Vol 16 No 11, November 2007
1009-1963/2007/16(11)/3168-08

Chinese Physics

© 2007 Chin. Phys. Soc.
and IOP Publishing Ltd

Hojman conserved quantity for nonholonomic systems of unilateral non-Chetaev type in the event space*

Jia Li-Qun(贾利群)[a)†], Zhang Yao-Yu(张耀宇)[b)], and Luo Shao-Kai(罗绍凯)[c)]

[a)] *School of Science, Jiangnan University, Wuxi* 214122, *China*
[b)] *Electric and Information Engineering College, Pingdingshan University, Pingdingshan* 467002, *China*
[c)] *Institute of Mathematical Mechanics and Mathematical Physics, Zhejiang Sci-Tech University, Hangzhou* 310018, *China*

(Received 26 February 2007; revised manuscript received 5 April 2007)

Hojman conserved quantities deduced from the special Lie symmetry, the Noether symmetry and the form invariance for a nonholonomic system of the unilateral non-Chetaev type in the event space are investigated. The differential equations of motion of the system above are established. The criteria of the Lie symmetry, the Noether symmetry and the form invariance are given and the relations between them are obtained. The Hojman conserved quantities are gained by which the Hojman theorem is extended and applied to the nonholonomic system of the unilateral non-Chetaev type in the event space. An example is given to illustrate the application of the results.

Keywords: event space, unilateral nonholonomic system, Hojman conserved quantity
PACC: 0320

1. Introduction

In recent years, analytical mechanics has made a rapid progress in research of symmetries and conserved quantities.[1−22] A higher achievement has also been gained about the study on both symmetry and conserved quantity for a holonomic system in an event space. By using the Noether theorem deduced from the Noether symmetry, the Noether conserved quantity is obtained. Similarly, by utilizing the Noether symmetry deduced from the Lie symmetry, the Noether conserved quantity is obtained.[23] By using the Noether symmetry deduced from the form invariance, the Noether conserved quantity is also found.[24] In Ref.[25], the Hojman conserved quantities deduced from the special Lie symmetry, Noether symmetry and form invariance for a holonomic system in the event space are studied. However, the symmetry and conserved quantity for a unilateral nonholonomic system in the event space have not been deeply studied. In this paper, the Hojman conserved quantities deduced from the special Lie symmetry, Noether symmetry and form invariance for an nonholonomic system of the unilateral non-Chetaev type in the event space are investigated.

2. Differential equations of motion of system

Suppose that the configuration of a mechanical system of an ideal unilateral non-Chetaev type whose motion is subjected to nonholonomic constraints is determined by n generalized coordinates $q_s(s = 1, 2, \cdots, n)$, and its corresponding $(n+1)$-degree event space is set up by the n generalized coordinates and time t. Introducing signs of $x_1 = t$ and $x_{s+1} = q_s(s = 1, \cdots, n)$, all variables $x_\alpha(\alpha = 1, \cdots, n+1)$ may be regarded as known functions of parameter τ. Let $x_\alpha = x_\alpha(\tau)$ is C^2-type curve, and $(n+1)$ variables $\dfrac{\mathrm{d}x_\alpha}{\mathrm{d}\tau} = x'_\alpha(\alpha = 1, \cdots, n+1)$ will not be equal to zero at the same time. Thus $\dot{x}_\alpha = \dfrac{\mathrm{d}x_\alpha}{\mathrm{d}t} = \dfrac{x'_\alpha}{x'_1}(\alpha = 1, \cdots, n+1)$. If the Lagrange function $L = L(t, q_s, \dot{q}_s)$ and the generalized non-potential forces $Q_s = Q_s(t, q_s, \dot{q}_s)$ of the system in the configuration space, then the Lagrange functions and the generalized non-potential forces respectively are

$$\Lambda(x_\alpha, x'_\alpha) = x'_1 L\left(x_\alpha, \frac{x'_2}{x'_1}, \cdots, \frac{x'_{n+1}}{x'_1}\right), \quad (1)$$

$$P_1 = -Q_s x'_{s+1}, \quad (s = 1, 2, \cdots, n),$$

*Project supported by the National Natural Science Foundation of China (Grant No 10572021).
†E-mail: jlq0@sina.com

$$P_{s+1} = x'_1 Q_s \left(x_\alpha, \frac{x'_2}{x'_1}, \cdots, \frac{x'_{n+1}}{x'_1} \right), \quad (s = 1, 2, \cdots, n). \tag{2}$$

The universal subscripts denote Einstein summation convention in the above equations as well as in the following text. Suppose that the motion of the system in the configuration space is subjected to g nonholonomic constraints of the ideal unilateral non-Chetaev type as follows:

$$f_\beta(t, q_s, \dot{q}_s) \geq 0, \quad (\beta = 1, \cdots, g; \quad s = 1, \cdots, n). \tag{3}$$

Inequality (3) can be expressed in the event space as

$$F_\beta(x_\alpha, x'_\alpha) \geq 0, \quad (\beta = 1, \cdots, g; \quad \alpha = 1, \cdots, n+1), \tag{4}$$

$$F_\beta(x_\alpha, x'_\alpha) = f_\beta \left(x_1, \cdots, x_{n+1}; \frac{x'_2}{x'_1}, \cdots, \frac{x'_{n+1}}{x'_1} \right). \tag{5}$$

Inequality (4) belongs to the nonholonomic constraints of the ideal unilateral non-Chetaev type. When the values of the left functions of inequalities (3) and (4) are positive, the constraints vanish from the system; when their values equal zero, the system is subjected to the constraints. When the system is subjected to the constraints, the restrictive condition of virtual displacement, to which the constraint equation (4) is subjected in the event space, is

$$F_{\beta\alpha}(x_\alpha, x'_\alpha)\delta x_\alpha = 0, \quad (\beta = 1, \cdots, g; \quad \alpha = 1, \cdots, n+1). \tag{6}$$

In general, $F_{\beta\alpha}$ has nothing to do with $\dfrac{\partial F_\beta}{\partial x'_\alpha}$. If both of them equal each other, the nonholonomic constraints of the non-Chetaev type turn into the nonholonomic constraints of the Chetaev type.

