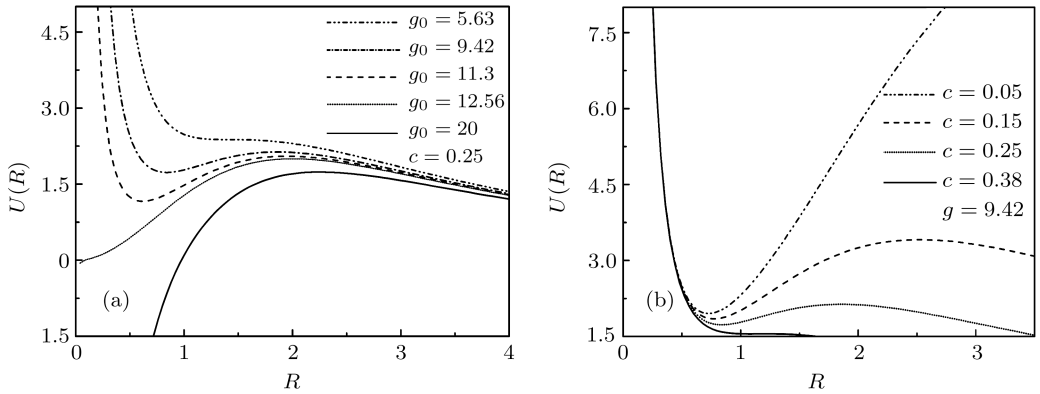


Fig.3. The effective potential well $U(R)$.

Now we study the oscillating frequency of the width. Replace R with $R = R_0 + R'$, where R_0 is defined in Eq.(7), then Eq.(12) will reduce to

$$\frac{\mathrm{d}^2R'}{\mathrm{d}t^2} = \left(-\frac{12}{R_0^4} + \frac{3g}{\pi R_0^4} - 4 + 48cR_0^2 - 180c^2R_0^4 \right) R', \tag{15}$$

thus we obtain the oscillating frequency of the width

$$\omega = \sqrt{\frac{12}{R_0^4} - \frac{3g}{\pi R_0^4} + 4 - 48cR_0^2 + 180c^2R_0^4}. \tag{16}$$

It is clear that the frequency ω is a function of parameters g and c. From Fig.4(a) we find that ω increases with g increasing. Because the width of BEC decreases with g increasing, the oscillation of the width becomes more and more rapid. When the value of g reaches g_{crit}, the frequency arrives at a maximum. Figure 4(a) also shows that with the value of g fixed, ω decreases with c because of the expanding of potential well. In Fig.4(b) we also plot the frequency ω for the repulsive case. It is clear that ω decreases with the value of $|g|$ increasing; as the value of $|g|$ increases, the effect of the repulsive interaction becomes more and more strong. Figure 4(b) also shows that for a fixed g, ω increases with c increasing because the potential becomes more and more deep.

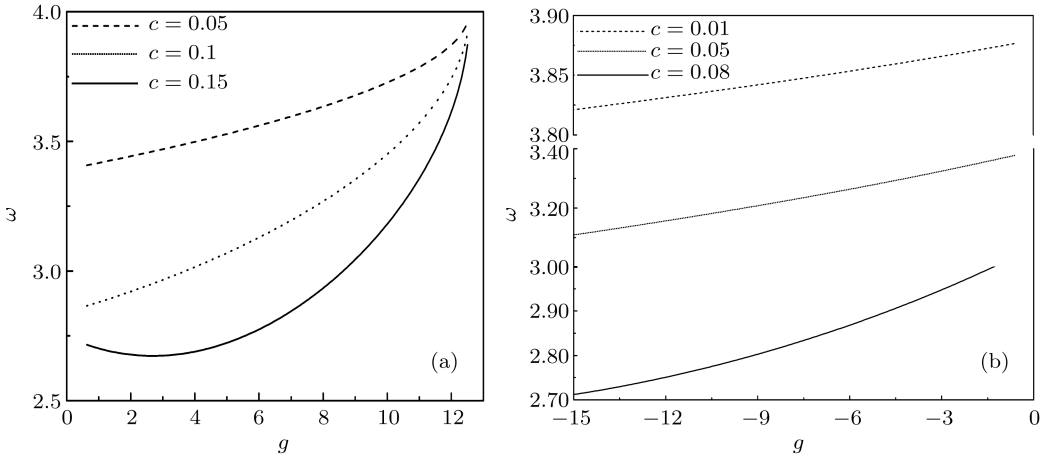


Fig.4. The frequency of the width versus g for different values of c in the attractive case (a) and repulsive case (b).

