Vol 16 No 10, October 2007
1009-1963/2007/16(10)/3072-08
Chinese Physics
© 2007 Chin. Phys. Soc.
and IOP Publishing Ltd

Proximity effects in normal mental/spin-splitting material/superconductor junctions*

Yu Hua-Ling(郁华玲)† and Dong Zheng-Chao(董正超)

Department of Physics, Huaiyin Teachers College, Huaian 223001, *China*

(Received 3 March 2007; revised manuscript received 16 April 2007)

We extend the Blonder, Tinkham and Klapwijk (BTK) theory to the study of the coexistence between ferromagnetism and s-wave superconductivity in ferromagnet/superconductor (F/S) structures. It is found that the ferromagnetism and s-wave superconductivity can coexist near the F/S interface, which is induced by proximity effects. On the F side, the density of states (DOS) exhibits some superconducting-like properties, and it displays a damped oscillation from '0' to 'π' states with increasing either the thickness of F film or the exchange energy. We also study the influences of the spin-polarized exchange splitting in the F and the spin-degeneracy by Rashba spin–orbit coupling (RSOC) in the two-dimensional electron gas (2DGE) on the proximity effects. It is shown that the case of Rashba spin-degeneracy is very different from that of the spin-polarized exchange splitting.

Keywords: proximity effects, 'π' state, Rashba spin–orbit coupling
PACC: 7210, 7450

1. Introduction

The possibility of coexistence between ferromagnetism and singlet superconductivity has attracted much attention in experimental and theoretical investigations for many years. Owing to their incompatible nature, i.e. the ferromagnetism and singlet superconductivity having different requirements for spin orientation of itinerant electrons, it is difficult for them to coexist in a bulk material. Pioneering studies in the 1960s have shown that their coexistence condition is given by $h_0/\Delta_0 < 0.707$ for zero centre-of-mass momentum Cooper pairs[1] and $0.707 < h_0/\Delta_0 < 0.754$ for the Fulde–Ferrell–Larkin–Ovchinikov (FFLO) states[2,3] with the Cooper pair having a finite centre-of-mass momentum $Q = 2h_0/\hbar v_{\rm F}$. Here $2h_0$ is the exchange energy corresponding to the difference in energy between spin-up and spin-down bands in ferromagnet (F), Δ_0 is the superconducting energy gap, and $v_{\rm F}$ is the Fermi speed. The exchange energy in a bulk F is typically at least two orders of magnitude larger than Δ_0 of a bulk superconductor (S), so that it is not easy to verify this prediction experimentally and the 'FFLO' state has never been observed in bulk materials. Until recently, whether or not the itinerant ferromagnetism can coexist with the singlet superconductivity is still in dispute.[4−7] The condition for the coexistence of the ferromagnetism and singlet superconductivity may be largely relaxed near the F/S interface. The Cooper pairs in S can penetrate into the interface over some distance inside the F, while the spin-polarized electrons in F can enter into the S, leading to the proximity effect[8] and inverse proximity effect.[9] As a result, special attention has been paid to the F/S systems, from which we can study the coexistence of the ferromagnetism and singlet superconductivity near the interface.

In the F/S structure the proximity effect can induce superconducting-like properties near the interface on the F side. The induced superconducting order parameter $F(x)$ near the interface exhibits damped oscillations with the distance from the interface.[10−14] The state with positive $F(x)$ is usually called the '0' state and that with negative $F(x)$ called the 'π' state.[12,14] Kontos *et al* [12] have studied differential conductance spectra in $Al/Al_2O_3/PdNi/Nb$ tunnel junctions where PdNi is a thin ferromagnetic film and Nb is an s-wave S. The observed density of states (DOS) in the F shows the '0' and 'π' state behaviours for the thicknesses of the ferromagnetic layer being, respectively, smaller and larger than $\xi_{\rm F}\pi/2$, where

*Project supported by the Natural Science Foundation of Education Bureau of Jiangsu Province, China (Grant No 05KJB140008), and the Program for Excellent Talents in Huaiyin Teachers College, China.
†E-mail: hlyu7505@yahoo.com.cn

$\xi_F = \hbar v_F/2h_0$ is the coherence length of the F. Near the Fermi energy the DOS of the 'π' state exhibits an inversion of that of the '0' state with respect to the normal state. Many theoretical studies have shown that inhomogeneous superconductivity can appear in the F, and the DOSs for induced '0' and 'π' states can be reproduced by their calculations.[8,9,13−21] However, there has been a dispute about the DOS oscillations observed in the experiment.[12] The point at issue is whether the DOS oscillations appear only in the diffusive case or also in a clean limit.[8,9]

Zareyan *et al*[13] applied the quasiclassical approach based on the Eilenberger equation to obtain DOS oscillations in the clean limit for the F film with rough surface. Sun, *et al*[14] reproduced oscillations of the DOS observed in the experiment[12] using theoretical approaches in the clean limit developed by McMillan[22] and Blonder, Tinkham and Klapwijk (BTK).[23] Halterman and Valls[16−18] performed extensive theoretical studies of the local DOS behaviour in F/S systems in the clean limit using the self-consistent Bogoliubov–de Gennes (BdG)[24] approach. They calculated the DOS spectra in abundant cases, in which reversions of maxima and minima in the subgap structure could be found. On the other hand, from the quasiclassical approach using Elienberger equations, it is concluded that damped oscillations of the DOS appear only in the diffusive case and they disappear in the clean limit.[8,9,15,19] As regards the DOS oscillations obtained in the clean limit from the Elienberger equation,[13] they were argued to depend essentially on the surface boundary conditions of the F film.[8] For the F film on the top of S, whose thickness is only of a few atomic layers, the calculations of the DOS should be more suitably performed within the ballistic regime than in the diffusive case. In the clean limit, however, the quasiclassical approach[8,9,15,19] based on Elienberger equations and the BdG approach[14,17,18,25−28] seem to give completely different results for the DOS oscillations. This raises a natural question: Which is valid? To clarify this point, in this work we apply the BdG approach to performing a study for the tunnelling conductance in N/I/F/S structures where N is a normal metal and I is a thin insulating film. Such a structure is just the same as that used in the experiment, in which the superconducting-like DOS in the thin F film was measured by tunnelling spectroscopy.[12] As regards the DOS oscillations obtained, it is found that the Andreev reflection (AR) at the F/S interface plays a dominant role, and the DOS oscillations in the clean limit arise from quantum interference effects between the electron and the Andreev-reflected hole wavefunctions. Their wavevector difference in magnitude is equal to $1/\xi_F$, leading to the oscillation period on the order of ξ_F.

