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Noether symmetry and non-Noether conserved quantity of the relativistic holonomic nonconservative systems in general Lie transformations*

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For a relativistic holonomic nonconservative system, by using the Noether symmetry, a new non-Noether conserved quantity is given under general infinitesimal transformations of groups. On the basis of the theory of invariance of differential equations of motion under general infinitesimal transformations, we construct the relativistic Noether symmetry, Lie symmetry and the condition under which the Noether symmetry is a Lie symmetry under general infinitesimal transformations. By using the Noether symmetry, a new relativistic non-Noether conserved quantity is given which only depends on the variables t, q_s and $\dot{q}_s$. An example is given to illustrate the application of the results.

Keywords: relativity, holonomic nonconservative system, Noether symmetry, non-Noether conserved quantity

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Relativistic analytical mechanics is an important aspect of the modern development in the field of theoretical physics. Since 1987, we have constructed the theory of analytical mechanics for the relativistic system, and given its basic theoretical frame.[1−8] The symmetry and the conserved quantity for a dynamical system play an important role in the fields of mathematics, mechanics and physics. There are three main symmetry methods used to seek the conserved quantities of the dynamical system: Noether symmetry,[9−14] Lie symmetry[15−21] and Mei's form invariance.[22−29] As is well known, the Noether symmetry can be led to the conserved quantity directly, and the conserved quantity is called Noether. In general, the Lie symmetry does not always imply the conserved quantity. If and only if the infinitesimal transformations of Lie symmetry satisfy the structure equation (i.e. Noether identity), the Lie symmetry can be led to a Noether conserved quantity. In 1992, Hojman[30] gave the direct method of finding conserved quantity by using Lie symmetry. This direct method is generalized by González-Gascón[31] and Lutzky.[32,33] Pillay and Leach[34] demonstrated that the invariant derived using Hojman's conservation law is trivial for all Noether symmetries. Therefore, using Hojman's method one can find the non-trivial conserved quantity which is led by the non-Noether symmetry and the conserved quantity is called non-Noether. In recent years, the Lie symmetrical Hojman conserved quantities of some dynamical systems have been studied by using this direct method in Refs.[11, 19, 35–40], and a series of important results have been obtained. Recently, Refs.[8, 39] gave the Hojman conserved quantity of the relativistic Hamiltonian system under the special infinitesimal transformations. But, the direct method of finding conserved quantity presented above is only utilized under the special infinitesimal transformations of groups in which the time is invariant. Recently, by using the Lie symmetry, Refs.[41, 42] gave directly the non-Noether conserved quantities of the classical dynamical system under general infinitesimal transformations.

In this paper, by using the Noether symmetry, we obtain directly the non-Noether conserved quantity of the relativistic holonomic nonconservative system under general infinitesimal transformations in which the time is variant. At first, the relativistic Noether symmetry, Lie symmetry and Noether conserved quantity

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under general infinitesimal transformations are given. Secondly, the condition under which the relativistic Noether symmetry is a Lie symmetry is obtained. Finally, a set of relativistic non-Noether conserved quantity of the system is given, which only depends on the variables t, q_s, and $\dot{q}_s$. An example is given to illustrate the application of the results.

Here we study a relativistic holonomic nonconservative system composed of N particles, and its configuration is determined by the n generalized coordinates $q_s(s=1,\cdots,n)$. Its motion can be described by the differential equations as follows:[1]

$$\begin{aligned}
&E_s(L^*)=Q_s, \quad (s=1,\cdots,n),\\
&E_s=\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial}{\partial\dot{q}_s}-\frac{\partial}{\partial q_s},\\
&L^*(t,\boldsymbol{q},\dot{\boldsymbol{q}})=T^*-V, \quad V=V(\boldsymbol{q}),\\
&T^*=\sum_{i=1}^{N}m_{\mathrm{o}i}c^2\left(1-\sqrt{1-\dot{\boldsymbol{r}}_i^2/c^2}\right),\\
&m_{\mathrm{o}i}=m_i\sqrt{1-\dot{\boldsymbol{r}}_i^2/c^2}, \quad (i=1,\cdots,N).
\end{aligned}\tag{1}$$

