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Oscillation of spatial solitons in a waveguide with a symmetrical refractive index profile*

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Dynamics of (1+1)D spatial solitons in a Kerr medium with a transversely symmetrical refractive index profile is investigated. Propagation of solitons is analysed theoretically by using an effective-particle approach. Analytical results show that the soliton oscillates periodically with a variable acceleration. The expression of oscillatory period is derived by introducing a concept of 'average acceleration'. Both acceleration and oscillatory period are determined by the parameters of the input soliton and the waveguide. Propagations of solitons are simulated numerically and good agreement is obtained between the theoretical and numerical results.

Keywords: effective-particle approach, effective 'acceleration', oscillatory period
PACC: 4265S, 4225B, 4280R

1. Introduction

During the past decades, the concept of soliton has penetrated into almost all areas of physics, such as hydrodynamics, plasma physics, optics,[1,2] Bose–Einstein condensates (BECs) in cold gases,[3,4] etc. Optical spatial solitons are self-trapped optical beams of finite spatial cross section that travel without the divergence associated with freely diffracting beams.[5] Due to their novel physics as well as potential applications, spatial solitons are under intensive study these days.[1,2,5−13] Some possible applications of spatial solitons such as optical pulse compression,[14] all optical switching,[15,16] logic devices,[17] etc have been proposed. To explore all possible applications, it is important to extract and understand the generic properties of spatial solitons.

One of the most important properties of spatial solitons is that their dynamics are like a particle moving in a light-induced potential well.[18,19] Aceves *et al* developed an effective particle theory and implemented it on the study of dynamics of spatial solitons due to interface separating nonlinear media.[18] Later, interesting properties of solitons in a waveguide with a transverse refractive index profile is reported.[19,20] More recently, Kartashov *et al*[21,22] have investigated the dynamics of the spatial soliton in a medium with a transverse periodically modulated refractive index by using the same method. The theoretical analysis of dynamics of spatial soliton in a waveguide with both parabolic transverse refractive index distribution and nonlocal nonlinearity is also presented by using this method.[23] We should note that this method is very frequently used in quantum mechanics and enables one to obtain trajectories of particle motion in different potentials if the evolution of the localized particle wavefunction is governed by the equation of modulated nonlinear Schrödinger equation (NLSE).

Recent studies have shown that a harmonic modulation of the linear refractive index (optical lattice) with tunable depth and period can be obtained by interfering two or more plane waves.[6] The soliton may be trapped in or moves along the lattice for the (1+1)D case.[21,22] In the present paper, we are interested in the analytical description of the propagation of soliton within one period of such a lattice. The medium is modulated by a transversely symmetrical refractive index distribution in the form of cosinusoidal function. Our analysis is restricted to the case of a wide index profile with respect to the beam width. The theoretical model is presented in Section 2. The acceleration and oscillatory period of solitons are analysed by using an effective-particle approach in Section 3. Simulations of the propagating dynamics of solitons are presented in Section 4 and Section 5 gives the conclusions.

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2. Model

Taking the paraxial approximation and neglecting the anisotropic properties of the medium, the propagation of optical beam along the z-axis in a slab waveguide with a refractive index modulation in the x-direction is described by the NLSE,

$$\mathrm{i}\frac{\partial A}{\partial z}+\frac{1}{2}\frac{\partial^2 A}{\partial x^2}+|A|^2A+pR(x)A=0, \tag{1}$$

where $A(x,z)=(L_{\mathrm{diff}}/L_{\mathrm{nl}})^{1/2}\,E(x,z)I_0^{-1/2}$, $E(x,z)$ is the slowly varying envelope, I_0 is the peak input intensity, $x=X/r_0$, r_0 is the transverse scale, $z=Z/L_{\mathrm{diff}}$, $L_{\mathrm{diff}}=n_0\omega r_0^2/c$, $L_{\mathrm{nl}}=2c/\omega n_2 I_0$, ω is the carrying frequency, n_0 and n_2 is the linear and nonlinear refractive indices of the medium respectively, $p=L_{\mathrm{diff}}/L_{\mathrm{ref}}$ is the guiding parameter, $L_{\mathrm{ref}}=c/(\delta n\omega)$, δn is the refractive index modulation depth, $R(x)$ describes the transverse refractive index profile. In this paper, we assume $R(x)=\cos(2\pi x/T)$ for $|x|\le T/2$ and $R(x)=0$ for $|x|>T/2$, T characterizes the width of transverse refractive index modulation.