Very recently, the effects of spin–orbit coupling on transport properties have attracted much attention both in experimental and theoretical investigations.[29−45] The pioneering theoretical work of Datta and Das[46] has suggested the way to control the precession of the electrical spins via the Rashba spin–orbit coupling (RSOC) in the ballistic F/2DEG/F junctions, where the interlayer is 2DEG for the semiconductor. This spin–orbit coupling can be tuned by a gate voltage and depends on the interfacial electric fields, which has been demonstrated subsequently by many studies. It is well known that the spin-degeneracy induced by the RSOC does not break the time reversal symmetry unlike a spin-polarized exchange-splitting in ferromagnet. These two kinds of splittings are illustrated in Fig.1. It is naturally

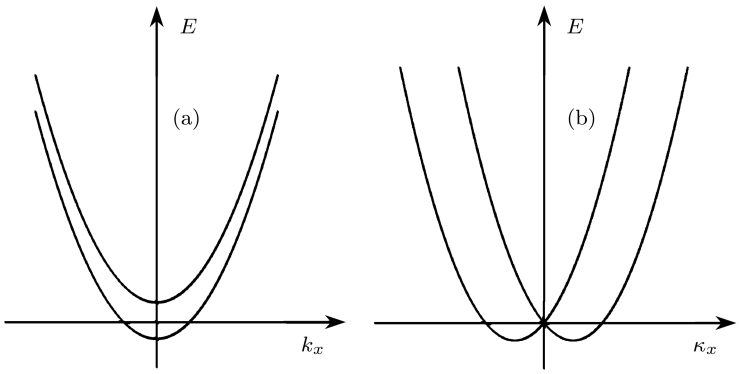


Fig.1. Schematic representation of the conduction band structure of (a) a spin-polarized exchange-split band ferromagnet and (b) 2DEG in which the spin-degeneracy is induced by RSOC.

expected that proximity effects in 2DEG/S junctions may be qualitatively different from those in F/S junctions, and they have not been studied well. In the present work, we use N/I/2DEG/S structures to study the proximity effects in 2DEG. The calculated results show that the 'π' state cannot be induced by RSOC in the 2DEG, which is very different from that induced by spin-polarized exchange-splitting in the F film.

2. Model and theory

Consider an N/I_1/F(2DEG)/I_2/S double-barrier junction as shown in Fig.2. The left and right sides are made of N (normal metal) and S (superconductor) and they are separated from the central F(2DEG) by two thin insulating interfaces, respectively. The two interfaces at $x = 0$ and $x = L$ are described by two δ-type barrier potentials $V(x) = U_1\delta(x) + U_2\delta(x-L)$, where L is the thickness of F(2DEG) interlayer and $U_{1(2)}$ depends on the product of barrier height and width. We use an effective single-particle Hamiltonian to describe the electrons (holes) in N, F and 2DEG,

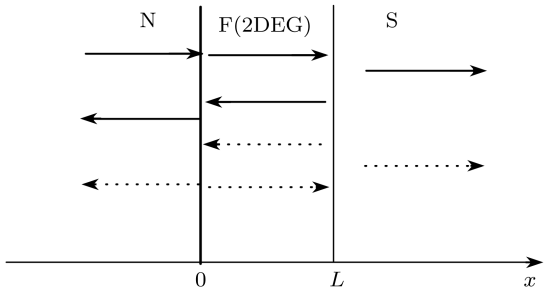


Fig.2. Schematic illustration of an N/I_1/F(2DEG)/I_2/S junction and reflections and transmissions of quasiparticles in the junction. The solid-line arrow denotes the electrons in N, F and 2DEG or ELQ in S with spin σ, while the dotted-line arrow represents the holes in N, F and 2DEG or HLQ in S with $\bar{\sigma}$.

and the S is assumed to be s-wave pairing and described by a Bardeen–Cooper–Schrieffer (BCS)-like Hamiltonian. In the one-dimensional BTK model the quasiparticle wavefunctions satisfy the following Bogoliubov–de Gennes (BdG) equation[24]

$$\begin{bmatrix} H_0(x) + H_{1\sigma}\Theta(x)\Theta(L-x) & \Delta(x) \\ \Delta^*(x) & -H_0(x) - H_{1\bar{\sigma}}\Theta(x)\Theta(L-x) \end{bmatrix} \begin{bmatrix} u_\sigma \\ v_{\bar{\sigma}} \end{bmatrix} = E \begin{bmatrix} u_\sigma \\ v_{\bar{\sigma}} \end{bmatrix}. \tag{1}$$