Introducing a Lagrange multiplier $\lambda_\beta (\beta = 1, \cdots, g)$, the differential equations of motion for the nonholonomic system of the unilateral non-Chetaev type in the event space are expressed as

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial \Lambda}{\partial x'_\alpha} - \frac{\partial \Lambda}{\partial x_\alpha} = P_\alpha + \lambda_\beta F_{\beta\alpha}, \\ &(\alpha = 1, \cdots, n+1) \quad \text{(subjected to constraints)}, \\ &\frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial \Lambda}{\partial x'_\alpha} - \frac{\partial \Lambda}{\partial x_\alpha} = P_\alpha, \\ &(\alpha = 1, \cdots, n+1) \quad \text{(constraints vanishing)}, \\ &\lambda_\beta \geq 0, \quad f_\beta \geq 0, \quad \lambda_\beta f_\beta = 0. \end{aligned} \tag{7}$$

Let

$$\Gamma_\alpha = \lambda_\beta F_{\beta\alpha}, \quad (\beta = 1, \cdots, g; \quad \alpha = 1, \cdots, n+1), \tag{8}$$

where Γ_α is the generalized nonholonomic constraint forces in the event space. Introduce an Euler operator in the event space

$$\mathrm{E}_\alpha = \frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial}{\partial x'_\alpha} - \frac{\partial}{\partial x_\alpha}, \quad (\alpha = 1, \cdots, n+1), \tag{9}$$

then Eq.(7) will be rewritten as

$$\begin{aligned} &\mathrm{E}_\alpha(\Lambda) = P_\alpha + \Gamma_\alpha, \quad (\alpha = 1, \cdots, n+1) \\ &\text{(subjected to the constraints)}, \\ &\mathrm{E}_\alpha(\Lambda) = P_\alpha, \quad (\alpha = 1, \cdots, n+1) \\ &\text{(constraints vanishing)}. \end{aligned} \tag{10}$$

Equations (10) are called the differential equations of motion corresponding to the nonholonomic system of the ideal unilateral non-Chetaev type, whose equation is Eqs.(7) and constraint condition is inequality (4). Obviously, the $(n+1)$ equations of the first equation in Eqs.(10) are not independent of each other, and the $(n+1)$ equations of the second equation of Eqs.(10) are not independent of each other, either. Suppose that the subsequent n x''_{s+1} are obtained from the first or second equation of Eqs.(10), then we will have

$$\begin{aligned} &x''_{s+1} = A_s(x_\alpha, x'_\alpha, x''_1), \quad (s = 1, \cdots, n) \\ &\text{(subjected to constraints)}, \\ &x''_{s+1} = B_s(x_\alpha, x'_\alpha, x''_1), \quad (s = 1, \cdots, n) \\ &\text{(constraints vanishing)}. \end{aligned} \tag{11}$$

3. Criteria of three kinds of symmetries of the system

Take a special infinitesimal transformation of groups

$$\begin{aligned} &\tau^* = \tau, \quad x_1^* = x_1 \\ \text{and} \quad &x_{s+1}^* = x_{s+1} + \varepsilon \xi_{s+1}(x_\alpha, x'_\alpha), \\ &(s = 1, \cdots, n), \end{aligned} \tag{12}$$

where ε is the infinitesimal transformation parameter, and ξ_{s+1} is the infinitesimal transformation generator. The following sections illustrate the criteria of the Lie symmetry, the Noether symmetry and the form invariance for the nonholonomic system of the unilateral non-Chetaev type in the event space in transformation (12).

3.1. Criteria of the Lie symmetry

Determining equations of the Lie symmetry under transformation (12) of Eqs.(10) are

$$\begin{aligned}&\mathrm{X}^{(2)}[\mathrm{E}_\alpha(\Lambda)]=\mathrm{X}^{(1)}(P_\alpha+\Gamma_\alpha),\\&(\alpha=1,\cdots,n+1)\quad\text{(subjected to constraints)},\\&\mathrm{X}^{(2)}[\mathrm{E}_\alpha(\Lambda)]=\mathrm{X}^{(1)}(P_\alpha),\\&(\alpha=1,\cdots,n+1)\quad\text{(constraints vanishing)},\end{aligned}\tag{13}$$

where

$$\begin{aligned}\mathrm{X}^{(1)}&=\xi_{s+1}\frac{\partial}{\partial x_{s+1}}+\frac{\mathrm{d}\xi_{s+1}}{\mathrm{d}\tau}\frac{\partial}{\partial x'_{s+1}},\\\mathrm{X}^{(2)}&=\mathrm{X}^{(1)}+\frac{\mathrm{d}^2\xi_{s+1}}{\mathrm{d}\tau^2}\frac{\partial}{\partial x''_{s+1}}.\end{aligned}\tag{14}$$

The $(n+1)$ equations of the first equation of Eqs.(13) are not independent of each other, and the $(n+1)$ equations of the second equations of (13) are not independent of each other, either. There exist the following relationships:

$$\begin{aligned}&\{\mathrm{X}^{(2)}[\mathrm{E}_\alpha(\Lambda)]-\mathrm{X}^{(1)}(P_\alpha+\Gamma_\alpha)\}x'_\alpha=0,\\&\text{(subjected to constraints)},\\&\{\mathrm{X}^{(2)}[\mathrm{E}_\alpha(\Lambda)]-\mathrm{X}^{(1)}(P_\alpha)\}x'_\alpha=0,\\&\text{(constraints vanishing)}.\end{aligned}\tag{15}$$

Proof From the first formula of Eqs.(10), we know $[\mathrm{E}_\alpha(\Lambda)-(P_\alpha+\Gamma_\alpha)]\,x'_\alpha=0$. Consider Eqs.(14), P_α, and Γ_α do not include clearly x''_{s+1}, then we will have

$$\begin{aligned}0&=\mathrm{X}^{(2)}\left\{[\mathrm{E}_\alpha(\Lambda)-(P_\alpha+\Gamma_\alpha)]\,x'_\alpha\right\}\\&=\left\{\mathrm{X}^{(2)}[\mathrm{E}_\alpha(\Lambda)-(P_\alpha+\Gamma_\alpha)]\right\}x'_\alpha\\&\quad+[\mathrm{E}_\alpha(\Lambda)-(P_\alpha+\Gamma_\alpha)]\,\mathrm{X}^{(2)}(x'_\alpha)\\&=\left\{\mathrm{X}^{(2)}[\mathrm{E}_\alpha(\Lambda)-(P_\alpha+\Gamma_\alpha)]\right\}x'_\alpha\\&=\left\{\mathrm{X}^{(2)}[\mathrm{E}_\alpha(\Lambda)]-\mathrm{X}^{(2)}(P_\alpha+\Gamma_\alpha)\right\}x'_\alpha\\&=\left\{\mathrm{X}^{(2)}[\mathrm{E}_\alpha(\Lambda)]-\left[\mathrm{X}^{(1)}(P_\alpha+\Gamma_\alpha)\right.\right.\\&\quad\left.\left.+\frac{\mathrm{d}^2\xi_{s+1}}{\mathrm{d}\tau^2}\frac{\partial(P_\alpha+\Gamma_\alpha)}{\partial x''_{s+1}}\right]\right\}x'_\alpha\\&=\left\{\mathrm{X}^{(2)}[\mathrm{E}_\alpha(\Lambda)]-\mathrm{X}^{(1)}(P_\alpha+\Gamma_\alpha)\right\}x'_\alpha,\end{aligned}$$

hence the first formula of Eqs.(15) is verified. In a similar way, the second formula of Eqs.(15) can also be proved.

