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Nuclear statistical equilibrium at core-collapse supernova*

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A new improved nuclear partition function is employed to calculate the nuclear statistical equilibrium (NSE) in core-collapse supernova environment. The results show that the change of nucleus abundance is slight even though the temperature is higher than 10^{11} K when shock propagates, which indicates that the effect of the nuclear partition function is not so important as shown in the previous calculations, but it can also be considered in detailed simulation if it is sensitive to weak interaction rates in core-collapse supernova.

Keywords: supernova, nucleosynthesis, abundances
PACC: 9760B, 9710C, 2340

1. Introduction

Nuclear statistical equilibrium (NSE) plays a very important role in the evolution of core-collapse supernova (C-C SN) when the silicon burning is complete. Both the temperature and the density are very high ($0.1 < T_9 < 5$, $10^5\,\mathrm{g/cm^3} < \rho < 10^{10}\,\mathrm{g/cm^3}$)[1] in the centre of the supernova (SN) progenitor (so called 'iron core'). The strong interaction and electromagnetic interaction are at equilibrium in this case and the weak interaction is otherwise. The explicit ingredients of the iron core are given by nuclear Saha equation, as a function of density, temperature, electron fraction, binding energy and nuclear partition function of each nucleus.[2] The subsequent nuclear reactions are influenced dramatically by the NSE, so the whole evolution process of C-C SN would be undergone. It is known that electron capture (EC) causes the electron number density and subsequently the electron degenerate pressure to decrease and the core to collapse.[3] Lots of energies are lost due to neutrino leakage in this process. The change rate of electron fraction Y_e is sensitive to the abundance of each nucleus as well as the electron capture rate. Consequently, the acceleration of the collapse is changed with NSE. Therefore, the compositions of C-C SN progenitor and other physical processes are essential to be treated carefully.

This problem has been addressed by many authors.[2,4−7] Clifford and Tayler[4] derived the NSE equation in stellar environment by using the nuclear Saha equation. Hartmann *et al*[2] studied the NSE in supernova ejecta and the freeze-out condition of supernova expansion. They have concluded that the NSE makes a good approximation for the composition after freezing. In 1994, Aufderheide *et al*[6] computed the weak interaction rates of 150 important nuclei under NSE in SN environment. The thermal condition has been developed by using the quasi equilibrium (QSE) theory and the results indicate that the QSE is similar to the NSE when $T > 5 \times 10^9\,\mathrm{K}$.[8,9] Newly observed data of SN 1987A by Chandra shows that the important synthesized radioactive nuclei are produced in normal freeze-out of NSE in the inner ejecta of C-C SN.[10]

The largest uncertainty in the calculation of NSE originates from the nuclear partition function because the highly excited state distribution of nuclei has not been well known hitherto. (the distribution of the highly excited states is almost continuous, particularly for the heavier nuclei). In the SN environment, the nuclei are usually unstable and different isotopes have quite different nuclear level distributions. So it is difficult to obtain an explicit distribution for each nucleus. Hartmann *et al*[2] adopted a ground state nuclear partition function in their calculation. Auferheide *et al*[6] adopted an energy level density parameter given by Thielemann[11] to describe the nuclear partition function. Dimarco *et al*[12] considered the limited experimental data of low-lying discrete nuclear states to calculate the nuclear partition function.

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Obviously, those methods should be corrected, especially for high temperature environments. Recently Rauscher and Thielemann [13−15] gave a new nuclear partition function based on the Fermi gas formula, which is relative to an energy-dependent level density parameter with microscopic correction to a nuclear mass model (finite-range droplet model (FRDM) and extended Thomas–Fermi mass formula with shell quenching effects (ETFSI-Q)). This work makes the level densities in a wider range acknowledged as compared with some other methods. This new method is reliable, especially for exploring a nuclear partition function of unknown isotopes at high temperatures, and makes it possible to obtain more accurate nucleus abundance in NSE. In the present paper, we recalculate the NSE by using the Rauscher's new method. Because the central area of presupernova is composed of the 'iron-group' nuclei after the silicon burning, and the abundance of the 'iron-group' is crucial to the change of electron fraction, so here we focus on the nuclei with A values in a range of 50–60 in core-collapse supernova environment.