Here $L^*=L^*(t,\boldsymbol{q},\dot{\boldsymbol{q}})$ is the relativistic Lagrangian, $Q_s=Q_s(t,\boldsymbol{q},\dot{\boldsymbol{q}})$ are the nonpotential generalized forces, and the repeated subscripts represent the summation. In general, it is supposed that system (1) is nonsingular, i.e.

$$\det\left(\frac{\partial^2 L^*}{\partial\dot{q}_s\partial\dot{q}_k}\right)\neq 0. \tag{2}$$

From Eq.(1), we can find all generalized accelerations as

$$\ddot{q}_s=h_s(t,\boldsymbol{q},\dot{\boldsymbol{q}}), \quad (s=1,\cdots,n). \tag{3}$$

We introduce the general infinitesimal transformations for t and q_s,

$$\begin{aligned}
\bar{t} &= t+\varepsilon\xi_0(t,\boldsymbol{q},\dot{\boldsymbol{q}}),\\
\bar{q}_s(\bar{t}) &= q_s(t)+\varepsilon\xi_s(t,\boldsymbol{q},\dot{\boldsymbol{q}}),
\end{aligned}\tag{4}$$

where ε is an infinitesimal parameter, and ξ_0 and ξ_s are infinitesimal generators. Equation (4) is a one-parameter Lie group of transformations.

If the generators ξ_0 and ξ_s of infinitesimal transformations (4) satisfy the relativistic Noether identity as follows:

$$\frac{\partial L^*}{\partial t}\xi_0+\frac{\partial L^*}{\partial q_s}\xi_s+\left(L^*-\frac{\partial L^*}{\partial\dot{q}_s}\dot{q}_s\right)\dot{\xi}_0+\frac{\partial L^*}{\partial\dot{q}_s}\dot{\xi}_s+Q_s(\xi_s-\dot{q}_s\xi_0)+\dot{G}_N=0, \tag{5}$$

or the following relativistic Killing equations have solution:

$$\begin{aligned}
&\frac{\partial L^*}{\partial t}\xi_0+\frac{\partial L^*}{\partial q_s}\xi_s+\left(L^*-\frac{\partial L^*}{\partial\dot{q}_s}\dot{q}_s\right)\left(\frac{\partial\xi_0}{\partial t}+\frac{\partial\xi_0}{\partial q_k}\dot{q}_k\right)+\frac{\partial L^*}{\partial\dot{q}_s}\left(\frac{\partial\xi_s}{\partial t}+\frac{\partial\xi_s}{\partial q_k}\dot{q}_k\right)+Q_s(\xi_s-\dot{q}_s\xi_0)\\
&=-\frac{\partial G_N}{\partial t}-\frac{\partial G_N}{\partial q_s}\dot{q}_s,\\
&\left(L^*-\frac{\partial L^*}{\partial\dot{q}_s}\dot{q}_s\right)\frac{\partial\xi_0}{\partial\dot{q}_k}+\frac{\partial L^*}{\partial\dot{q}_s}\frac{\partial\xi_s}{\partial\dot{q}_k}=-\frac{\partial G_N}{\partial\dot{q}_k}, \quad (k=1,\cdots,n),
\end{aligned}\tag{6}$$

then the symmetry is relativistic Noether symmetry.

Taking the infinitesimal generator vector

$$X^{(0)}=\xi_0\frac{\partial}{\partial t}+\xi_s\frac{\partial}{\partial q_s}, \tag{7}$$

and its extensions

$$X^{(1)}=X^{(0)}+(\dot{\xi}_s-\dot{q}_s\dot{\xi}_0)\frac{\partial}{\partial\dot{q}_s}, \tag{8}$$

$$X^{(2)}=X^{(1)}+(\ddot{\xi}_s-2\ddot{q}_s\dot{\xi}_0-\dot{q}_s\ddot{\xi}_0)\frac{\partial}{\partial\ddot{q}_s}, \tag{9}$$

the invariance of Eq.(1) under the infinitesimal transformations (4) can lead the satisfaction of the relativistic Lie symmetrical determining equations

$$X^{(2)}\{E_s(L^*)\}-X^{(1)}(Q_s)=0. \tag{10}$$

By using the relativistic Noether theorem, from the Noether symmetry or the Lie symmetry, we can obtain the relativistic Noether conserved quantity as

$$I_{\mathrm{N}}=L^*\xi_0+\frac{\partial L^*}{\partial\dot{q}_s}(\xi_s-\dot{q}_s\xi_0)+G_{\mathrm{N}}=\mathrm{const}. \tag{11}$$