Because of the existence of Eqs.(15), we take the first equation of Eqs.(13) or the subsequent n equations of the second equation of Eqs.(13)

$$\begin{aligned}&\mathrm{X}^{(2)}[\mathrm{E}_{s+1}(\Lambda)]=\mathrm{X}^{(1)}(P_{s+1}+\Gamma_{s+1}),\\&(s=1,\cdots,n)\quad\text{(subjected to constraints)}\\\text{and}\quad&\mathrm{X}^{(2)}[\mathrm{E}_{s+1}(\Lambda)]=\mathrm{X}^{(1)}(P_{s+1}),\\&(s=1,\cdots,n)\quad\text{(constraints vanishing)}\end{aligned}\tag{16}$$

as the determining equations of the Lie symmetry, namely the criterion of the Lie symmetry. The total derivatives along the trajectory of the system with respect to parameter τ are

$$\begin{aligned}&\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}=x'_\alpha\frac{\partial}{\partial x_\alpha}+x''_1\frac{\partial}{\partial x'_1}+A_s\frac{\partial}{\partial x'_{s+1}},\\&\text{(subjected to constraints)},\\&\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}=x'_\alpha\frac{\partial}{\partial x_\alpha}+x''_1\frac{\partial}{\partial x'_1}+B_s\frac{\partial}{\partial x'_{s+1}},\\&\text{(constraints vanishing)}.\end{aligned}\tag{17}$$

Since Eqs.(11) need to be substituted into Eqs.(16), calculating Eqs.(16) yields the result the same as that obtained by replacing $\dfrac{\mathrm{d}}{\mathrm{d}\tau}$ of the operators $\mathrm{X}^{(1)}$ and $\mathrm{X}^{(2)}$ with $\dfrac{\bar{\mathrm{d}}}{\mathrm{d}\tau}$. Thus the determining equations of Eqs.(11) under transformation (12) can be written as

$$\begin{aligned}&\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\frac{\bar{\mathrm{d}}\xi_{s+1}}{\mathrm{d}\tau}=\frac{\partial A_s}{\partial x_{k+1}}\xi_{k+1}+\frac{\partial A_s}{\partial x'_{k+1}}\frac{\bar{\mathrm{d}}\xi_{k+1}}{\mathrm{d}\tau},\\&(s=1,\cdots,n)\quad\text{(subjected to constraints)},\\&\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\frac{\bar{\mathrm{d}}\xi_{s+1}}{\mathrm{d}\tau}=\frac{\partial B_s}{\partial x_{k+1}}\xi_{k+1}+\frac{\partial B_s}{\partial x'_{k+1}}\frac{\bar{\mathrm{d}}\xi_{k+1}}{\mathrm{d}\tau},\\&(s=1,\cdots,n)\quad\text{(constraints vanishing)}.\end{aligned}\tag{18}$$

Equations (18) are another criterion of the Lie symmetry.

3.2. Criteria of the Noether symmetry

Noether equations of Eqs.(10) under transformation (12) are

$$\begin{aligned}&\frac{\partial\Lambda}{\partial x_{s+1}}\xi_{s+1}+\frac{\partial\Lambda}{\partial x'_{s+1}}\xi'_{s+1}+(P_{s+1}+\Gamma_{s+1})\xi_{s+1}\\&+G'_{\mathrm{N}}=0,\quad\text{(subjected to constraints)},\\&\frac{\partial\Lambda}{\partial x_{s+1}}\xi_{s+1}+\frac{\partial\Lambda}{\partial x'_{s+1}}\xi'_{s+1}+P_{s+1}\xi_{s+1}+G'_{\mathrm{N}}=0,\\&\text{(constraints vanishing)},\end{aligned}\tag{19}$$

where $G_{\mathrm{N}}=G_{\mathrm{N}}(x_\alpha,x'_\alpha)$ is the gauge function. Killing equations responding to the first and second formulae

of Eqs.(19) are

$$\begin{aligned}
&\frac{\partial \Lambda}{\partial x_{s+1}}\xi_{s+1}+\frac{\partial \Lambda}{\partial x'_{s+1}}\frac{\partial \xi_{s+1}}{\partial x_{k+1}}x'_{k+1}\\
&+(P_{s+1}+\Gamma_{s+1})\xi_{s+1}=-\frac{\partial G_{\mathrm{N}}}{\partial x_{s+1}}x'_{s+1},\\
&\text{(subjected to constraints)},\\
&\frac{\partial \Lambda}{\partial x'_{s+1}}\frac{\partial \xi_{s+1}}{\partial x'_{k+1}}=-\frac{\partial G_{\mathrm{N}}}{\partial x'_{k+1}},\\
&(k=1,\cdots,n)\quad\text{(subjected to constraints)};\\
&\frac{\partial \Lambda}{\partial x_{s+1}}\xi_{s+1}+\frac{\partial \Lambda}{\partial x'_{s+1}}\frac{\partial \xi_{s+1}}{\partial x_{k+1}}x'_{k+1}+P_{s+1}\xi_{s+1}\\
=\ &-\frac{\partial G_{\mathrm{N}}}{\partial x_{s+1}}x'_{s+1},\quad\text{(constraints vanishing)},\\
&\frac{\partial \Lambda}{\partial x'_{s+1}}\frac{\partial \xi_{s+1}}{\partial x'_{k+1}}=-\frac{\partial G_{\mathrm{N}}}{\partial x'_{k+1}},\\
&(k=1,\cdots,n)\quad\text{(constraints vanishing)}.
\end{aligned}\tag{20}$$

Equations (19) or (20) are criteria of the Noether symmetry.