2. Calculation and analysis of NSE

For the convenience of the explanation of the following results, it is necessary to introduce some formulae into our calculation. The mass abundance χ_k for the kth nucleus (Z_k, A_k) in NSE at a temperature T, density ρ, and electron fraction Y_e must conserve the mass and charge,[6] so

$$\sum_k \chi_k = 1, \tag{1}$$

$$\sum_k \frac{Z_k}{A_k}\chi_k = Y_\mathrm{e}. \tag{2}$$

According to nuclear Saha formula, the abundance for the kth nucleus is given by

$$\chi_k = \frac{G(Z,A,T)}{2}\left(\frac{\rho N_\mathrm{A}\lambda^3}{2}\right)^{A-1} \times A^{5/2} X_\mathrm{n}^\mathrm{N} X_\mathrm{p}^\mathrm{Z} \exp(Q_k/kT) \tag{3}$$

where $G(Z,A,T)$ is the nuclear partition function for the kth nucleus, N_A is Avogadro's number, λ is the thermal wavelength, $\lambda = (h^2/2\pi m_\mathrm{H} kT)^{1/2}$, k is Boltzmann's constant. X_p and X_n are abundances of proton and neutron respectively. Q_k is the binding energy. $Q_k = (Zm_\mathrm{p} - Nm_\mathrm{n} - m_k)c^2$, m_p, m_n and m_k are the masses of the proton, neutron and the kth nucleus respectively, the masses come from the experiment if available, otherwise from Ref.[16]. $G(Z,A,T)$ is defined as

$$G(Z,A,T) = \sum_i (2J_i+1)\exp\left(-\frac{E_i}{kT}\right), \tag{4}$$

where J_i and E_i are the spin and energy of the ith state respectively. As $G(Z,A,T)$ is normalized to the ground state spin of nucleus, it is usually defined as

$$G(Z,A,T) = (2J_0+1)G(T) = \sum_{\mu=0}^{\mu_\mathrm{m}} (2J_\mu+1)\mathrm{e}^{-E_\mu/kT} + \int_{E^{\mu_\mathrm{m}}}^{E^{\max}} \sum_{J_\mu,\pi_\mu} 2(J_\mu+1)\mathrm{e}^{-\varepsilon/kT}\rho(\varepsilon,J_\mu,\pi_\mu)\mathrm{d}\varepsilon, \tag{5}$$

where J_0 is the spin of ground state, ρ is the level density and μ_m is the label of the involved last state which is known experimentally. The sum over Boltzmann-weighted discrete states from the ground state to state μ_m is performed using experimental levels as listed in Ref.[12]. Above the last known state an integration over the nuclear level density is used instead of a summation as described in Ref.[14]. Especially, the high temperature correction in expression (5), which is based on the independent particle model, is naturally applied to both bound and continuous nuclear excited states.

In order to investigate the effect of nuclear partition function on the NSE, we investigate the NSE for some important temperature and density at the presupernova stage. Figures 1 and 2 show a comparison of the species mass abundances at $T = 5\times10^9$ K and $0.45 < Y_\mathrm{e} < 0.5$ for different densities. The solid line is for the nucleus abundance while the ground state partition function is adopted; the dotted line is for the nucleus abundance while improved partition function is adopted. One can find the abundance of each

nuclide peaks around its specific Y_e-value Z/A (which is corresponding to that derived by Hartmann *et al*[2]) and the widths of the peaks are the same for different partition functions, but the corrections are dramatically different for seven most abundant nuclei. For example, there is almost no change for ^{58}Fe, but more than 5% for ^{52}Cr at $\rho = 1 \times 10^7\,\mathrm{g\cdot cm^{-3}}$. Note that this difference is not determined by just one nucleus, it is the contribution of all nuclei involved in the calculation. In fact, the corrections to partition functions of ^{58}Fe and ^{52}Cr are 2.05 and 1.266 times larger than that of their ground state respectively. ^{56}Ni and ^{54}Fe are two of most important nuclei when $Y_e = 0.5$ (their abundance is about 69%), so the total change rate of electron fraction $\dot{Y}_e^{ec}$ is strongly influenced by them. The EC rates of ^{56}Ni and ^{54}Fe are $10^{-1.8}(\mathrm{s}^{-1})$ and $10^{-3}(\mathrm{s}^{-1})$ at collapse stage, calculated by detailed shell model, respectively.[17] The $\dot{Y}_e^{ec}$ has the following relation with EC rate:

$$\dot{Y}_e^{ec} = \sum_k -\chi_k/A_k\lambda_k^{ec}, \tag{6}$$

where λ_k^{ec} is the EC rate for the kth nucleus. We find from Figs.1 and 2 that the abundance decreases for ^{56}Ni, but increases for ^{54}Fe at $Y_e < 0.49$, the total $\dot{Y}_e^{ec}$ should decrease. By making the analyses similar to those for other nuclei (^{56}Fe, ^{58}Fe, ^{52}Cr), we find that $\dot{Y}_e^{ec}$ decreases and then increases with EC when $0.45 < Y_e < 0.5$. Certainly, the collapse process of C-C SN will be changed correspondingly. Surely there are many feedback effects in the numerical simulations, these effects need to be confirmed by detailed numerical simulations. Figure 2 shows the results at a higher density $\rho = 1\times10^9\,\mathrm{g\cdot cm^{-3}}$; obviously the abundances change significantly but the corrections become lower, which indicates the influence of partition function is not sensitive to density. Table 1 gives the concrete data of nuclei abundance at different temperatures. It also can be seen that, in general, corrections to abundances increase slightly with temperature. The reason is that the increase of the temperature tends to enhance the nuclear partition function. But the increase of partition function is on the average 3 times less than compared with that for a ground state when the temperature $T < 5 \times 10^9\,\mathrm{K}$.