4.2. BEC with modulation of atom interaction ($\varepsilon \neq 0$)

Now we consider the modulating of the nonlinear coefficient g with $g(t) = g_0[1 + \varepsilon \sin(\omega t)]$, where g_0 corresponds to the constant part of the scattering length and $g_1 = g_0\varepsilon$ to the oscillating part, and ω is the frequency of oscillating part. Thus, we can separate $R(t)$ into a slowly varying part $A(t)$ and a rapidly varying part $B(t)$ by $R(t) = A(t) + B(t)$. Substituting this into Eq.(12) and retaining terms of the order of ω^{-2} in $B(t)$ yield the following equations of motion for $B(t)$ and $A(t)$:

$$\frac{\mathrm{d}^2B}{\mathrm{d}t^2} = \frac{3g_0B - 12B\pi}{\pi A^4} - \frac{g_0\varepsilon\sin(\omega t)}{\pi A^3} - 4B + 48cA^2B - 180c^2A^4B, \tag{17}$$

and

$$\frac{\mathrm{d}^2A}{\mathrm{d}t^2} = \frac{4\pi - g_0}{\pi A^3} - 4A + 16cA^3 + 48cA\langle B^2\rangle - 36c^2A^5 - 360c^2A^3\langle B^2\rangle + \frac{24\langle B^2\rangle}{A^5} - \frac{6g_0\langle B^2\rangle}{\pi A^5} + \frac{3g_0\langle\varepsilon\sin(\omega t)B\rangle}{\pi A^4}. \tag{18}$$

Where $\langle\cdots\rangle$ denotes the time average over rapid oscillations. Taking the slowly varying part A as constant, a solution to Eq.(17) is obtained as

$$B(t) = [Ag_0\varepsilon\sin(\omega t)]/(\omega^2\pi A^4 - 12\pi + 3g_0 - 4\pi A^4 + 48c\pi A^6 - 180c^2\pi A^8). \tag{19}$$

Substituting expression (19) into Eq.(18) yields the final evolution equation for the slow variable, and then the equation of motion for $A(t)$ becomes

$$\frac{\mathrm{d}^2A}{\mathrm{d}t^2} = [(12\pi g_0^2\varepsilon^2 - 3\varepsilon^2g_0^3 + 24c\pi g_0^2\varepsilon^2A^6$$

$$-180\pi c^2 g_0^2 \varepsilon^2 A^8)/[\pi(\pi\omega^2 A^4 - 12\pi$$
$$+3g_0 - 4\pi A^4 + 48c\pi A^6 - 180\pi c^2 A^8)^2]$$
$$+3g_0^2\varepsilon^2/[2\pi(\pi\omega^2 A^4 - 12\pi + 3g_0$$
$$-4\pi A^4 + 48c\pi A^6 - 180\pi c^2 A^8) - 4A^4$$
$$+16cA^6 - 36c^2 A^8 + (4\pi - g_0)/\pi]A^{-3}]. \quad (20)$$

To examine whether the collapse is controlled by the temporal modulation of the nonlinear coefficient, we consider Eq.(20) in the case of limit $A \to 0$. In this limit, the equation reduces to

$$\frac{\mathrm{d}^2 A}{\mathrm{d}t^2} = A^{-3}\left[\frac{4\pi - g_0}{\pi} + \frac{g_0^2\varepsilon^2}{6\pi(g_0 - 4\pi)}\right]. \quad (21)$$

It is clear from Eq.(21) that if the amplitude of the rapidly oscillating part is large enough, $\varepsilon^2 > 6\left(\frac{g_0 - 4\,\pi}{g_0}\right)^2$, the stability of the BEC is changed acutely from that without modulation of g. In the case $(g_0 - 4\,\pi) > 0$, the bounce should occur rather than the collapse, and they are reversed in the case $(g_0 - 4\,\pi) < 0$. Thus, these asymptotic results guarantee that Eq.(20) gives a stable behaviour of the condensate. Both the collapse and decay are ruled out if the following condition is satisfied:

$$\varepsilon > \frac{\sqrt{6}(g_0 - 4\pi)}{g_0} > 0. \quad (22)$$

For example, if we give a value of g_0, say, $g_0 = 5\,\pi$, the value of the ε should be larger than 0.4898, i.e. $\varepsilon > 0.4898$. In fact, the direct numerical simulation of Eq.(12) confirms condition (22). In Fig.5 we plot the time variation of width R by the direct numerical simulation of Eq.(12). We can see that when $\varepsilon = 0.3$,

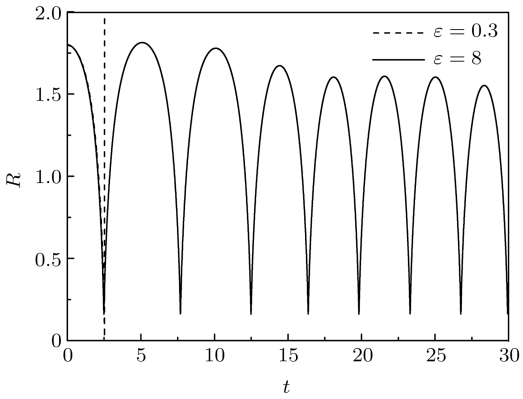


Fig.5. Variation of R with rapidly modulation g for different ε with $c = 0.3$.

the BEC still collapses at $t \approx 2.5$, because condition (22) is not satisfied in this case. But for $\varepsilon > 0.4898$, namely, $\varepsilon = 8$, we find that the collapse will be controlled.