Here $H_0(x) = \hat{p}_x \dfrac{1}{2m^*(x)} \hat{p}_x + V(x) - E_{\rm F}$, E is the quasi-particle energy relative to the Fermi energy $E_{\rm F}$, $\hat{p}_x$ is the momentum operator, $m^*(x) = m_0$ in the N/F/S junctions and $m^*(x) = m_0[\Theta(-x)+\Theta(x-L)]+ m^*_{\rm D}\Theta(x)\Theta(L-x)$ in the N/2DEG/S junctions with $\Theta(x)$ being a unit step function and $m^*_{\rm D}(m_0)$ being an effective mass in 2DEG (others), $H_{1\sigma} = -\eta_\sigma h_0$ for N/F/S junctions with h_0 being the exchange energy in F, $\eta_\sigma = 1$ for $\sigma = \uparrow$ and $\eta_\sigma = -1$ for $\sigma = \downarrow$, $\bar{\sigma}$ denotes the spin opposite to σ, and $H_{1\sigma} = \dfrac{1}{2\hbar}[\alpha\hat{p}_x + \hat{p}_x\alpha] + \delta E_c$ for N/2DEG/S junctions with α being the RSOC parameter in 2DEG, and δE_c being the conduction band mismatch between the 2DEG and the others. In general, Eq.(1) should be a four-component BdG equation for u_σ and v_σ with $\sigma = \uparrow$ and $\downarrow$, where u_σ and v_σ represent the spin-σ electron-like quasiparticle (ELQ) and hole-like quasiparticle (HLQ), respectively. After the spin-flip process has been neglected in S near the interface, the four-component BdG equation may be decomposed into two sets of two-component equations: $(u_\sigma, v_{\bar{\sigma}})$ describing the spin-σ ELQ and spin-$\bar{\sigma}$ HLQ.

Consider a spin-σ electron incident upon the interface at $x = 0$ from the left N as shown in Fig.2. There are four possible trajectories: normal reflection (b_σ), AR[47] ($a_{\bar{\sigma}}$), transmission to S as ELQ (c_σ) and transmission as HLQ ($d_{\bar{\sigma}}$). With general solutions of BdG equation (1), the wavefunctions in the three regions have the following forms:

$$\psi_\sigma(x) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} {\rm e}^{{\rm i}q_{\rm e}^\sigma x} + a_{\bar{\sigma}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} {\rm e}^{{\rm i}q_{\rm h}^{\bar{\sigma}} x} + b_\sigma \begin{pmatrix} 1 \\ 0 \end{pmatrix} {\rm e}^{-{\rm i}q_{\rm e}^\sigma x} \tag{2}$$

in the left region ($x \le 0$);

$$\psi_\sigma(x) = c_\sigma \begin{pmatrix} u \\ v \end{pmatrix} {\rm e}^{{\rm i}q_{\rm e}x} + d_{\bar{\sigma}} \begin{pmatrix} v \\ u \end{pmatrix} {\rm e}^{-{\rm i}q_{\rm h}x} \tag{3}$$

in the right region ($x \geq L$); and in the middle region ($0 \leq x \leq L$),

$$\psi_\sigma(x) = \mathrm{e}\begin{pmatrix}1\\0\end{pmatrix}\mathrm{e}^{\mathrm{i}k_{\mathrm{e}}^{\sigma}x} + f\begin{pmatrix}1\\0\end{pmatrix}\mathrm{e}^{-\mathrm{i}k_{\mathrm{e}}^{\sigma}x} + g\begin{pmatrix}0\\1\end{pmatrix}\mathrm{e}^{\mathrm{i}k_{\mathrm{h}}^{\bar{\sigma}}x} + h\begin{pmatrix}0\\1\end{pmatrix}\mathrm{e}^{-\mathrm{i}k_{\mathrm{h}}^{\bar{\sigma}}x} \tag{4}$$

for the F, and

$$\psi_\sigma(x) = \mathrm{e}\begin{pmatrix}1\\0\end{pmatrix}\mathrm{e}^{\mathrm{i}\kappa_{\mathrm{e}}^{\sigma}x} + f\begin{pmatrix}1\\0\end{pmatrix}\mathrm{e}^{-\mathrm{i}\kappa_{\mathrm{e}}^{\bar{\sigma}}x} + g\begin{pmatrix}0\\1\end{pmatrix}\mathrm{e}^{\mathrm{i}\kappa_{h}^{\sigma}x} + h\begin{pmatrix}0\\1\end{pmatrix}\mathrm{e}^{-\mathrm{i}\kappa_{\mathrm{h}}^{\bar{\sigma}}x} \tag{5}$$

for the 2DEG. Here $q_{\mathrm{e(h)}}^{\sigma} = k_{\mathrm{F}} \pm m_0 E/(\hbar^2 k_{\mathrm{F}})$, $k_{\mathrm{e(h)}}^{\sigma} = k_{\mathrm{F}}^{\sigma} \pm m_0 E/(\hbar^2 k_{\mathrm{F}}^{\sigma})$ with $k_{\mathrm{F}}^{\sigma} = k_{\mathrm{F}}\sqrt{1+\eta_\sigma h_0/E_{\mathrm{F}}}$ and $k_{\mathrm{F}} = \sqrt{2m_0 E_{\mathrm{F}}}/\hbar$, $\kappa_{\mathrm{e(h)}}^{\sigma} = \dfrac{\sqrt{2m_{\mathrm{D}}^*}}{\hbar}\left[-\eta_\sigma\gamma + \sqrt{\gamma^2 + E_{\mathrm{F}} - \delta E_c \pm E}\right]$ with $\gamma = \alpha\sqrt{\dfrac{m_{\mathrm{D}}^*}{2\hbar^2}}$, $q_{\mathrm{e(h)}} = k_{\mathrm{F}} \pm m_0\Omega/(\hbar^2 k_{\mathrm{F}})$ with $u = \sqrt{(1+\Omega/E)/2}$, $v = \sqrt{(1-\Omega/E)/2}$ and $\Omega = \sqrt{E^2 - \Delta_0^2}$. Coefficients $a_{\bar{\sigma}}$, b_σ, c_σ and $d_{\bar{\sigma}}$ correspond to the probability amplitudes for the AR, normal reflection, transmissions within the same branch and between different branches, respectively. All the coefficients can be determined by matching boundary conditions at $x = 0$ and $x = L$:

$$\begin{aligned}
&\psi_\sigma(0_-) = \psi_\sigma(0_+),\\
&\mu\frac{\mathrm{d}\psi_\sigma(x)}{\mathrm{d}x}|_{x=0_+} = \frac{\mathrm{d}\psi_\sigma(x)}{\mathrm{d}x}|_{x=0_-} + (-\mathrm{i}\eta_\sigma k_{\mathrm{F}}\sqrt{\frac{\mu}{E_{\mathrm{F}}}}\gamma + 2k_{\mathrm{F}}Z_1)\psi_\sigma(0),\\
&\psi_\sigma(L_-) = \psi_\sigma(L_+),\\
&\frac{\mathrm{d}\psi_\sigma(x)}{\mathrm{d}x}|_{x=L_+} = \mu\frac{\mathrm{d}\psi_\sigma(x)}{\mathrm{d}x}|_{x=L_-} + (\mathrm{i}\eta_\sigma k_{\mathrm{F}}\sqrt{\frac{\mu}{E_{\mathrm{F}}}}\gamma + 2k_{\mathrm{F}}Z_2)\psi_\sigma(L),
\end{aligned} \tag{6}$$

with $\mu = 1$ and $\gamma = 0$ for the N/I_1/F/I_2/S junctions, and $\mu = \dfrac{m_0}{m_{\mathrm{D}}^*}$ for the N/I_1/2DEG/I_2/S junctions, where $Z_1 = m_0 U_1/(\hbar^2 k_{\mathrm{F}})$ and $Z_2 = m_0 U_2/(\hbar^2 k_{\mathrm{F}})$ are the dimensionless barrier strength. From them we obtain the probabilities $A_{\bar{\sigma}} = |a_{\bar{\sigma}}|^2 \mathrm{Re}(q_{\mathrm{h}}^{\bar{\sigma}}/q_{\mathrm{e}}^{\sigma})$, $B_\sigma = |b_\sigma|^2$, $C_\sigma = |c_\sigma|^2(|u|^2 - |v|^2)\mathrm{Re}(q_{\mathrm{e}}/q_{\mathrm{e}}^{\sigma})$, and $D_{\bar{\sigma}} = |d_{\bar{\sigma}}|^2(|u|^2 - |v|^2)\mathrm{Re}(q_{\mathrm{h}}/q_{\mathrm{e}}^{\sigma})$, respectively, corresponding to the AR and normal reflection coefficients, the transmission coefficients of electron-like and hole-like quasiparticles. Finally, the tunnelling conductance is given by

$$G = \sum_{\sigma=\uparrow,\downarrow} G_\sigma = \sum_{\sigma=\uparrow,\downarrow} \frac{2e^2}{h}(1 - B_\sigma + A_{\bar{\sigma}}). \tag{7}$$

We give the expressions for $a_{\bar{\sigma}}$ and b_σ in the Appendix. In the clean limit, the quasiparticle in the middle layer (F or 2DEG) undergoes no scattering by breaking up the phase coherence, thus the wavevector of the quasiparticle remains unchanged. In the dirty case the impurity scattering needs considering and the wavevector in the exponents should be extended, in magnitude, to a complex number with a small imaginary part being inversely proportional to the quasiparticle lifetime. In the following calculations we use a simple approximation by replacing $k_{\mathrm{e,h}}^{\sigma}$ and $\kappa_{\mathrm{e,h}}^{\sigma}$ in the exponents by $k_{\mathrm{e,h}}^{\sigma} \pm \dfrac{\mathrm{i}}{2\xi_{\mathrm{F}}}$ and $\kappa_{\mathrm{e,h}}^{\sigma} \pm \dfrac{\mathrm{i}}{2\xi_{\mathrm{D}}}$, where ξ_{D} is the coherence length in 2DEG and $\xi_{\mathrm{D}} = \xi_{\mathrm{S}}$ with $\xi_{\mathrm{S}} = \hbar v_{\mathrm{F}}/2\Delta_0$, and the coherence length is in S. Here $\dfrac{1}{\xi_{\mathrm{F}}}$ or $\dfrac{1}{\xi_{\mathrm{S}}}$ is much smaller than the magnitude of wavevector $k_{\mathrm{e,h}}^{\sigma}$ in F or $\kappa_{\mathrm{e,h}}^{\sigma}$ in 2DEG.

3. Results and discussion

We first calculate the tunnelling conductance in the N/I_1/F/S double junctions to study the coexistence between ferromagnetism and s-wave superconductivity in the F film, in which I_1 is assumed to be a delta-type barrier and the F/S interface is of the ohmic contact. Owing to the superconducting proximity effect, a few Cooper pairs are injected into F from S side and these Cooper pairs can survive for a time corresponding to a travelled length on the order of ξ_{F}. Together with the interference between the electron and the AR hole in the F film, the conductance displays a damped oscillation with increasing L/ξ_{F}. For a fixed coherence length ξ_{F}, when the thickness L is smaller

