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Magnetotransport in a dual waveguide coupled by a finite barrier: Energy filter and directional coupler*

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We propose in this paper that a dual waveguide coupled by a finite barrier be able to serve as an energy filter under a perpendicular magnetic field. In the waveguide direction, the conductance exhibits a periodic square-wave pattern in which the miniband is controlled by the magnetic and potential modulation. The electrons with energies in the miniband can completely transfer along one waveguide while the other electrons undergo filtration. Compared with the coupled waveguide without magnetic modulation, the structure under magnetic field is found to be a good directional coupler. By adjusting the potential barrier and magnetic field, the electrons input from one port of waveguide can transfer to any other ports.

Keywords: magnetotransport, dual waveguide
PACC: 7320D, 7220F

1. Introduction

With the rapid development of semiconductor nanotechnology, coupled electron waveguides have been experimentally realized by split gate technology[1] and cleaved-edge overgrowth technology.[2] This renews the interest in studying the peculiar transport properties of electrons in interacting one-dimensional systems.[3−5] Theoretical and experimental studies on a dual waveguide coupled by a barrier have been extensively carried out.[6−13] Such a structure can serve as a quantum field-effect directional coupler (QFED),[6−11] which was first proposed by del Alamo and Eugster.[6] A QFED is comprised of two parallel identical electron waveguide separated by a narrow barrier whose height could be controlled by the voltage applied to it. Due to the tunnelling effect, an electron wave packet initially confined in one of the waveguide will transfer to another while propagating along the device. A complete transfer may occur if the electrons have just travelled a certain distance defined as transfer length.[6] Since the physical dimension of the device's active region is fixed and the transfer length varies with gate voltage, the portion of the electron wave transferred from the input channel to the output channel can be modulated by the gate voltage.

Various interesting magnetotransport properties are exhibited in the waveguide structures, with a magnetic field applied to it.[12−15] Electronic transmission in a quasi-one-dimensional waveguide in the presence of a thin potential barrier is investigated in high magnetic fields.[12,13] The coupling between the reflected edge states on both sides of the barrier by the tunnelling effect makes an effective loop in the barrier region and leads to an Aharonov–Bohm-type oscillation. The numerical analysis of an electronic waveguide coupler based on two quantum wires coupled by a magnetically defined barrier is presented by the use of the scattering-matrix method.[14] The magnetic field in the coupling region drastically enhances the coupling between the two quantum wires for one specific input port while it weakens the coupling for the other input ports. The system also behaves as a good QFED. Shi and Gu[15] studied magnetoconductance oscillations of two parallel quantum wires coupled through a infinite potential barrier. It is found that in the resonant-tunnelling regions a pair of edge states around the barrier region with countermoving directions is coupled and forms a circulating localized state, leading to the quenching of the relevant propagation modes.

In this paper, we study the magnetotransport of a

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dual waveguide coupled by a finite barrier in the presence of a perpendicular magnetic field. Our calculations indicate that the structure can be used as a fine energy filter. The parallel conductance of the coupled waveguide exhibits a periodic square-wave pattern. The electrons with energies in the miniband transfer along one waveguide with transmittance $T = 1$, while other electrons with energies in the minigap undergo filtration. The widths of the minibands and minigaps are related to the potential height and magnetic field. However, the conductance quenching because of coupling of edge states is not found. It is also found that the coupled waveguide is a better directional coupler, with a magnetic field applied to it, because electrons input from one port of the structure can transfer to any other ports under the external modulation, while the electrons in a QFED without magnetic field cannot transfer to the port which is perpendicular to the input port with a big probability. In addition, the edge states in the structure are displayed by plots of probability density to verify the analysis of transport mechanism.

2. Model and method

We consider a dual waveguide coupled by a finite barrier as shown in Fig.1, which can be realized by

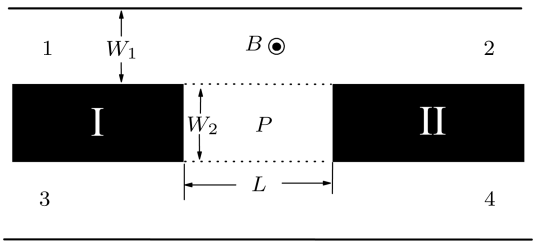


Fig.1. Schematic diagram of the dual waveguide where two waveguides with the same width W_1 are coupled through a finite potential barrier with a size $W_2 \times L$ and height P. A magnetic field B is perpendicularly applied to the structure.