We now study the relation between the relativistic Noether symmetry and Lie symmetry, and obtain the condition under which the relativistic Noether symmetry is a Lie symmetry.

For

$$X^{(2)}\{E_s(L^*)\} = E_s\left\{X^{(1)}(L^*)+L^*\dot{\xi}_0\right\}-\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left[\frac{\partial L^*}{\partial \dot{\boldsymbol{q}}_k}\frac{\partial \xi_k}{\partial \dot{q}_s}-\left(\frac{\partial L^*}{\partial \dot{q}_k}\dot{q}_k-L^*\right)\frac{\partial \xi_0}{\partial \dot{q}_s}\right]+E_s(\xi_k-\dot{q}_k\xi_0)E_k(L^*)$$
$$+\left(\frac{\partial \xi_k}{\partial \dot{q}_s}-\dot{\boldsymbol{q}}_k\frac{\partial \xi_0}{\partial \dot{q}_s}\right)\frac{\mathrm{d}}{\mathrm{d}t}E_k(L^*), \tag{12}$$

$$E_s(\dot{G}_{\mathrm{N}}) = \frac{\mathrm{d}^2}{\mathrm{d}t^2}\frac{\partial G_{\mathrm{N}}}{\partial \dot{q}_s}, \tag{13}$$

we have

$$X^{(2)}\{E_s(L^*)\}-X^{(1)}(Q_s) = E_s\left\{X^{(1)}(L^*)+L^*\dot{\xi}_0+\dot{G}_{\mathrm{N}}\right\}-\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left[\frac{\partial L^*}{\partial \dot{q}_k}\frac{\partial \xi_k}{\partial \dot{q}_s}-\left(\frac{\partial L^*}{\partial \dot{q}_k}\dot{q}_k-L^*\right)\frac{\partial \xi_0}{\partial \dot{q}_s}+\frac{\partial G_{\mathrm{N}}}{\partial \dot{q}_s}\right]$$
$$+E_s(\xi_k-\dot{q}_k\xi_0)Q_k+\left(\frac{\partial \xi_k}{\partial \dot{q}_s}-\dot{q}_k\frac{\partial \xi_0}{\partial \dot{q}_s}\right)\dot{Q}_k-X^{(1)}(Q_s). \tag{14}$$

Adding and subtracting a function $E_s\{(\xi_k-\dot{q}_k\xi_0)Q_k\}$, we obtain

$$X^{(2)}\{E_s(L^*)\}-X^{(1)}(Q_s) = E_s\left\{X^{(1)}(L^*)+L^*\dot{\xi}_0+(\xi_k-\dot{q}_k\xi_0)Q_k+\dot{G}_{\mathrm{N}}\right\}$$
$$-\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left[\frac{\partial L^*}{\partial \dot{q}_k}\frac{\partial \xi_k}{\partial \dot{q}_s}-\left(\frac{\partial L^*}{\partial \dot{q}_k}\dot{q}_k-L^*\right)\frac{\partial \xi_0}{\partial \dot{q}_s}+\frac{\partial G_{\mathrm{N}}}{\partial \dot{q}_s}\right]+\left(\frac{\partial Q_k}{\partial q_s}-\frac{\partial Q_s}{\partial q_k}-\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial Q_k}{\partial \dot{q}_s}\right)$$
$$\times(\xi_k-\dot{q}_k\xi_0)-\left(\frac{\partial Q_s}{\partial \dot{q}_k}-\frac{\partial Q_k}{\partial \dot{q}_s}\right)(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0-\ddot{q}_k\xi_0). \tag{15}$$