3.3. Criterion of the form invariance

The criterion equations of the form invariance of Eqs.(10) under transformation (12) are

$$\begin{aligned}
&\mathrm{E}_{s+1}[\mathrm{X}^{(1)}(\Lambda)]=\mathrm{X}^{(1)}(P_{s+1}+\Gamma_{s+1}),\\
&(s=1,\cdots,n)\quad\text{(subjected to constraints)},\\
&\mathrm{E}_{s+1}[\mathrm{X}^{(1)}(\Lambda)]=\mathrm{X}^{(1)}(P_{s+1}),\\
&(s=1,\cdots,n)\quad\text{(constraints vanishing)}.
\end{aligned}\tag{21}$$

4. The Noether symmetry and form invariance being conditions of the Lie symmetry

4.1. The Noether symmetry is a condition of the Lie symmetry

The following relationship can be verified:

$$\begin{aligned}
&\mathrm{E}_{s+1}[\mathrm{X}^{(1)}(\Lambda)]-\frac{\mathrm{d}^2}{\mathrm{d}\tau^2}\left(\frac{\partial \Lambda}{\partial x'_{k+1}}\frac{\partial \xi_{k+1}}{\partial x'_{s+1}}\right)\\
&+\mathrm{E}_{s+1}(\xi_{k+1})\mathrm{E}_{k+1}(\Lambda)+\frac{\partial \xi_{k+1}}{\partial x'_{s+1}}\frac{\mathrm{dE}_{k+1}(\Lambda)}{\mathrm{d}\tau}\\
=\ &\mathrm{X}^{(2)}[\mathrm{E}_{s+1}(\Lambda)],\quad(s=1,\cdots,n).
\end{aligned}\tag{22}$$

Note that

$$\mathrm{E}_{s+1}\left(G'_{\mathrm{N}}\right)=\frac{\mathrm{d}^2}{\mathrm{d}\tau^2}\frac{\partial G_{\mathrm{N}}}{\partial x'_{s+1}},\qquad(s=1,\cdots,n),\tag{23}$$

which is classified into two kinds of cases in research, one is that the system is subjected to the constraints and the other is that the constraints vanish from the system. Substituting Eqs.(23) into Eqs.(22), and adding and subtracting $\mathrm{X}^{(1)}(P_{s+1}+\Gamma_{s+1})$ and $\mathrm{E}_{s+1}[(P_{k+1}+\Gamma_{k+1})\xi_{k+1}]$, or adding and subtracting $\mathrm{X}^{(1)}(P_{s+1})$ and $\mathrm{E}_{s+1}(P_{k+1}\xi_{k+1})$, and utilizing the first and second equations of Eqs.(10), and being simplified, respectively, yield

$$\begin{aligned}
&\mathrm{X}^{(2)}\left[\mathrm{E}_{s+1}\left(\Lambda\right)-\left(P_{s+1}+\Gamma_{s+1}\right)\right]\\
=\ &\mathrm{E}_{s+1}\left[\mathrm{X}^{(1)}\left(\Lambda\right)+\left(P_{k+1}+\Gamma_{k+1}\right)\xi_{k+1}+G'_{\mathrm{N}}\right]\\
&-\frac{\mathrm{d}^2}{\mathrm{d}\tau^2}\left(\frac{\partial \Lambda}{\partial x'_{k+1}}\frac{\partial \xi_{k+1}}{\partial x'_{s+1}}+\frac{\partial G_{\mathrm{N}}}{\partial x'_{s+1}}\right)\\
&-\left(\frac{\partial\left(P_{s+1}+\Gamma_{s+1}\right)}{\partial x'_{k+1}}+\frac{\partial\left(P_{k+1}+\Gamma_{k+1}\right)}{\partial x'_{s+1}}\right)\xi'_{k+1}\\
&+\left(\frac{\partial\left(P_{k+1}+\Gamma_{k+1}\right)}{\partial x_{s+1}}-\frac{\partial\left(P_{s+1}+\Gamma_{s+1}\right)}{\partial x_{k+1}}\right.\\
&\left.-\frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial\left(P_{k+1}+\Gamma_{k+1}\right)}{\partial x'_{s+1}}\right)\xi_{k+1},\\
&(s=1,2,\cdots,n)\quad\text{(subjected to constraints)};\\
&\mathrm{X}^{(2)}\left[\mathrm{E}_{s+1}\left(\Lambda\right)-P_{s+1}\right]\\
=\ &\mathrm{E}_{s+1}\left[\mathrm{X}^{(1)}\left(\Lambda\right)+P_{k+1}\xi_{k+1}+G'_{\mathrm{N}}\right]\\
&-\frac{\mathrm{d}^2}{\mathrm{d}\tau^2}\left(\frac{\partial \Lambda}{\partial x'_{k+1}}\frac{\partial \xi_{k+1}}{\partial x'_{s+1}}+\frac{\partial G_{\mathrm{N}}}{\partial x'_{s+1}}\right)\\
&-\left(\frac{\partial P_{s+1}}{\partial x'_{k+1}}+\frac{\partial P_{k+1}}{\partial x'_{s+1}}\right)\xi'_{k+1}\\
&+\left(\frac{\partial P_{k+1}}{\partial x_{s+1}}-\frac{\partial P_{s+1}}{\partial x_{k+1}}-\frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial P_{k+1}}{\partial x'_{s+1}}\right)\xi_{k+1},\\
&(s=1,2,\cdots,n)\quad\text{(constraints vanishing)}.
\end{aligned}\tag{24}$$