using split gate technology. Two waveguide with the same width W_1 are coupled through a finite potential barrier with a size $W_2 \times L$ and potential height P. The ports of upper and lower waveguide are respectively denoted as ports 1, 2 and 3, 4. I and II represent two infinite potential regions. A magnetic field $\boldsymbol{B}$ is perpendicularly applied to the whole structure. To describe the electronic properties of the system, by using a discretized lattice model the tight-binding Hamiltonian of the system is written as[16−18]

$$\begin{aligned} H = & \sum_{i,j} \varepsilon_{i,j} |i,j\rangle \langle i,j| + \sum_{i,j} V(|i,j\rangle \langle i,j+1| + \text{H.c.}) \\ & + \sum_{i,j} \exp(\text{i}eBa^2 j/\hbar) V(|i+1,j\rangle \langle i,j| + \text{H.c.}), \end{aligned} \tag{1}$$

where $\varepsilon_{i,j}$ is the site energy at (i,j) site,

$$\begin{aligned} \varepsilon_{i,j} &= -4V && \text{sites in two waveguides,} \\ \varepsilon_{i,j} &= -4V + P && \text{sites in potential barrier,} \\ \varepsilon_{i,j} &= \infty && \text{sites in regions I and II,} \end{aligned} \tag{2}$$

and $V = -\hbar^2/2m^*a^2$ (m^* is the effective mass of electron and one sets $m^* = 0.067m_0$) is hopping matrix element between the nearest neighbouring sites. In order to calculate transmission amplitude between each two leads, we first regard the four-terminal system as a two-terminal system, i.e. leads 1, 3 and region I are regarded as the left terminal, while the right terminal consists of leads 2, 4 and region II. Using recursive Green's function sketch,[16] one can obtain the total Green's function $\mathcal{G}$ of the system. The total Green's function $\mathcal{G}$ has the following form:

$$\mathcal{G} = \begin{pmatrix} \mathcal{G}_{2,1} & \mathcal{G}_{\text{II},1} & \mathcal{G}_{4,1} \\ \mathcal{G}_{2,\text{I}} & \mathcal{G}_{\text{II,I}} & \mathcal{G}_{4,I} \\ \mathcal{G}_{2,3} & \mathcal{G}_{\text{II},3} & \mathcal{G}_{4,3} \end{pmatrix}, \tag{3}$$

where $\mathcal{G}_{ba}$ represents the Green's function from lead (or region) b to lead (or region) a. Once the Green's function $\mathcal{G}_{ba}$ is obtained, the transmission coefficient T_{ba} from lead a to lead b can be calculated.[16] According to Launder formula, the conductance G_{ba} from lead a to lead b is

$$G_{ba} = \frac{2e^2}{h} T_{ba}. \tag{4}$$

To calculate the eigen energy E of the system and corresponding wave function Ψ, one can write the Hamiltonian of the system as

$$H = H_0 + \Sigma', \tag{5}$$

where H_0 is the Hamiltonian of the scattering region without leads, $\Sigma' = V^2(\mathcal{G}_{11} + \mathcal{G}_{22} + \mathcal{G}_{33} + \mathcal{G}_{44})$ is the sum of self-energies of the four leads ($\mathcal{G}_{ii}$ is the Green's function of the lead i). Solving the eigen equation $H\Psi = E\Psi$, one can obtain the eigen energy and wave function.[16]

3. Results and discussion

In Fig.2, the calculated conductance is presented as a function of electron energy for the coupled waveguide in Fig.1 with $P = 4\,\mathrm{meV}$ and $B = 8T$. The solid