If the symmetry is Noether, then Eqs.(5) and (6) hold. Substituting Eqs.(5) and (6) into Eq.(15), we obtain

$$X^{(2)}\{E_s(L^*)\}-X^{(1)}(Q_s)=\left(\frac{\partial Q_k}{\partial q_s}-\frac{\partial Q_s}{\partial q_k}-\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial Q_k}{\partial \dot{q}_s}\right)(\xi_k-\dot{q}_k\xi_0)-\left(\frac{\partial Q_s}{\partial \dot{q}_k}-\frac{\partial Q_k}{\partial \dot{q}_s}\right)(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0-\ddot{q}_k\xi_0). \tag{16}$$

Hence, we have the following theorem.

Theorem 1 For the relativistic holonomic nonconservative system (1), if the relativistic Noether symmetrical generators ξ_0 and ξ_s satisfy the following condition:

$$\left(\frac{\partial Q_k}{\partial q_s}-\frac{\partial Q_s}{\partial q_k}-\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial Q_k}{\partial \dot{q}_s}\right)(\xi_k-\dot{q}_k\xi_0)-\left(\frac{\partial Q_s}{\partial \dot{q}_k}-\frac{\partial Q_k}{\partial \dot{q}_s}\right)(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0-\ddot{q}_k\xi_0)=0, \tag{17}$$

then the generators ξ_0 and ξ_s certainly are Lie symmetric under the infinitesimal transformations (4); otherwise, it is uncertain.

We now obtain directly the relativistic non-Noether conserved quantities from the Noether symmetry of the relativistic holonomic nonconservative system, which only depend on the variables t, q_s and $\dot{q}_s$.

Theorem 2 For the relativistic holonomic nonconservative system (1), if the relativistic Noether symmetrical generators ξ_0 and ξ_s satisfy the identity (17) and there is a function $\mu=\mu(t,\ \boldsymbol{q},\ \dot{\boldsymbol{q}})$ such that

$$\frac{\partial h_s}{\partial \dot{q}_s}+\frac{1}{\mu}\left(\frac{\partial \mu}{\partial t}+\dot{q}_s\frac{\partial \mu}{\partial q_s}+h_s\frac{\partial \mu}{\partial \dot{q}_s}\right)=0,$$
$$(s=1,\cdots,n), \tag{18}$$

then the system possesses the conserved quantity as follows:

$$I_{\mathrm{L}} = \frac{1}{\mu}\frac{\partial}{\partial t}(\mu\xi_0)$$
$$+\frac{1}{\mu}\frac{\partial}{\partial q_s}(\mu\xi_s)+\frac{1}{\mu}\frac{\partial}{\partial \dot{q}_s}[\mu(\dot{\xi}_s-\dot{q}_s\dot{\xi}_0)]-\dot{\xi}_0$$
$$= \mathrm{const}. \tag{19}$$

In fact, if the Lie symmetrical generators ξ_0 and ξ_s of infinitesimal transformations (4) satisfy the condition (18), then system (1) possesses the conserved quantity (19). So we can obtain directly Theorem 2 from Theorem 1.

From Theorem 2, we have the following corollaries.

Corollary 1 For the relativistic Lagrangian sys-

tem (1) satisfying $Q_s \equiv 0$, if the relativistic Noether symmetrical generators ξ_0 and ξ_s satisfy the identity (17) and there is a function $\mu = \mu(t, \boldsymbol{q}, \dot{\boldsymbol{q}})$ such that condition (18) holds, then the relativistic system possesses the conserved quantity (19).