For a vanishing refractive index distribution ($p=0$), the travelling nonlinear wave solutions of Eq.(1) are proportional to the elliptic functions $\mathrm{cn}(u)$ or $\mathrm{dn}(u)$ with $u\propto(x-vt-x_0)$. In addition to these extended wavelike solutions, localized soliton solutions such as $A(x,z)=\eta\mathrm{sech}(\eta x)\exp(\mathrm{i}vx+\mathrm{i}\phi)$ are also possible. For a general potential, the NLSE may become unintegrable. Nevertheless, the dynamics still possesses two integrals of motion, i.e. 'power' $P(z)=\int_{-\infty}^{\infty}AA^*\mathrm{d}x$ and 'Hamiltonian' $H=\int_{-\infty}^{\infty}\left[|A_x|^2-|A|^4-p\,|A|^2\,R(x)\right]\mathrm{d}x$.

In the following discussion, we assume that the potential width is much wider than that of soliton, i.e. $\eta T\gg 1$. The medium is nearly homogeneous to the soliton profile but inhomogeneous to the motion of soliton in the transverse direction when the refractive index distribution due to the symmetrical modulation is small. By applying the quasi-homogeneous approximation, the perturbed sech-type solution becomes[20]

$$\begin{aligned}A(x,z)=\;&\eta\mathrm{sech}[\eta(x-\bar{x})]\\&\times\exp[\mathrm{i}\,(v(z)(x-\bar{x})+\sigma(z))],\end{aligned} \tag{2}$$

where η is the beam amplitude and characterizes the inverse beam width as well, the beam amplitude increases and beam width decreases with the increase of η,[20] $\bar{x}$ is the centre of the soliton defined in Section 3. Note that the speed is now a function of propagation distance z in that $v(z)=\mathrm{d}\bar{x}/\mathrm{d}z$ and $\mathrm{d}\sigma(z)/\mathrm{d}z=[\eta^2-v(z)^2]/2$ compared with the case of uniform medium.

3. Effective particle approach—analytical description

A qualitative picture of the soliton properties in a waveguide with a refractive index profile can be grasped by implementing the effective-particle approach method[18] on such solitons on condition that the beam width is much smaller than the width of refractive index modulation. In this approach, the light beam can be considered as a particle whose position is given by $\bar{x}(z)$, where z is treated as the 'time' variable. To find an equation of motion for the particle (beam), we begin by defining the centre of the beam as[18]

$$\bar{x}(z)=\frac{\int_{-\infty}^{\infty}xAA^*\mathrm{d}x}{P(z)}, \tag{3}$$

where $P(z)$ is the power of the soliton, 'speed'

$$v(z)=\frac{\mathrm{d}\bar{x}}{\mathrm{d}z}=\frac{1}{2}\mathrm{i}\frac{\int_{-\infty}^{\infty}(A\partial_xA^*-A^*\partial_xA)\mathrm{d}x}{P(z)}, \tag{4}$$

and 'acceleration'

$$a(\bar{x})=\frac{\mathrm{d}^2\bar{x}}{\mathrm{d}z^2}=\frac{\mathrm{d}v}{\mathrm{d}z}=-\frac{1}{P(z)}\int_{-\infty}^{\infty}\frac{\mathrm{d}V}{\mathrm{d}x}AA^*\mathrm{d}x. \tag{5}$$

In the present model, $V=-p\cos(2\pi x/T)$ is the symmetrical refractive index distribution.

According to the theory of effective-particle approach, one can assume that AA^* is only a function of $x-\bar{x}$. Since power P is a conservative quantity, i.e. $\mathrm{d}P/\mathrm{d}z=0$, expression (5) can be written as follows:

$$a(\bar{x})=\frac{\mathrm{d}^2\bar{x}}{\mathrm{d}z^2}=-\frac{\partial U(\bar{x})}{\partial\bar{x}}. \tag{6}$$

Expression (6) depicts the movement of a particle in a Newtonian potential U.