Hence, we have

Proposition 1. Under transformation (12), if infinitesimal transformation generator ξ_{s+1} of the nonholonomic system of the unilateral non-Chetaev type in the event space is of the Noether symmetry, and ξ_{s+1}, P_{s+1} and Γ_{s+1} satisfy the equations

$$\begin{aligned}
&\left(\frac{\partial\left(P_{k+1}+\Gamma_{k+1}\right)}{\partial x_{s+1}}-\frac{\partial\left(P_{s+1}+\Gamma_{s+1}\right)}{\partial x_{k+1}}\right.\\
&\left.-\frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial\left(P_{k+1}+\Gamma_{k+1}\right)}{\partial x'_{s+1}}\right)\xi_{k+1}\\
&-\left(\frac{\partial\left(P_{s+1}+\Gamma_{s+1}\right)}{\partial x'_{k+1}}+\frac{\partial\left(P_{k+1}+\Gamma_{k+1}\right)}{\partial x'_{s+1}}\right)\xi'_{k+1}=0,\\
&(s=1,\cdots,n)\quad\text{(subjected to constraints)},
\end{aligned}$$

$$\left(\frac{\partial P_{k+1}}{\partial x_{s+1}}-\frac{\partial P_{s+1}}{\partial x_{k+1}}-\frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial P_{k+1}}{\partial x'_{s+1}}\right)\xi_{k+1}$$
$$-\left(\frac{\partial P_{s+1}}{\partial x'_{k+1}}+\frac{\partial P_{k+1}}{\partial x'_{s+1}}\right)\xi'_{k+1}=0,$$
$$(s=1,\cdots,n)\quad(\text{constraints vanishing})\,. \tag{25}$$

The infinitesimal transformation generator ξ_{s+1} must be of the Lie symmetry. Whereas, it is uncertain.

Proof The situation can be divided into two kinds, one is that the system is subjected to the constraints and the other is that the constraints vanish from the system. If ξ_{s+1} is an infinitesimal transformation generator of the Noether symmetry, then Eqs.(19) and (20) are tenable. Substituting Eqs.(19), (20) and (25) into Eqs.(24), we obtain

$$\begin{aligned}&\mathrm{X}^{(2)}\left[\mathrm{E}_{s+1}\left(\Lambda\right)\right]-\mathrm{X}^{(1)}\left(P_{s+1}+\Gamma_{s+1}\right)\\&=\mathrm{X}^{(2)}\left[\mathrm{E}_{s+1}\left(\Lambda\right)-\left(P_{s+1}+\Gamma_{s+1}\right)\right]\\&=0,\quad(s=1,\cdots,n)\quad(\text{subjected to constraints})\,,\\&\mathrm{X}^{(2)}\left[\mathrm{E}_{s+1}\left(\Lambda\right)\right]-\mathrm{X}^{(1)}\left(P_{s+1}\right)\\&=\mathrm{X}^{(2)}\left[\mathrm{E}_{s+1}\left(\Lambda\right)-P_{s+1}\right]\\&=0,\quad(s=1,\cdots,n)\quad(\text{constraints vanishing})\,.\end{aligned} \tag{26}$$

From Eqs.(16), we know ξ_{s+1} to be also of the Lie symmetry. *QED*

As for the nonholonomic conservative mechanical system of the unilateral non-Chetaev type in the event space, exists $P_{s+1}=0$, the second equation of Eqs.(25) is automatically tenable, hence the following corollary from Eqs.(25) can be obtained:

Corollary If the infinitesimal transformation generator ξ_{s+1} of the nonholonomic conservative mechanical system of the unilateral non-Chetaev type is of the Noether symmetry, and ξ_{s+1} and Γ_{s+1} satisfy the following equation:

$$\left(\frac{\partial \Gamma_{k+1}}{\partial x_{s+1}}-\frac{\partial \Gamma_{s+1}}{\partial x_{k+1}}-\frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial \Gamma_{k+1}}{\partial x'_{s+1}}\right)\xi_{k+1}$$
$$-\left(\frac{\partial \Gamma_{s+1}}{\partial x'_{k+1}}+\frac{\partial \Gamma_{k+1}}{\partial x'_{s+1}}\right)\xi'_{k+1}=0,\quad(s=1,\cdots,n)$$
$$(\text{subjected to constraints})\,, \tag{27}$$

the Noether symmetry is of the Lie symmetry.

4.2. The form invariance being a condition of the Lie symmetry

We can verify that the formula

$$\begin{aligned}&\mathrm{E}_{s+1}\left[\mathrm{X}^{(1)}\left(\Lambda\right)\right]-\mathrm{X}^{(2)}\left[\mathrm{E}_{s+1}\left(\Lambda\right)\right]\\&=\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\partial\xi_{k+1}}{\partial x'_{s+1}}\frac{\partial\Lambda}{\partial x_{k+1}}\right)-\frac{\partial\xi_{k+1}}{\partial x_{s+1}}\frac{\partial\Lambda}{\partial x_{k+1}}\\&\quad+\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\partial\xi'_{k+1}}{\partial x'_{s+1}}\frac{\partial\Lambda}{\partial x'_{k+1}}\right)-\frac{\partial\xi'_{k+1}}{\partial x_{s+1}}\frac{\partial\Lambda}{\partial x'_{k+1}},\\&\quad(s=1,\cdots,n)\end{aligned} \tag{28}$$

is tenable. Hence we have

Proposition 2. Under transformation (12), a full and necessary condition under which the form invariance of the nonholonomic system in the event space is of the Lie symmetry is that the infinitesimal transformation generator ξ_{s+1} satisfies

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\partial\xi_{k+1}}{\partial x'_{s+1}}\frac{\partial\Lambda}{\partial x_{k+1}}\right)-\frac{\partial\xi_{k+1}}{\partial x_{s+1}}\frac{\partial\Lambda}{\partial x_{k+1}}$$
$$+\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\partial\xi'_{k+1}}{\partial x'_{s+1}}\frac{\partial\Lambda}{\partial x'_{k+1}}\right)-\frac{\partial\xi'_{k+1}}{\partial x_{s+1}}\frac{\partial\Lambda}{\partial x'_{k+1}}=0,$$
$$(s=1,\cdots,n)\,. \tag{29}$$

Proof Firstly, verify Proposition 2 to be tenable when the system is subjected to the constraints. If the form invariance is of the Lie symmetry, then we have

$$\begin{aligned}&\mathrm{E}_{s+1}[\mathrm{X}^{(1)}(\Lambda)]-\mathrm{X}^{(1)}(P_{s+1}+\Gamma_{s+1})\\&=\mathrm{X}^{(2)}[\mathrm{E}_{s+1}(\Lambda)]-\mathrm{X}^{(1)}(P_{s+1}+\Gamma_{s+1}),\\&(s=1,\cdots,n)\quad(\text{subjected to constraints})\,.\end{aligned} \tag{30}$$

Hence

$$\begin{aligned}&\mathrm{E}_{s+1}[\mathrm{X}^{(1)}(\Lambda)]-\mathrm{X}^{(2)}[\mathrm{E}_{s+1}(\Lambda)]=0,\\&(s=1,\cdots,n)\quad(\text{subjected to constraints})\,.\end{aligned} \tag{31}$$