than ξ_F the conductance presents the '0' state and when L is larger than ξ_F the case is reversed and it corresponds to 'π' state, which has already studied clearly in our previous work.[48] Here we calculate the effects of exchange energy h_0 on the tunnelling conductance for a fixed thickness of F film. The parameters used in the calculations are taken as follows: $Z_1 = 3.0$ for the dimensionless potential strength of I_1, $Z_2 = 0$ for the F/S ohmic contact, the Fermi energy $E_F = 1.4\,\text{eV}$, the superconducting energy gap $\Delta_0 = 1.4\,\text{meV}$, and the thickness of F film is 50 Å. As shown in Fig.3(a), the energy dependences of the tunnelling conductance for different values of exchange energy h_0 are plotted. The tunnelling conductance is normalized by its value at $E = 3\Delta_0$ beyond which the conductance is almost constant. For the case of $h_0 = 0$, the normalized conductance exhibits two peaks at $E = \pm\Delta_0$ and nearly appears a gap within $|E| < \Delta_0$, which is very close to that in the bulk S, corresponding to '0' state. With increasing the exchange energy h_0, the two peaks shift gradually towards the centre and so the normalized conductance shows a minigap and its width decreases. At the same time, the amplitudes of peaks decrease slowly as they go towards Fermi level ($E = 0$) and the values within the minigap increase, but there remain two small dips at $E = \pm\Delta_0$, indicating that the superconducting gap in bulk S is not the significantly influenced due to the presence of the F layer. When the exchange energy h_0 is larger than $10\Delta_0$, the state of the normalized conductance is reversed and two peaks arrive at zero energy ($E = 0$), exhibiting two dips on the energy gap edges ($E = \pm\Delta_0$) and a maximum at zero energy ($E = 0$), which corresponds to the 'π' state. These are illuminated clearly by the insets. The left shows that the normalized conductance at the Fermi level displays a damped oscillation with increasing h_0 and the right shows the position of the peak E_g/Δ_0 as a function of h_0/Δ_0. All of these are the same as those with increasing the thickness of F film for a fixed exchange energy. Now, we further get at the root of this case. To see through the physical picture, we consider a simplest case of $Z_1 \gg 1$ and $Z_2 = 0$ under the assumption that all the N, S and F have the same Fermi wavevector except in the exponential. In this case, the tunnelling conductance can be approximately reduced to

$$G_\sigma = \frac{2e^2}{h}\frac{1}{Z_1^4 \sin^2[(k_e^\sigma - k_h^{\bar\sigma})L - \phi(E)]} \qquad (8)$$

for $|E| \le \Delta_0$, where $\phi(E) = \tan^{-1}\left(\sqrt{(\Delta_0/E)^2 - 1}\right)$. How the divergent peak emerges and how its position changes have been examined in our previous work. Here we only analyse the oscillation of the tunnelling conductance with increasing the exchange energy. For a fixed E, $\phi(E)$ is a certain phase. We see clearly from Eq.(8) that the tunnelling conductance displays the oscillation and it is determined by $\cos^2(k_e^\sigma - k_h^{\bar\sigma})L$ for $E = 0$. In the F film $k_e^\sigma - k_h^{\bar\sigma} = 2h_0/\hbar v_F = 1/\xi_F$, thus $\cos^2(k_e^\sigma - k_h^{\bar\sigma})L = \cos^2 L/\xi_F$, from which we conclude that this oscillation depends on L/ξ_F with $\xi_F = \hbar v_F/2h_0$ the coherent length in the F film. The coherent length ξ_F is a certain value for a fixed h_0, so an increase in L will increase L/ξ_F. For a fixed L, an increase in h_0 will decrease ξ_F, thus L/ξ_F also increases.

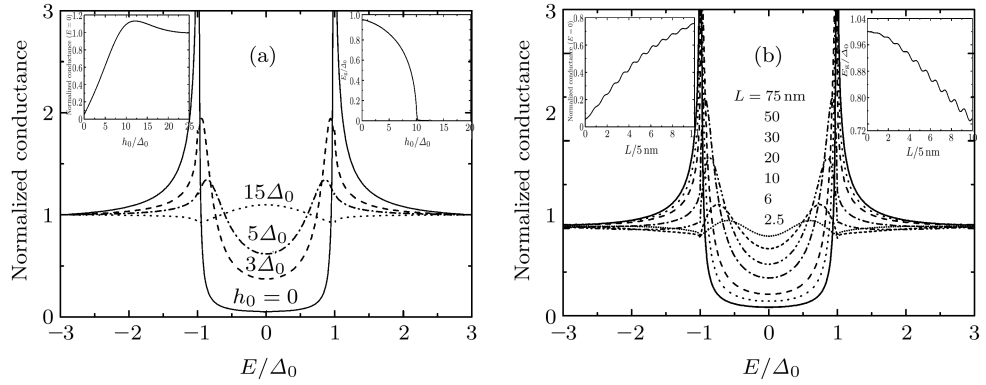


Fig.3. Energy dependence of normalized conductance (a) for different values of exchange energy h_0 in the N/I_1/F/S junctions. It is gradually reversed from '0' to 'π' states with increasing h_0. The inset on the left shows the normalized conductance at Fermi level as a function of h_0/Δ_0 and that on the right shows the position of the maximal conductance E_g/Δ_0. (b)for different thicknesses of 2DEG in the N/I_1/2DEG/S junctions. The inset on the right shows the position of the maximal conductance E_g/Δ_0 and that on the left shows the normalized conductance at Fermi level as a function of L.