The power of Sech-type soliton (expression (2)) can be integrated easily as $P=2\eta$. Substituting the soliton profile (expression (2)) into expression (5), we have the explicit formula of the acceleration,

$$a(\bar{x})=\frac{\mathrm{d}^2\bar{x}}{\mathrm{d}z^2}=-\frac{p\pi^2}{T}\frac{\pi^2/T\eta}{\sinh(\pi^2/T\eta)}\sin\left(\frac{2\pi\bar{x}}{T}\right). \tag{7}$$

We show in Fig.1 the profiles of Newtonian potential, Sech-soliton and acceleration against the position of beam for different values of soliton amplitude η. We should note that the width of soliton is assumed to be zero when the effective-particle approach is applied. If the input beam is launched in the vicinity

of the potential margin, this method will fail because of the practical width of soliton (see Fig.1(a)). The role of Newtonian potential induced by refractive index distribution is to imprison the soliton around the waveguide centre. Obviously, the acceleration is a sinusoidal function of $\bar{x}$. The transverse acceleration tends to zero when the beam moves to the margin of potential. The profiles of acceleration remain a similar shape and approach to an asymptotic curve with the increase of η. One can infer from Fig.1(b) that the acceleration of the 'effective particle' has a maximum value at a certain $\bar{x}$ and the maximum value increases with the increase of η. This position is determined by $\bar{x}_{\max} = \pm T/4$ which can be derived by setting $\partial a/\partial \bar{x} = 0$. We can also predict the movement of soliton from the acceleration profile according to Newton mechanics. If an input beam is launched at a position x_0 deviating from the centre of waveguide with a zero initial 'speed' (defined by expression (4)), it will be subjected to an antisymmetrical ' centripetal force' and moves back and forth towards the waveguide centre periodically within the scope of $[-x_0, x_0]$.

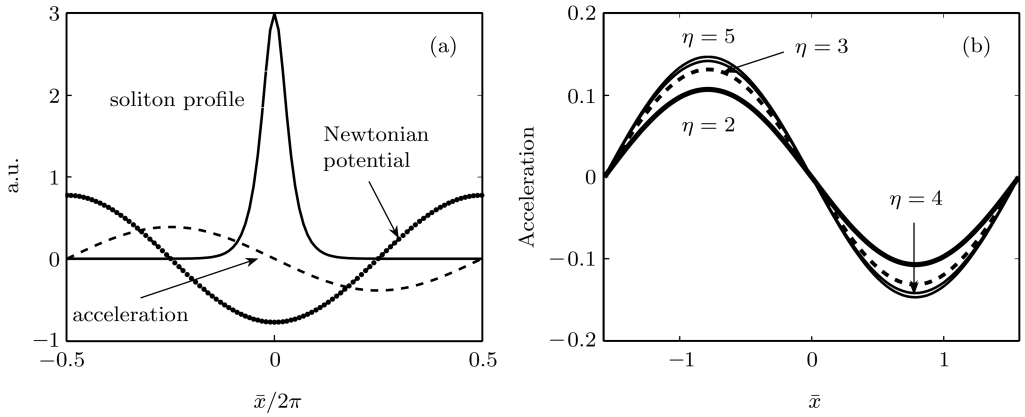


Fig.1. Profiles of acceleration, Newtonian potential and Sech-soliton (a) and acceleration profiles of solitons with different values of η (b). (a) $T = 4\pi$, $p = 0.5$; (b) $T = \pi$, $p = 0.05$.