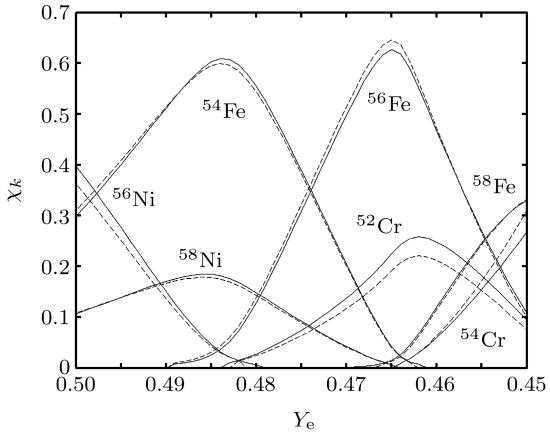


Fig.1. Mass fraction of the most abundant nuclei in NSE as a function of electron fraction at $T = 5 \times 10^9\,\mathrm{K}$, $\rho = 1 \times 10^7\,\mathrm{g\cdot cm^{-3}}$.

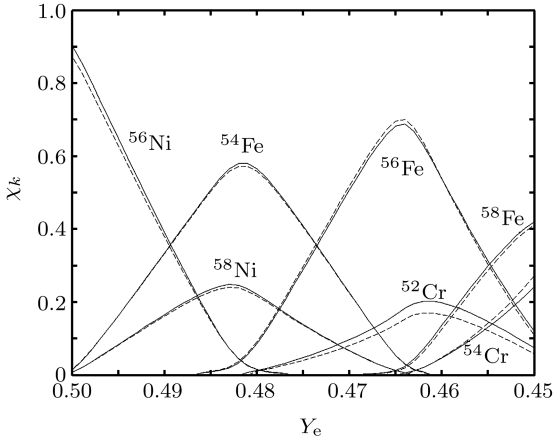


Fig.2. Mass fraction of the most abundant nuclei in NSE as a function of electron fraction at $T = 5 \times 10^9\,\mathrm{K}$, $\rho = 1 \times 10^9\,\mathrm{g\cdot cm^{-3}}$.

Table 1. Mass fraction of the most abundant nuclei in NSE for different temperatures at $\rho = 1\times10^7\,\mathrm{g\cdot cm^{-3}}$ and $Y_e = 0.5$. The left panel shows the results of ground state partition function and the right panel gives the results of the improved partition function.

$T/10^9$K	3	4	5	3	4	5
^{56}Ni	9.941×10^{-1}	9.081×10^{-1}	3.633×10^{-1}	9.948×10^{-1}	9.182×10^{-1}	3.978×10^{-1}
^{52}Fe	4.721×10^{-3}	2.859×10^{-2}	4.093×10^{-2}	3.977×10^{-3}	2.016×10^{-2}	2.581×10^{-2}
^{54}Fe	3.379×10^{-4}	2.834×10^{-2}	3.101×10^{-1}	3.365×10^{-4}	2.760×10^{-2}	2.984×10^{-1}
^{58}Ni	1.593×10^{-4}	1.156×10^{-2}	1.080×10^{-1}	1.591×10^{-4}	1.140×10^{-2}	1.065×10^{-1}
^{55}Co	2.877×10^{-4}	6.865×10^{-3}	2.514×10^{-2}	2.903×10^{-4}	7.124×10^{-3}	2.868×10^{-2}
^{53}Fe	2.801×10^{-4}	8.633×10^{-3}	4.024×10^{-2}	2.684×10^{-4}	7.893×10^{-3}	3.653×10^{-2}

3. Conclusion

By investigating the behaviour of different partition functions, we find that the NSE weakly depends on partition function (it is below 10%) at $T < 5\times10^9$ K and $0.45 < Y_e < 0.5$. In the whole collapse process of C-C SN, low entropy is a remarkable characteristic feature due to the neutrino energy loss.[18] Although new results provide a better abundance estimate for the NSE, the correction to partition function is limited for the collapse stage of C-C SN. As the central density reaches a maximum (2–3 times the nuclear density), the infalling outer core collides with the stiff inner core and produces the bounce shock, and the shock temperature is more than 10^{11} K (for example, the maximal temperature $T > 10^{12}$ K for 15Msolar Model). At such a high temperature, the energy of photo is much larger than the nucleus binding energy, the iron nuclei are photodisintegrated into free nuclei, and there appear behind shock only protons, neutrons and electrons, which is the so-called 'three particle model'.[19] We also investigate the NSE under such conditions, but similar conclusion is obtained. So the correction to nucleus abundance caused by improved partition function for C-C SN is not so important as shown in the previous prediction.[14] Weak interactions (such as electron captures and beta decays), however, might be sensitive to the partition function. Therefore, more detailed numerical simulation will be considered in our future work.

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