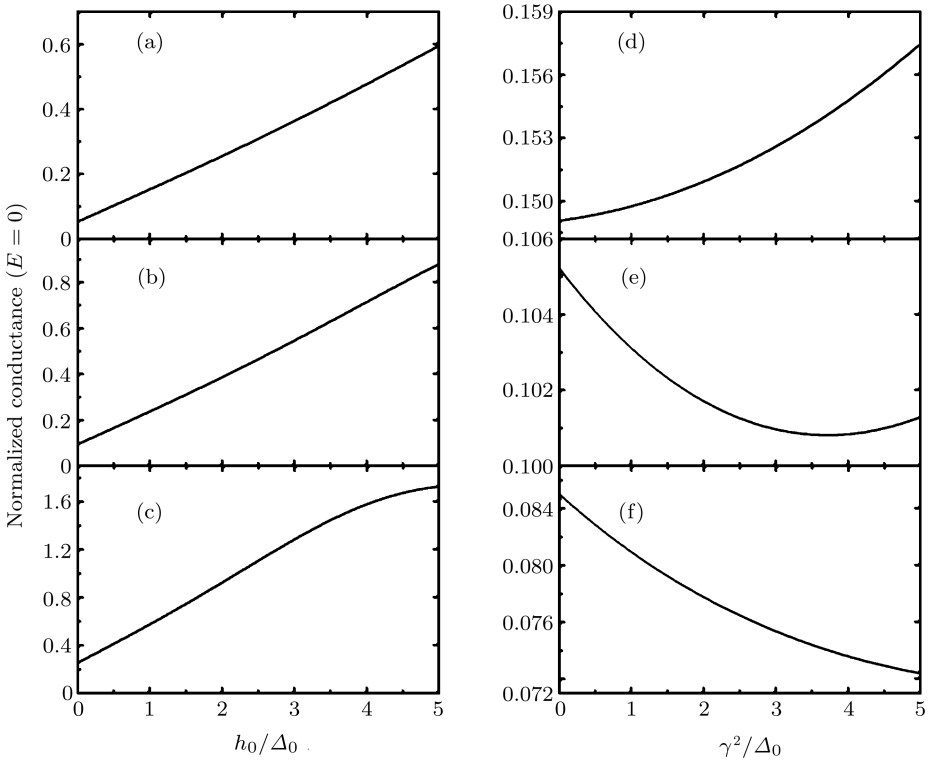


Fig.4. Normalized conductance at Fermi level as a function of exchange energy h_0/Δ_0 in the $N/I_1/F/I_2/S$ junctions in the left panels and RSOC γ_0^2/Δ_0 in the $N/I_1/2DEG/I_2/S$ junctions in the right panels for different values of Z_2.

Now, we study the energy dependences of normalized conductance for different thicknesses of 2DEG in $N/I_1/2DEG/S$ junctions with $\gamma^2 = \Delta_0$, $m_D^* = 0.036m_0$, $\delta E_c = 1.35\,\text{eV}$ and the other parameters being the same as those in Fig.3(a). As shown in Fig.3(b), they exhibit BCS-like minigap behaviours. They are similar to our previous results in the $N/I_1/N/S$ junctions, and there are no 'π' states, which also can be understood well by the insets. From the right, we find that the position of the peak E_g cannot arrive at $E_g = 0$ till the L becomes very larger. From the left, the normalized conductance at $E = 0$ does not display a damped oscillation, which is very different from that in the $N/I_1/F/S$ junction. In the F film, the damped oscillation from '0' state to 'π' state of DOS is primarily determined by the phase coherence between the electron and the AR hole. This phase coherence in the 2DEG film is similar to that in the N but different from that in the F. Here the small waves in the insets are due to the conduction band mismatch δE_c and the effective mass m_D^* in the 2DEG.

Next we present the different effects on the normalized conductance at $E = 0$ between the spin-polarized exchange- splitting in the F and the Rashba splitting in the 2DEG. The microscopic mechanism responsible for the proximity effect is the quantum coherence between the electron near E_F and its corresponding AR hole. In F/S junctions, the spin-polarized exchange splitting in the F film breaks the time reversal symmetry, which leads to the difference in the perpendicular wavevector component between the injected electron and AR hole, i.e. $k_e^\sigma - k_h^{\bar\sigma} = 2h_0/\hbar v_F$. In the 2DEG/S junctions, the spin-degeneracy by RSOC in 2DEG do not break the time reversal symmetry and the AR wavevector is κ_h^σ, which results in the zero difference between the injected electron and the AR hole, i.e. $\kappa_e^\sigma - \kappa_h^\sigma = 0$ at Fermi level. The wavevector of the normal reflection is no more equal to the negative wavevector of injection κ_e^σ but equal to $\kappa_e^{\bar\sigma}$, which is also unlike that in the F/S structure. All of these will lead to the different results in the 2DEG/S junctions from those in the F/S junctions. We plot the normalized conductance for $N/I_1/F/I_2/S$ junction as a function of the exchange energy h_0/Δ_0 in Figs.4(a)–(c) and for $N/I_1/2DEG/I_2/S$ junction as a function of γ^2/Δ_0 in Figs.4(d)–(f). Here, the dimensionless potential strength of I_2 is $Z_2 = 0$ in (a) and (d), $Z_2 = 0.2$ in (b) and (e), and $Z_2 = 0.5$ in (c) and (f). The other

parameters are the same as those in Fig.3(b). We find that the exchange energy h_0 always enhances the normalized conductance independent of Z_2 for small values of h_0 in Figs.4(a)–(c), i.e. h_0 always suppresses the AR in the F/S junctions. But the dependence of RSOC on the normalized conductance is apparently different, which is shown in Figs.4(d)–(f) for different values of Z_2. For $Z_2 = 0$, it increases with increasing RSOC. For $Z_2 = 0.2$, it has a reentrant behaviour as a function of γ^2/Δ_0. For $Z_2 = 0.5$, it decreases as increasing RSOC, indicating that RSOC may enhance the AR in the 2DEG/S junctions.

4. Conclusion

The BTK theory has been extended to the calculation of the tunnelling conductance for the N/I_1/F(2DEG)/I_2/S double junctions. Owing to the proximity effects, on the F side near the F/S interface, the DOS exhibits some superconducting-like properties, and it displays a damped oscillation from '0' to 'π' states with increasing either the thickness of the F film or the exchange energy, exhibiting the coexistence of the ferromagnetism and s-wave superconductivity. In the 2DEG in the proximity of a superconductor, the 'π' state cannot be induced by superconducting proximity effects, and the AR may be enhanced by RSOC, which is very different from that in the F/S structures.