From Eqs.(28) and (31), Eqs.(29) are obtained. Whereas, if Eqs.(29) are tenable, from Eqs.(28) we know that Eqs.(31) and (30) are all tenable. Hence, Proposition 2 is verified to be tenable when the system is subjected to the constraints. When the constraints vanish from the system, Proposition 2 can also be verified to be tenable in the same way. *QED*

In calculating Eqs.(21), the $\dfrac{\mathrm{d}}{\mathrm{d}\tau}$ of the operators in $\mathrm{X}^{(1)}$ and E_s is required to be replaced by $\dfrac{\bar{\mathrm{d}}}{\mathrm{d}\tau}$, then the $\dfrac{\mathrm{d}}{\mathrm{d}\tau}$ in Eqs.(29) is expressed as $\dfrac{\bar{\mathrm{d}}}{\mathrm{d}\tau}$, ξ'_{s+1} is expressed as $\dfrac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_{s+1}$.

5. Hojman conserved quantities resulting from the three kinds of symmetries

In the event space, the Hojman conserved quantities resulting from the Lie symmetry, the Noether symmetry and the form invariance for a nonholonomic system are given respectively by the following three propositions.

Proposition 3 If the generator ξ_{s+1} satisfies (18) under transformation (12) and a certain function $\mu=\mu(x_\alpha,x'_\alpha)$ exists, which make

$$\frac{\partial A_s}{\partial x'_{s+1}}+\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\ln\mu=0 \quad \text{(subjected to constraints)}$$

and

$$\frac{\partial B_s}{\partial x'_{s+1}}+\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\ln\mu=0 \quad \text{(constraints vanishing)} \tag{32}$$

tenable, then the Hojman conserved quantity resulting from the Lie symmetry in the event space is

$$\begin{aligned} I_{\mathrm{H}} &= \frac{1}{\mu}\frac{\partial}{\partial x_{s+1}}\left(\mu\xi_{s+1}\right)+\frac{1}{\mu}\frac{\partial}{\partial x'_{s+1}}\left(\mu\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_{s+1}\right)\\ &= \text{const.} \end{aligned} \tag{33}$$

Proof Take the total derivative of Eqs.(17) with respect to τ, then we will obtain

$$\begin{aligned} \frac{\bar{\mathrm{d}}I_{\mathrm{H}}}{\mathrm{d}\tau} &= \frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\left(\frac{1}{\mu}\frac{\partial\mu}{\partial x_{s+1}}\xi_{s+1}\right)+\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\frac{\partial\xi_{s+1}}{\partial x_{s+1}}\\ &\quad+\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\left(\frac{1}{\mu}\frac{\partial\mu}{\partial x'_{s+1}}\frac{\bar{\mathrm{d}}\xi_{s+1}}{\mathrm{d}\tau}+\frac{\partial}{\partial x'_{s+1}}\frac{\bar{\mathrm{d}}\xi_{s+1}}{\mathrm{d}\tau}\right). \end{aligned} \tag{34}$$

We can easily verify the following operational equations:

$$\begin{aligned} &\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\frac{\partial}{\partial x'_{s+1}}=\frac{\partial}{\partial x'_{s+1}}\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}-\frac{\partial}{\partial x_{s+1}}-\frac{\partial A_{k+1}}{\partial x'_{s+1}}\frac{\partial}{\partial x'_{k+1}}\\ &\text{(subjected to constraints)},\\ &\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\frac{\partial}{\partial x_{s+1}}=\frac{\partial}{\partial x_{s+1}}\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}-\frac{\partial A_{k+1}}{\partial x_{s+1}}\frac{\partial}{\partial x'_{k+1}}\\ &\text{(subjected to constraints)};\\ &\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\frac{\partial}{\partial x'_{s+1}}=\frac{\partial}{\partial x'_{s+1}}\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}-\frac{\partial}{\partial x_{s+1}}-\frac{\partial B_{k+1}}{\partial x'_{s+1}}\frac{\partial}{\partial x'_{k+1}}\\ &\text{(constraints vanishing)},\\ &\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\frac{\partial}{\partial x_{s+1}}=\frac{\partial}{\partial x_{s+1}}\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}-\frac{\partial B_{k+1}}{\partial x_{s+1}}\frac{\partial}{\partial x'_{k+1}}\\ &\text{(constraints vanishing)}. \end{aligned} \tag{35}$$

Take the partial derivative of (18) with respect to x'_{s+1} and the summation of s, then we will have

$$\begin{aligned} &\frac{\partial}{\partial x'_{s+1}}\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\frac{\bar{\mathrm{d}}\xi_{s+1}}{\mathrm{d}\tau}\\ &=\frac{\partial}{\partial x'_{s+1}}\left(\frac{\partial A_s}{\partial x_{k+1}}\xi_{k+1}\right)+\frac{\partial}{\partial x'_{s+1}}\left(\frac{\partial A_s}{\partial x'_{k+1}}\frac{\bar{\mathrm{d}}\xi_{k+1}}{\mathrm{d}\tau}\right)\\ &\text{(subjected to constraints)},\\ &\frac{\partial}{\partial x'_{s+1}}\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\frac{\bar{\mathrm{d}}\xi_{s+1}}{\mathrm{d}\tau}\\ &=\frac{\partial}{\partial x'_{s+1}}\left(\frac{\partial B_s}{\partial x_{k+1}}\xi_{k+1}\right)+\frac{\partial}{\partial x'_{s+1}}\left(\frac{\partial B_s}{\partial x'_{k+1}}\frac{\bar{\mathrm{d}}\xi_{k+1}}{\mathrm{d}\tau}\right)\\ &\text{(constraints vanishing)}. \end{aligned} \tag{36}$$