The control of collapse can also be revealed by using an effective potential. In order to obtain the effective potential, we reduce Eqs.(17) and (18), respectively, to

$$\frac{\mathrm{d}^2 B}{\mathrm{d}t^2} = -\frac{g_0\varepsilon\sin(\omega t)}{\pi A^3}, \quad (23)$$

and

$$\frac{\mathrm{d}^2 A}{\mathrm{d}t^2} = \frac{4\pi - g_0}{\pi A^3} + \frac{3g_0\varepsilon\langle\sin(\omega t)B\rangle}{\pi A^4} - 4A$$
$$+16cA^3 - 36c^2A^5. \quad (24)$$

According to the solution $B(t) = g_0\varepsilon\sin(\omega t)/(\pi\omega^2 A^3)$, the equation of motion for $A(t)$ becomes

$$\frac{\mathrm{d}^2 A}{\mathrm{d}t^2} = \frac{4\pi - g_0}{\pi A^3} + \frac{3g_0^2\varepsilon^2}{2\pi^2\omega^2 A^7} - 4A + 16cA^3 - 36c^2A^5$$
$$= -\frac{\partial}{\partial A}\left[\frac{2}{A^2} - \frac{g_0}{2\pi A^2} + \frac{g_0^2\varepsilon^2}{4\pi^2\omega^2 A^6} + 2A^2 - 4cA^4 + 6c^2A^6\right]. \quad (25)$$

The part in the square bracket of Eq.(25) is the effective potential $U(A)$ of the equation of motion, i.e.

$$U(A) = \frac{4\pi - g_0}{2\pi A^2} + \frac{g_0^2\varepsilon^2}{4\pi^2\omega^2 A^6} + 2A^2 - 4cA^4 + 6c^2A^6. \quad (26)$$

Small oscillations around a stable configuration may be possible when there exists a minimum in this effective potential. In Figs.6(a) and 6(b) we plot the effective potential $U(A)$ for different values of g_0 with $\omega = 10\,\pi$ and $\omega = 20\,\pi$, respectively. We find that as the value of g_0 increases, the effective potential becomes deeper and deeper, and there is no upper limit for g_0 and stabilization is possible for an arbitrarily large values of g_0. The depth of the effective potential increases as ω increases. Comparing Figs.6(a) and 6(b) with Figs.3(a) and 3(b), respectively, we conclude that the BEC is controlled.

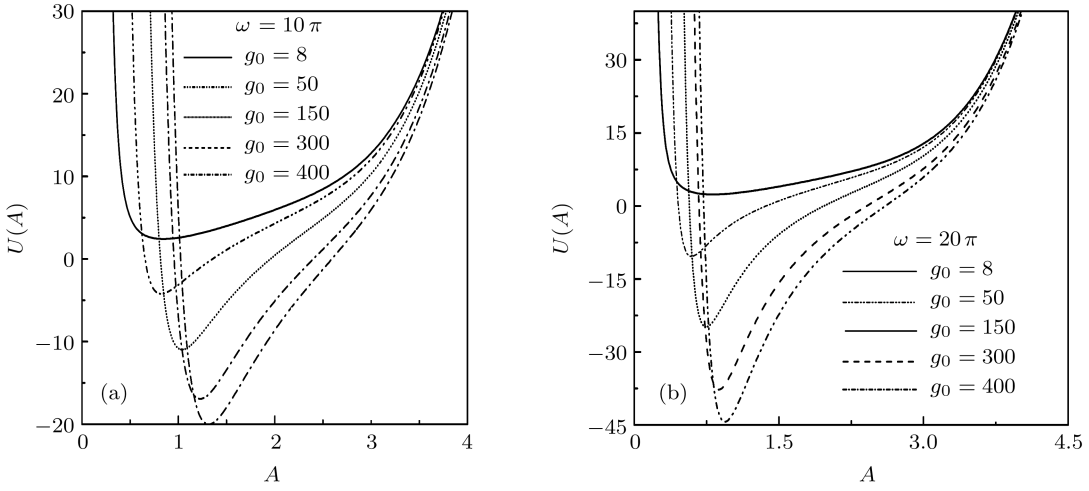


Fig.6. The effective potential $U(A)$ versus A for $c = 0.05$.

5. Conclusion

In the present work, by using the variational method, we first investigate the ground state properties for an interacting BEC in a shallow trap. And the stability diagram of the ground state in parameter space is obtained. We find that the values of g and c play an important role in stabilizing the BEC. We also have studied the dynamics of 2D BEC when the scattering length is modulated periodically. Applying the variational approximation and the averaging procedure to the GP equation, we have demonstrated that, in the 2D case, the time-variable parts of the nonlinearity makes it possible to maintain the condensate in a stable self-confined state in a shallow well. These predictions can be verified experimentally.

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