Acknowledgment

We are especially grateful to Professor D. Y. Xing for pointing out the problem in this paper.

Appendix Expressions for normal and Andreev reflection coefficients

Using the boundary conditions in Eq.(6) of the wavefunctions given by Eqs.(2)–(5), we obtain the Andreev and normal reflection coefficients in the N/I_1/2DEG/I_2/S junction as

$$a_{\bar\sigma} = 2uv\mu^2 q_{\rm e}^{\sigma}(q_{\rm e}+q_{\rm h})(\kappa_{\rm e}^{\sigma}+\kappa_{\rm e}^{\bar\sigma})(\kappa_{\rm h}^{\sigma}+\kappa_{h}^{\bar\sigma})\mathrm{e}^{\mathrm{i}(\kappa_{\rm e}^{\sigma}-\kappa_{\rm e}^{\bar\sigma})L}/D_{\sigma}, \tag{A1}$$

$$b_{\sigma} = F_{\sigma}/D_{\sigma}, \tag{A2}$$

with

$$D_{\sigma} = (N_{1\sigma}^{+}-\mu\kappa_{\rm e}^{\bar\sigma})D_{1\sigma} + (N_{1\sigma}^{+}-\mu\kappa_{\rm e}^{\bar\sigma})D_{2\sigma} - (N_{1\sigma}^{+}+\mu\kappa_{\rm e}^{\sigma})D_{3\sigma} - (N_{1\sigma}^{+}+\mu\kappa_{\rm e}^{\sigma})D_{4\sigma}, \tag{A3}$$

$$F_{\sigma} = -(N_{1\sigma}^{-}-\mu\kappa_{\rm e}^{\bar\sigma})D_{1\sigma} - (N_{1\sigma}^{-}-\mu\kappa_{\rm e}^{\bar\sigma})D_{2\sigma} + (N_{1\sigma}^{-}+\mu\kappa_{\rm e}^{\sigma})D_{3\sigma} + (N_{1\sigma}^{-}+\mu\kappa_{\rm e}^{\sigma})D_{4\sigma}, \tag{A4}$$

$$D_{1\sigma} = (N_{2\sigma}-\mu\kappa_{\rm h}^{\bar\sigma})[N_3N_{5\sigma}^{h} - N_4N_{5\sigma}^{\rm e} - (v^2-u^2)(q_{\rm e}q_{\rm h} - N_{5\bar\sigma}^{\rm e}N_{5\bar\sigma}^{\rm h})]\mathrm{e}^{\mathrm{i}(\kappa_{\rm e}^{\sigma}+\kappa_{\rm h}^{\sigma})L}, \tag{A5}$$

$$D_{2\sigma} = (N_{2\sigma}+\mu\kappa_{\rm h}^{\sigma})[N_4N_{5\sigma}^{\rm e} + N_3N_{5\bar\sigma}^{\rm h} + (v^2-u^2)(q_{\rm e}q_{\rm h} + N_{5\sigma}^{\rm e}N_{5\bar\sigma}^{\rm h})]\mathrm{e}^{\mathrm{i}(\kappa_{\rm e}^{\sigma}-\kappa_{\rm h}^{\bar\sigma})L}, \tag{A6}$$

$$D_{3\sigma} = (N_{2\sigma}-\mu\kappa_{\rm h}^{\bar\sigma})[N_4N_{5\bar\sigma}^{\rm e} + N_3N_{5\sigma}^{\rm h} - (v^2-u^2)(q_{\rm e}q_{\rm h} + N_{5\sigma}^{\rm h}N_{5\bar\sigma}^{\rm e})]\mathrm{e}^{-\mathrm{i}(\kappa_{\rm e}^{\bar\sigma}-\kappa_{\rm h}^{\sigma})L}, \tag{A7}$$

$$D_{4\sigma} = (N_{2\sigma}+\mu\kappa_{\rm h}^{\sigma})[N_3N_{5\bar\sigma}^{\rm h} - N_4N_{5\bar\sigma}^{\rm e} + (v^2-u^2)(q_{\rm e}q_{\rm h} - N_{5\bar\sigma}^{\rm e}N_{5\bar\sigma}^{\rm h})]\mathrm{e}^{-\mathrm{i}(\kappa_{\rm e}^{\bar\sigma}+\kappa_{\rm h}^{\bar\sigma})L}. \tag{A8}$$

Here, $N_{1\sigma}^{\pm} = \pm q_{\rm e}^{\sigma} + \eta_\sigma k_{\rm F}\sqrt{\dfrac{\mu}{E_{\rm F}}}\gamma + 2\mathrm{i}k_{\rm F}Z_1$, $N_{2\sigma} = -q_{\rm h}^{\bar\sigma} + \eta_\sigma k_{\rm F}\sqrt{\dfrac{\mu}{E_{\rm F}}}\gamma + 2\mathrm{i}k_{\rm F}Z_1$, $N_3 = (q_{\rm h}v^2 + q_{\rm e}u^2)$, $N_4 = (q_{\rm h}u^2 + q_{\rm e}v^2)$, and $N_{5\sigma}^{\rm e(h)} = \mu\kappa_{\rm e(h)}^{\sigma} + k_{\rm F}\sqrt{\dfrac{\mu}{E_{\rm F}}}\gamma - 2\eta_\sigma\mathrm{i}k_{\rm F}Z_2$ have been considered. Those coefficients in the N/I_1/F/I_2/S can be obtained by replacing $\kappa_{\rm e}^{\sigma}$ and $\kappa_{\rm e}^{\bar\sigma}$ in above expressions by $k_{\rm e}^{\sigma}$, and replacing $\kappa_{\rm h}^{\sigma}$ and $\kappa_{\rm h}^{\bar\sigma}$ by $k_{\rm h}^{\bar\sigma}$, and letting $\gamma = 0$ and $\mu = 1$.