We are interested in the analytical description of oscillatory period that a soliton beam propagates along the z-axis with a zero initial speed. Since the acceleration varies with $\bar{x}$, we introduce a concept of 'mean acceleration' to depict the motion of soliton,

$$a_{\mathrm{M}} = \frac{1}{w}\int_0^w a(\bar{x})\mathrm{d}\bar{x}, \tag{8}$$

where w is the position of the initial input beam with respect to the centre of the waveguide. Thus the transverse displacement can be expressed as $\bar{x} = a_{\mathrm{M}} z^2/2$. If we consider the beam to be propagating in a quarter of oscillatory period, i.e. the beam to be moving from w to 0, the equation of motion can be written as

$$\frac{1}{2}a_{\mathrm{M}} z^2 - w = 0. \tag{9}$$

The z-direction oscillatory period now is

$$S_z = 4\left(\frac{2w}{a_{\mathrm{M}}}\right)^{1/2}. \tag{10}$$

Substituting expression (7) into expression (8), we obtain

$$\begin{aligned} a_{\mathrm{M}} &= \frac{1}{w}\int_0^w a(\bar{x})\mathrm{d}\bar{x} \\ &= \frac{1}{w}\int_{-w}^0 a(\bar{x})\mathrm{d}\bar{x} \\ &= \frac{\pi p}{2w}\frac{\pi^2/T\eta}{\sinh(\pi^2/T\eta)}\left(1-\cos(2\pi w/T)\right). \end{aligned} \tag{11}$$

Thus, the oscillatory period is determined by

$$S_z = \frac{8w}{\pi}\sqrt{\frac{\pi\sinh(\pi^2/T\eta)}{p\left(1-\cos(2\pi w/T)\right)\pi^2/T\eta}}. \tag{12}$$

Expression (12) is the main result given in this paper. The oscillatory period depends on the soliton amplitude η, refractive index modulation width T, depth parameter p and the initial input position w. Numerical simulations in next section prove that expression (12) is very accurate in describing the oscillatory period if the condition $\eta T \gg 1$ is satisfied.

4. Numerical simulation

To prove the analytical results, we simulate the dynamics of the solitons in a waveguide with a cosinusoidal refractive index modulation by using the stan-

dard beam propagation method (BPM). Periodical oscillations indeed occur. An input beam is launched at a position deviating from the centre of waveguide for a distance w with a zero initial 'speed'. Figure 2 gives the periodical oscillation of soliton with the same parameters η and p, but different values of T and w. We make two comments here. First, the oscillatory frequency increases with the decrease of T. Second, the oscillatory region in the transverse direction is only determined by w.

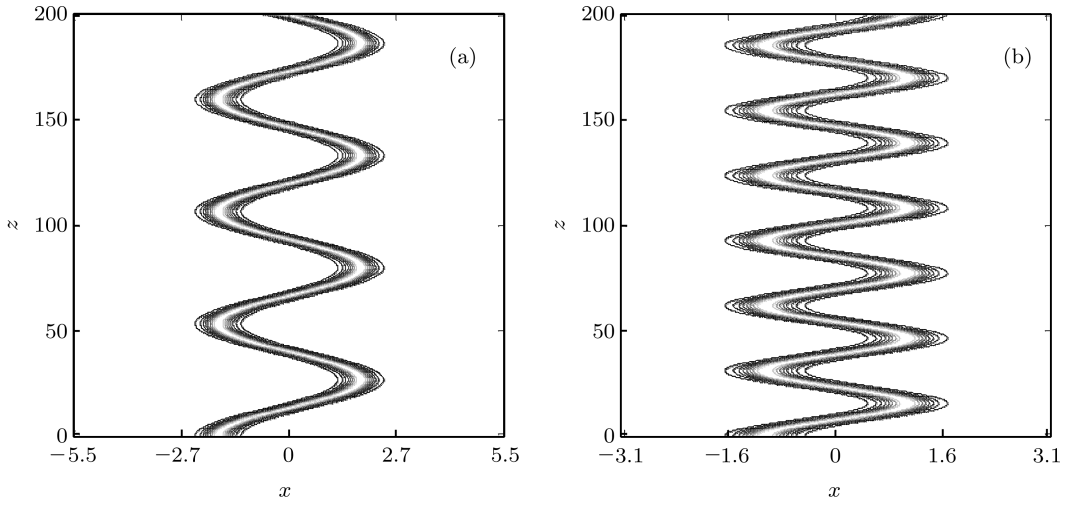


Fig.2. Oscillations of solitons with a cosinusoidal refractive index distribution, $w = T/6, \eta = 3, p = 0.05$. (a) $T = 3.5\pi$, (b) $T = 2\pi$.