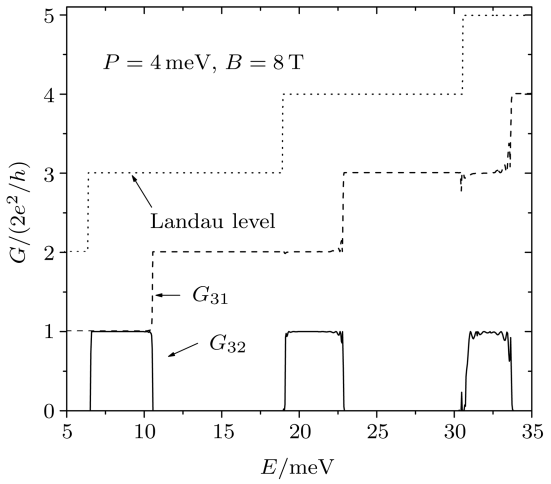


Fig.2. Conductances G_{21} and G_{31} as functions of electron energy for the coupled waveguide in Fig.1 with parameters $W_1 = W_2 = 50\,\mathrm{nm}$, $L = 150\,\mathrm{nm}$, $P = 4\,\mathrm{meV}$ and $B = 8\,\mathrm{T}$. The dotted line represents the quantitative conductance G_L corresponding to the Landau level of a perfect waveguide with a width W_1 under a magnetic field B. The dashed and dotted lines have been vertically offset by one unit for clarity.

and dashed lines represent the conductances G_{21} and G_{31} respectively. For comparison, the dotted line shows the quantitative conductance G_L corresponding to the Landau level of one perfect waveguide under a magnetic field B. The dashed and dotted lines have been vertically offset by one unit for clarity. One can see that the conductance G_{21} exhibits a nearly perfect square-wave pattern. Minibands with transmittance $T = 1$ and minigaps with $T = 0$ periodically appear in the conductance profile. Only the electrons with energies in the miniband of G_{21} are permitted to transfer along upper waveguide while other electrons undergo filtration (prohibited). Thus, in the parallel direction, the coupled waveguide under the magnetic modulation serve as a fine energy filter. It is better than the energy filters by means of modulated superlattice,[19,20] because almost no oscillation is superposed on the minibands of the conductance we calculated (especially on the two lower minibands). At the same time, the coupled waveguide becomes a divider. This can be illustrated by conductance profile of G_{31} which shows a step-like quantitative plateau. The quantitative plateau of G_{31} has a right shift and the shift distance is equal to the width of miniband of G_{21} as compared with that of G_L. Moreover, one can see that $G_{21}+G_{31} \approx G_\mathrm{L}$, due to conductances G_{41} and G_{11} being very small. Thus, for the electrons with energies in the miniband of G_{21}, part of them longitudinally flow from port 1 into port 2 while the others transversely flow into port 3. The ratio between the flows is related to the index of Landau level. For example, in the (second) miniband of G_{21} ($19.14\,\mathrm{meV} < E < 22.88\,\mathrm{meV}$) corresponding to the second Landau level, electrons input from port 1 transfer to ports 2 and 3 with the same probabilities; while the fluence flowing into port 3 is twice the fluence flowing into port 2, as the electron energy lies in the (third) miniband of G_{21} ($30.62\,\mathrm{meV} < E < 33.78\,\mathrm{meV}$) corresponding to the third Landau level.

The conductance profiles in Fig.2 can be well understood by the analysing of edge states and coupling between them. It is well known that a magnetic field will produce a pair of the left- and right-moving edge states. There is only one edge state along each boundary of structure as the electron energy E is just enough to satisfy the first Landau level E_1. When the energy difference $\triangle E_1 = E - E_1$ is lower than the potential height P, the edge state appears along the parallel direction as shown in Fig.3(a). The electrons in the state have to travel from port 1 to port 2, due to the transverse tunnelling effect of electron being suppressed by the strong magnetic field. The energies of these electrons will be restricted within the miniband of G_{21}. However, when the energy difference $\triangle E_1$ is higher than the potential height, electrons can pass over the barrier and thus form a new edge state along the hard wall of the barrier as shown in Fig.3(b). Then the electrons input from port 1 will transversely transfer to port 3 while the parallel transfer is prohibited, i.e. the energies of the electrons will lie in the minigap of G_{21}. To verify the explanation, we depict in Figs.3(c) and 3(d) the probability density plots of the electrons with energies $E = 8\,\mathrm{meV}$ (in the miniband) and $E = 12\,\mathrm{meV}$ (in the minigap) in Fig.2, respectively. One can find that the plots are respectively consistent with those in Figs.3(a) and 3(b).