References

[1] Clogston A M 1962 *Phys. Rev. Lett.* **3** 266
[2] Fulde P and Ferrell 1964 *Phys. Rev.* **135** A550
[3] Larkin A and Ovchinnikov Y 1965 *Sov. Phys.* JETP **20** 726
[4] Karchev N I, Blagoev K B, Bedell K S and Littlewood P B 2001 *Phys. Rev. Lett.* **86** 846
[5] Shen R, Zheng Z M, Liu S and Xing D Y 2003 *Phys. Rev.* B **67** 24514
[6] Sun G Y, Xing D Y, Shen R and Lin H Q 2002 *Eur. Phys. J.* **30** 33
[7] Drew A J, Lee S L, Charalambous D, Potenza A, Marrows C, Luetkens H, Suter A, Prokscha T, Khasanov R, Morenzoni E, Ucko D and Forgan E M 2005 *Phys. Rev. Lett.* **95** 197201
[8] Buzdin A I 2005 *Rev. Mod. Phys.* **77** 935
[9] Bergeret F S, Volkov, A F and Efetov K B 2005 *Rev. Mod. Phys.* **77** 1321
[10] Demler E A, Arnold G B and Beasley M R 1997 *Phys. Rev.* B **55** 15174
[11] Andreev A V, Buzdin A and Osgood III R M 1991 *Phys. Rev.* B **43** 10124
[12] Kontos T, Aprili M, Lesueur J and Grison X 2001 *Phys. Rev. Lett.* **86** 304
[13] Zareyan M, Belzig W and Nazarov Yu V 2001 *Phys. Rev. Lett.* **86** 308
[14] Sun G Y, Xing D Y, Dong J M and Liu M 2002 *Phys. Rev.* B **65** 174508
[15] Baladie I and Buzdin A 2001 *Phys. Rev.* B **64** 224514
[16] Halterman K and Valls O T 2001 *Phys. Rev.* B **65** 014509
[17] Halterman K and Valls O T 2002 *Phys. Rev.* B **66** 224516
[18] Halterman K and Valls O T 2004 *Phys. Rev.* B **69** 014517
[19] Bergeret F S, Volkov A F and Efetov K B 2002 *Phys. Rev.* B **65** 134505
[20] Zareyan M, Belzig W and Nazarov Yu V 2002 *Phys. Rev.* B **65** 184505
[21] Golubov A A, Kupriyanov M Yu and ll'ichev E 2004 *Rev. Mod. Phys.* **76** 411
[22] McMillan W L 1968 *Phys. Rev.* **175** 559
[23] Blonder G E, Tinkham M and Klapwijk T M 1982 *Phys. Rev.* B **25** 4515
[24] de Gennes P G 1966 *Superconductivity of Metals and Alloys* (New York: The Benjamin Press)
[25] Dong Z C, Xing D Y and Dong J M 2001 *Acta Phys. Sin.* **50** 556 (in Chinese)
[26] Li X W and Liu S J 2006 *Acta Phys. Sin.* **55** 834 (in Chinese)
[27] Dong Z C 2002 *Acta Phys. Sin.* **51** 894 (in Chinese)
[28] Dong Z C 2005 *Chin. Phys.* **14** 1209
[29] Nikolić B K and Souma S 2005 *Phys. Rev.* B **71** 195328
[30] Batov I E, Schäpers Th, Golubov A A and Ustinov A V 2004 *J. Appl. Phys.* **96** 3366
[31] Debald S and Kramer B 2005 *Phys. Rev.* B **71** 115322
[32] Li Y X, Guo Y and Li B Z 2005 *Phys. Rev.* B **71** 012406
[33] Hu C M and Matsuyama T 2001 *Phys. Rev. Lett.* **87** 066803
[34] Feve G, Oliver W D, Aranzana M and Yamamoto Y 2002 *Phys. Rev.* B **66** 155328
[35] Wu H C, Guo Y, Chen X Y and Gu B L 2003 *J. Appl. Phys.* **93** 5316
[36] Matsuyama T, Hu C M , Grundler D, Meier G and Merkt U 2002 *Phys. Rev.* B **65** 155322
[37] Mireles F and Kirczenow G 2001 *Phys. Rev.* B **64** 024426
[38] Molenkamp L W, Schmidt G and Bauer G E W 2001 *Phys. Rev.* B **64** 121202(R)
[39] Zülicke U and Schroll C 2002 *Phys. Rev. Lett.* **88** 029701
[40] Bezuglyi E V, Rozhavsky A S, Vagner I D and Wyder P 2002 *Phys. Rev.* B **66** 052508
[41] Středa P and Šeba P 2003 *Phys. Rev. Lett.* **90** 256601
[42] Schliemann J and Loss D 2003 *Phys. Rev.* B **68** 165311
[43] Yokoyama T and Tanaka Y cond-mat/051008
[44] Li Y X and Li B Z 2005 *Chin. Phys.* **14** 1021
[45] Duan S Q, Song H Z, Zhang P and Zhao X G 2006 *Chin. Phys.* **15** 3091
[46] Datta S and Das B 1990 *Appl. Phys. Lett.* **56** 665
[47] Andreev A F 1964 *Sov. Phys.* JEPT **19** 1228
[48] Yu H L, Sun G Y, Yang L Y and Xing D Y 2004 *Physica* C **402** 98