Substituting Eqs.(35) and (36) into Eq.(34), and utilizing Eqs.(32) and (18), we obtain

$$\begin{aligned} \frac{\bar{\mathrm{d}}I_H}{\mathrm{d}\tau} &= \frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\left(\frac{1}{\mu}\frac{\partial\mu}{\partial x_{s+1}}\xi_{s+1}\right)+\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\left(\frac{1}{\mu}\frac{\partial\mu}{\partial x'_{s+1}}\frac{\bar{\mathrm{d}}\xi_{s+1}}{\mathrm{d}\tau}\right)\\ &\quad+\frac{\partial^2 A_s}{\partial x'_{s+1}\partial x_{k+1}}\xi_{k+1}+\frac{\partial^2 A_s}{\partial x'_{s+1}\partial x'_{k+1}}\frac{\bar{\mathrm{d}}\xi_{k+1}}{\mathrm{d}\tau}\\ &= \frac{1}{\mu}\frac{\partial\mu}{\partial x'_{s+1}}\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\frac{\bar{\mathrm{d}}\xi_{s+1}}{\mathrm{d}\tau}-\frac{\partial A_s}{\partial x_{k+1}}\xi_{k+1}\right.\\ &\quad\left.-\frac{\partial A_s}{\partial x'_{k+1}}\frac{\bar{\mathrm{d}}\xi_{k+1}}{\mathrm{d}\tau}\right)=0\\ &\text{(subjected to constraints)},\\ \frac{\bar{\mathrm{d}}I_H}{\mathrm{d}\tau} &= \frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\left(\frac{1}{\mu}\frac{\partial\mu}{\partial x_{s+1}}\xi_{s+1}\right)+\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\left(\frac{1}{\mu}\frac{\partial\mu}{\partial x'_{s+1}}\frac{\bar{\mathrm{d}}\xi_{s+1}}{\mathrm{d}\tau}\right)\\ &\quad+\frac{\partial^2 B_s}{\partial x'_{s+1}\partial x_{k+1}}\xi_{k+1}+\frac{\partial^2 B_s}{\partial x'_{s+1}\partial x'_{k+1}}\frac{\bar{\mathrm{d}}\xi_{k+1}}{\mathrm{d}\tau}\\ &= \frac{1}{\mu}\frac{\partial\mu}{\partial x'_{s+1}}\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\frac{\bar{\mathrm{d}}\xi_{s+1}}{\mathrm{d}\tau}-\frac{\partial B_s}{\partial x_{k+1}}\xi_{k+1}\right.\\ &\quad\left.-\frac{\partial B_s}{\partial x'_{k+1}}\frac{\bar{\mathrm{d}}\xi_{k+1}}{\mathrm{d}\tau}\right)=0\\ &\text{(constraints vanishing)}. \end{aligned} \tag{37}$$

QED

Using Proposition 3, the Hojman conserved quantity may be directly obtained through the Lie symmetry of the system.

Proposition 4 If the infinitesimal transformation generator ξ_{s+1}, dynamical functions P_{s+1} and Γ_{s+1} of the Noether symmetry of the system under transformation (12) satisfy relationships (15) and exists a certain function $\mu=\mu(x_\alpha,x'_\alpha)$ to make Eqs.(32) tenable, then the Noether symmetry will result in Eq.(33) of the Hojman conserved quantity.

Proof By virtue of Propositions 1 and 3, Proposition 4 can be verified.

Using Proposition 4, by virtue of the Noether symmetry of the system, the Hojman conserved quantity can be obtained through the Lie symmetry.

Proposition 5 If the infinitesimal transformation generator ξ_{s+1} of the form invariance of the system under transformation (12) satisfies Eqs.(29) and exists a certain function $\mu = \mu(x_\alpha, x'_\alpha)$ to make Eqs.(32) tenable, then the form invariance will indirectly result in Eq.(33) of the Hojman conserved quantity.

Proof By virtue of Propositions 2 and 3, Proposition 5 can be verified.

Using the Proposition 5 and the form invariance, the Hojman conserved quantity can be obtained through the Lie symmetry.

6. Illustrative example

The Lagrange function, the generalized nonpotential forces, the constraint equations and the restricting conditions that the virtual displacement is subjected to the constraints for the unilateral nonholonomic system in the event space, respectively, are

$$\Lambda = x'_1 \left\{ \frac{1}{2} \left[\left(\frac{x'_2}{x'_1} \right) \right]^2 + \left[\left(\frac{x'_3}{x'_1} \right) \right]^{\mathbf{2}} - x_3 \right\}, \quad (38)$$

$$P_1 = P_2 = P_3 = 0, \quad (39)$$

$$F = x'_3 + x_1 x'_2 \geq 0, \quad (40)$$

$$(x'_1 - 2x'_2)\, \delta x_{\mathbf{2}} + x_1 x'_2 \delta x_3 = 0. \quad (41)$$

Now, we try to deduce the Hojman conserved quantities by means of the Lie symmetry, the Noether symmetry and the form invariance of the system.

Comparing conditions (6) and (41), we have

$$F_{12} = (x'_1 - 2x'_2) \quad \text{and} \quad F_{13} = x_1 x'_2. \quad (42)$$

Substituting expressions (38) and (42) into the first equation of Eqs.(7), we obtain

$$x''_2 = \lambda_1 x'_1 (x'_1 - 2x'_2) + \frac{x'_2 x''_1}{x'_1}$$

$$\text{and} \quad x''_3 = x'_1 (\lambda_1 x_1 x'_2 - x'_1) + \frac{x'_3 x''_1}{x'_1}$$

$$\text{(subjected to constraints)}, \quad (43)$$

and from inequality (40) and Eqs.(43), we have

$$\lambda_1 = 1/x_1. \quad (44)$$

Substituting expression (44) into Eqs.(43), and substituting expressions (38) and (42) into the second equation of Eqs.(7), we obtain

$$x''_2 = \frac{x'_1 (x'_1 - 2x'_2)}{x_1} + \frac{x'_2 x''_1}{x'_1} = A_1,$$

$$\text{and} \quad x''_3 = x'_1 (x'_2 - x'_1) + \frac{x'_3 x''_1}{x'_1} = A_2$$

$$\text{(subjected to constraints)};$$

$$x''_2 = \frac{x'_2 x''_1}{x'_1} = B_1, \quad x''_3 = \frac{x'_3 x''_1}{x'_1} - (x'_1)^2 = B_2$$

$$\text{(constraints vanishing)}. \quad (45)$$

From Eqs.(18) and (45), we obtain

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau} \frac{\bar{\mathrm{d}}\xi_2}{\mathrm{d}\tau} = \frac{x''_1}{x'_1} \frac{\bar{\mathrm{d}}\xi_2}{\mathrm{d}\tau}, \quad \frac{\bar{\mathrm{d}}}{\mathrm{d}\tau} \frac{\bar{\mathrm{d}}\xi_3}{\mathrm{d}\tau} = x'_1 \frac{\bar{\mathrm{d}}\xi_2}{\mathrm{d}\tau} + \frac{x''_1}{x'_1} \frac{\bar{\mathrm{d}}\xi_3}{\mathrm{d}\tau}$$