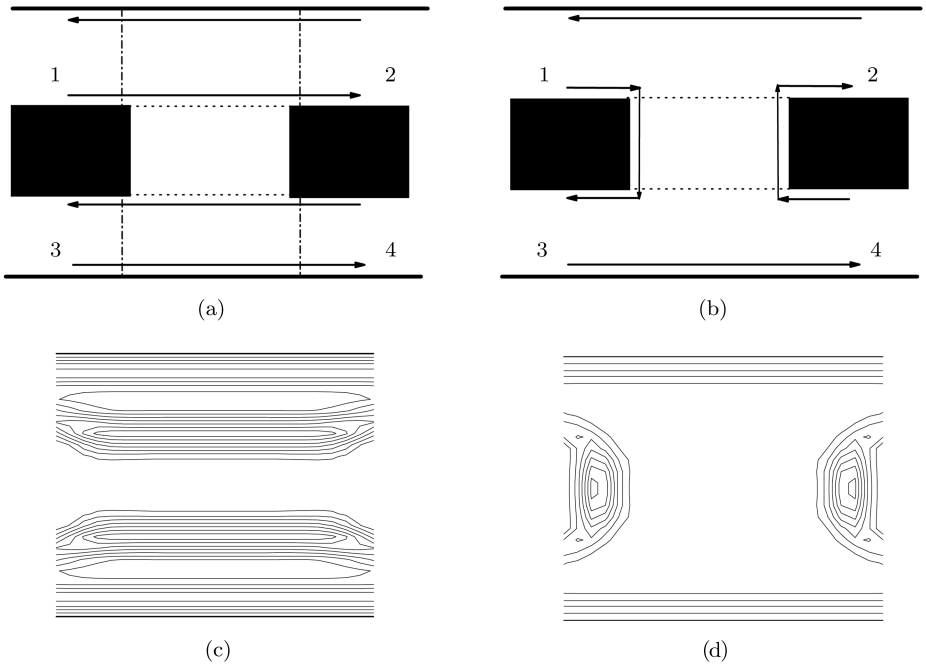


Fig.3. Schematic diagram of the edge states for electrons transferring along (a) the parallel direction and (b) the transverse direction. Subfigures (c) and (d) are for the plots of probability density of the electrons with energies $E = 8\,\mathrm{meV}$ and $E = 12\,\mathrm{meV}$ in Fig.2 respectively. The regions displayed in subfigures (c) and (d) are the middle region between the two dash-dot lines in Fig.3(a).

As the electron energy is more than sufficient to satisfy the nth Landau level, n edge states will appear. The electrons in the lower-lying $(n-1)$ states will transfer from port 1 to port 3, due to the energy difference $\triangle E_i = E - E_i$ (E_i is the energy of the ith Landau level) begin higher than the potential height P, while the nth edge states will undergo a change from Fig.3(a) to Fig.3(b) with the increase of electron energy. As the energy difference $\triangle E_n = E - E_n$ is lower than the potential barrier, the electron in the nth edge state will transfer along the parallel direction as shown in Fig.3(a). In this case, the electron energy lies in the miniband of G_{21}. As $\triangle E_n > P$, the electron in the nth edge state will pass over the barrier and flow into port 3. In this case, the electron energy lies in the minigap of G_{21} and all electrons will transversely transfer to port 3. As a result, the conductance G_{21} forms a periodic miniband while G_{31} exhibits a quantitative plateau.

The analyses indicate that the width of miniband of G_{21} is related to the potential height P. To show the dependence of the miniband on external modulation, we calculate the conductance for the coupled waveguide with two different sets of values for potential height P and magnetic field B as shown in Fig.4. When the magnetic field B is increased from 8 T in Fig.2 to 10 T, the conductance as a function of electron energy is shown in Fig.4(a). Compared with Fig.2, minibands of G_{21} and conductance G_{31} shift toward high energies and the periods of curves increase which originates from the variation of the Landau level with higher magnetic fields. However, one can also find that the miniband width of G_{21} does not change due to the invariance of P. Figure 4(b) shows the conductance as a function of electron energy as P increases to 8meV. It is clear that the width of the miniband becomes wider accordingly and the quantitative plateau of G_{31} shifts toward high energies. From Fig.4, we conclude that the magnetic field and potential barrier have obvious modulation to the miniband of G_{21}. In other words, as an energy filter, whether the electrons in it are filtered can be controlled by the external modulation. In addition, the coupled waveguide under an external modulation acts as a better directional coupler than a QFED without modulation. On the one hand, the coupled waveguide we considered is a QFED in the absence of magnetic field. The electrons input from port 1 can transfer to port 2 or 4 by adjusting potential height.[6,9,10] On the other hand, the electrons which transfer along the parallel direction can be controlled to output from port 3 under magnetic modulation. For example, in Fig.4(b) the

electrons with energies 12meV $< E <$ 16meV parallelly transfer from ports 1 to 2. By changing the potential height, these electrons transversely transfer to port 3 as shown in Fig.4(a). So the electrons input from one port of the coupled waveguide can transfer to any other ports by using the magnetic and potential modulations, while the electrons input from port 1 of a QFED without magnetic field cannot reach port 3 with a big probability.