If we only introduce the special infinitesimal transformations for q_s, then we have $\Delta t = 0$ and $\xi_0 = 0$, Eqs.(4), (7)–(9), (17) and (19) become, respectively,

$$\bar{t} = t, \quad \bar{q}_s(\bar{t}) = q_s(t) + \varepsilon\xi_s(t, q, \dot{q}), \tag{20}$$

$$X^{(0)} = \xi_s \frac{\partial}{\partial q_s}, \ X^{(1)} = X^{(0)} + \dot{\xi}_s \frac{\partial}{\partial \dot{q}_s},$$

$$X^{(2)} = X^{(1)} + \ddot{\xi}_s \frac{\partial}{\partial \ddot{q}_s}, \tag{21}$$

$$\left(\frac{\partial Q_k}{\partial q_s} - \frac{\partial Q_s}{\partial q_k} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial Q_k}{\partial \dot{q}_s}\right)\xi_k - \left(\frac{\partial Q_s}{\partial \dot{q}_k} - \frac{\partial Q_k}{\partial \dot{q}_s}\right)\dot{\xi}_k = 0, \tag{22}$$

$$I_{\mathrm{L}} = \frac{1}{\mu}\frac{\partial}{\partial q_s}(\mu\xi_s) + \frac{1}{\mu}\frac{\partial}{\partial \dot{q}_s}(\mu\dot{\xi}_s) = \text{const}. \tag{23}$$

Corollary 2 For the relativistic holonomic nonconservative system (1), if the relativistic Noether symmetrical generator ξ_s of infinitesimal transformations (20) satisfy the identity (22), and there is a function $\mu = \mu(t, \boldsymbol{q}, \dot{\boldsymbol{q}})$ such that condition (18) holds, then the relativistic system possesses the conserved quantity (23).

In the classical approximation $\dot{\boldsymbol{r}}_i \ll c$, we have $m_i = m_{oi}$. For T^*, choosing the former two items of the power-series expression of $(1 - \dot{\boldsymbol{r}}_i^2/c^2)^{1/2}$ with respect to $\dot{\boldsymbol{r}}_i^2/c^2$, we can obtain

$$T^* \approx \frac{1}{2}\sum_{i=1}^{N} m_i \dot{r}_i^2 = T, \quad L^* \approx T - V = L. \tag{24}$$

Therefore, from theorem 2, we have also the following corollary.

Corollary 3 For the classical holonomic nonconservative system (1) satisfying $\dot{\boldsymbol{r}}_i \ll c$, if the Noether symmetrical generators ξ_0 and ξ_s of infinitesimal transformation (4) satisfy the identity (17), and there is a function $\mu = \mu(t, \boldsymbol{q}, \dot{\boldsymbol{q}})$ such that conditions (18) hold, then the system possesses the classical conserved quantity (19).

The above corollaries are also new results.

As an example, using the Noether symmetry, we study the Lie symmetrical non-Noether conserved quantities of a relativistic holonomic nonconservative system, of which the Lagrangian and the nonpotential generalized forces are, respectively,

$$L^* = m_0c^2\left(1 - \sqrt{1 - \dot{q}^2/c^2}\right) - V,$$
$$Q = k_1\left(1 - \dot{q}^2/c^2\right)^{-1/2}, \tag{25}$$

where $V = \text{const}$ and $k_1 = \text{const}$. The equation of motion is

$$\ddot{q} = \frac{k_1}{m_0}\left(1 - \dot{q}^2/c^2\right) = h. \tag{26}$$

The Noether identity (5) gives

$$\xi_0 = 1, \quad \xi = \dot{q}^2,$$
$$G_{\mathrm{N}} = -\int [(2m_0 + k_1)\dot{q}^2 - k_1\dot{q}] \times \left(1 - \dot{q}^2/c^2\right)^{-1/2} \mathrm{d}t. \tag{27}$$

The generator (27) satisfies the conditions (17). The condition (18) gives

$$-\frac{2k_1\dot{q}}{m_0c^2} + \frac{\mathrm{d}}{\mathrm{d}t}\ln\mu = 0. \tag{28}$$

And the above equation has the following solution

$$\mu = \exp\left(\frac{2k_1q}{m_0c^2}\right). \tag{29}$$

According to theorem 2, Eqs.(27) and (29) give

$$I_{\mathrm{L}} = 2(1 + \frac{k_1\dot{q}^2}{m_0c^2}) = \text{const}. \tag{30}$$

Using the identity (11), we know that the conserved quantity (30) is non-Noether.

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