(subjected to constraints);

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau} \frac{\bar{\mathrm{d}}\xi_2}{\mathrm{d}\tau} = \frac{x''_1}{x'_1} \frac{\bar{\mathrm{d}}\xi_2}{\mathrm{d}\tau}, \quad \frac{\bar{\mathrm{d}}}{\mathrm{d}\tau} \frac{\bar{\mathrm{d}}\xi_3}{\mathrm{d}\tau} = \frac{x''_1}{x'_1} \frac{\bar{\mathrm{d}}\xi_3}{\mathrm{d}\tau}$$

$$\text{(constraints vanishing)}. \quad (46)$$

Equations (46) have the following solutions:

$$\xi_2 = 1, \quad \xi_3 = 0; \quad (47)$$

$$\xi_2 = 0, \quad \xi_3 = 1; \quad (48)$$

$$\xi_2 = \left(\frac{x'_3}{x'_1} - x_2 + x_1 \right)^2, \quad \xi_3 = 0. \quad (49)$$

From Eqs.(32), we have

$$2\frac{x''_1}{x'_1} + \frac{\bar{\mathrm{d}}}{\mathrm{d}\tau} \ln \mu = 0, \quad (50)$$

which has the following solutions:

$$\mu = (x'_1)^{-2} \left(\frac{x'_3}{x'_1} - x_2 + x_1 \right), \quad (51)$$

$$\mu = (x'_1)^{-2} \left(\frac{x'_2}{x'_1} + x_3 \right), \quad (52)$$

and

$$\mu = (x'_1)^{-2}. \quad (53)$$

Utilize Proposition 3 and solutions of three groups of equations, i.e. Eqs.(47) and (51), (48) and (52), and (49) and (53), then the three Hojman conserved quantities deduced from the Lie symmetry of the system will be respectively obtained as follows:

$$I_{\mathrm{H}} = -\left(\frac{x'_3}{x'_1} - x_2 + x_1 \right)^{-1} = \text{const}, \quad (54)$$

$$I_{\mathrm{H}} = \left(\frac{x'_2}{x'_1} + x_3 \right)^{-1} = \text{const}, \quad (55)$$

$$I_{\mathrm{H}} = -2 \left(\frac{x'_3}{x'_1} - x_2 + x_1 \right) = \text{const}. \quad (56)$$

Notes Generators (47) and (48) are of the Noether symmetry and all satisfy Eqs.(25), hence, the Hojman conserved quantities (54) and (55) deduced from the Noether symmetry can also be obtained by virtue of Proposition 4. Generators (47), (48) and (49) are the form invariances and all satisfy (29), hence, the Hojman conserved quantities (54), (55) and (56) deduced from the form invariances can also be obtained by virtue of Proposition 5.

7. Conclusion

The Hojman conserved quantities deduced by using the special Lie symmetry, the Noether symmetry and the Mei symmetry for nonholonomic systems of the unilateral non-Chetaev type in the event space are investigated. The differential equations of motion of the system in the event space are established. The criteria of the Lie symmetry, the Noether symmetry and the form invariance are given, and the relations between them are studied. The conditions and expressions of the Hojman conserved quantities deduced from the three kinds of symmetries are obtained.

The research conclusions indicate that if the generalized nonholonomic constraint $\Gamma_\alpha = 0$, then the conclusion of research will degenerate to a conclusion of the Hojman conserved quantity for a holonomic non-conservative mechanical system in the event space; If $P_\alpha = \Gamma_\alpha = 0$, the conclusion of research will degenerate to a conclusion of the Hojman conserved quantity for a nonholonomic mechanical system of the unilateral Chetaev type in the event space. Hence, the conclusions of research in this paper will be of general significance.

References

[1] Mei F X 2000 *Acta Phys. Sin.* **49** 1207 (in Chinese)
[2] Mei F X and Shang M 2000 *Acta Phys. Sin.* **49** 1901 (in Chinese)
[3] Qiao Y F and Zhao S H 2001 *Acta Phys. Sin.* **50** 1 (in Chinese)
[4] Li Y C, Zhang Y, Liang J H and Mei F X 2001 *Chin. Phys.* **10** 376
[5] Wang S Y and Mei F X 2002 *Chin. Phys.* **11** 5
[6] Zhang Y 2002 *Acta Phys. Sin.* **51** 461 (in Chinese)
[7] Zhang H B 2002 *Chin. Phys.* **11** 765
[8] Ge W K 2002 *Acta Phys. Sin.* **51** 1156 (in Chinese)
[9] Xu Z X 2002 *Acta Phys. Sin.* **51** 2423 (in Chinese)
[10] Luo S K 2003 *Chin. Phys.* **12** 140
[11] Fu J L and Chen L Q 2003 *Chin. Phys.* **12** 695
[12] Mei F X 2003 *Acta Phys. Sin.* **52** 1048 (in Chinese)
[13] Chen X W and Li YM 2003 *Chin. Phys.* **12** 936
[14] Zhang Y and Mei F X 2003 *Chin. Phys.* **12** 1058
[15] Ge W K and Zhang Y 2003 *Acta Phys. Sin.* **52** 2105 (in Chinese)
[16] Fu J L and Chen L Q 2003 *Acta Phys. Sin.* **52** 2664 (in Chinese)
[17] Luo S K 2003 *Acta Phys. Sin.* **52** 2941 (in Chinese)
[18] Fang J H, Chen P S, Zhang J and Li H 2003 *Acta Phys. Sin.* **52** 2945 (in Chinese)
[19] Luo S K 2004 *Acta Phys. Sin.* **53** 5 (in Chinese)
[20] Zhang Y 2004 *Acta Phys. Sin.* **53** 331 (in Chinese)
[21] Jia L Q and Zheng S W 2006 *Acta Phys. Sin.* **55** 3829 (in Chinese)
[22] Zheng S W, Jia L Q and Yu H S 2006 *Chin. Phys.* **15** 1039
[23] Mei F X 1999 *Applications of Lie Groups and Lie Algebras to Constrained Mechanical Systems* (Beijing: Science Press) (in Chinese)
[24] Mei F X 2003 *J. Jiangxi Normal Univ.* **27** 1 (in Chinese)
[25] Xu X J, Mei F X and Qin M C 2005 *Acta Phys. Sin.* **54** 1009 (in Chinese)