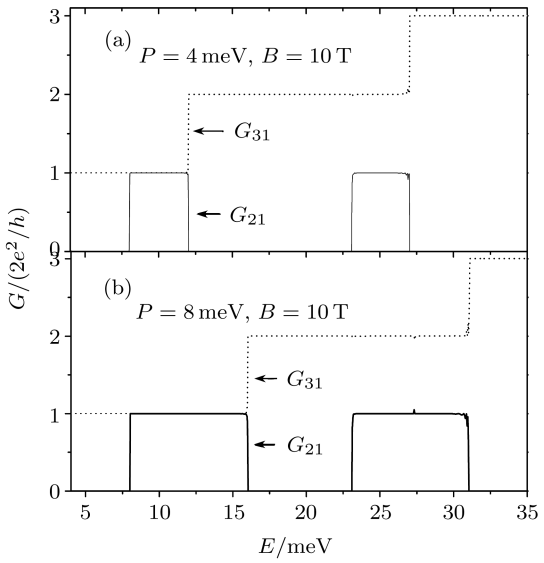


Fig.4. Conductances G_{21} and G_{31} as functions of electron energy for the coupled waveguides with (a) $P = 4\,\mathrm{meV}, B = 10\,\mathrm{T}$; (b) $P = 8\,\mathrm{meV}, B = 10\,\mathrm{T}$. Other parameters of the structure are the same as those in Fig.2. The solid and dotted lines represent the conductances G_{21} and G_{31} respectively. The dotted line has been vertically offset by one unit for clarity.

It is noted that, to form a good square-wave pattern of G_{21}, the length L of the barrier should be sufficiently long, otherwise the coupling of the edge states around the barrier with countermoving directions will destroy the regular conductance profiles. Figure 5 shows the calculated conductance for the coupled waveguide with different values of L. It is seen from Figs.5(a) to 5(c) that the periodic square-wave of G_{21} and the quantitative plateau of G_{31} are gradually broken with the decrease of L. The said reason reminds us that the width W_2 of the barrier should be sufficiently wide if we want to obtain a fine energy filter.

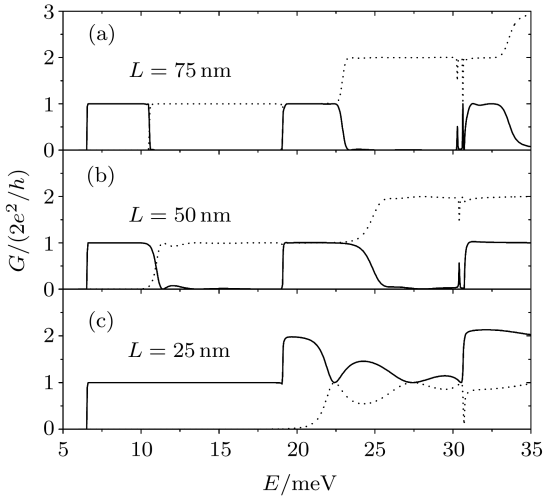


Fig.5. Conductances G_{21} and G_{31} as functions of electron energy for the coupled waveguide in Fig.1 with different values of length L of the potential barrier: $L = 75$ (a), $L = 50$ (b), and $L = 25\,\mathrm{nm}$ (c). The solid and dotted lines represent the conductances G_{21} and G_{31} respectively. Other parameters are same as those in Fig.2.

4. Conclusions

In conclusion, we have proposed that a dual waveguide coupled by a finite barrier be able to serve as an energy filter under magnetic modulation. Our calculations indicate that the parallel conductance exhibits a periodic square-wave pattern. The location and the width of miniband can be controlled by the potential and magnetic modulation. It is also found that the structure under a magnetic field acts as a better directional coupler than a QFED without modulation. Electrons input from one port of the coupled waveguide can transfer to any other ports by adjusting the bias voltage and magnetic field. To form a regular square-wave pattern of parallel conductance, the width and length of the potential barrier should be sufficiently wide and long, otherwise the coupling of edge states will destroy the regular conductance profiles.

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