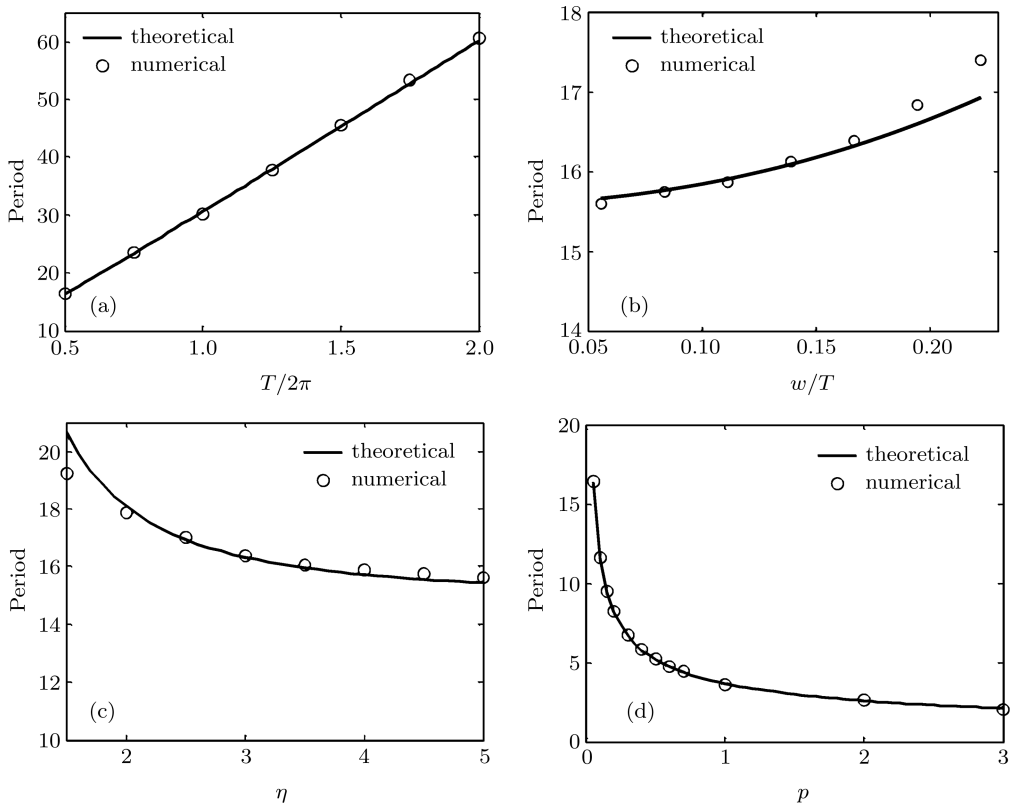


Fig.3. Comparison between oscillatory periods theoretically predicted by expression (12) and numerically calculated by BPM. (a) $\eta = 3$, $w = T/6$, $p = 0.05$; (b) $\eta = 3$, $T = \pi$, $p = 0.05$; (c) $T = \pi$, $w = T/6$, $p = 0.05$; (d) $\eta = 3$, $T = \pi$, $w = T/6$.

To illustrate the influences of all parameters on the oscillatory period, we simulate the propagation of solitons with various parameters and the results are summarized in Fig.3. One can see clearly that the theoretical predictions according to expression (12) are in good agreement with the numerical results. The oscillatory period is almost linearly increasing with the increase of the refractive index modulation width if the deviation of the soliton is fixed as $w = T/6$ (Fig.3(a)). Figure 3(b) shows that the oscillatory period increases with the increase of deviation of the input beam from the centre of waveguide, with other parameters fixed. One may question the discrepancy between the numerical and theoretical results for larger values of w, especially for $w \to T/4$. The cause responsible for the discrepancy can be attributed to the fact that the wing of soliton covers the marginal region of refractive index modulation. Thus partial energy of the beam is located outside the potential well (see Fig.1(a)), and expression (12) does not hold true any more. An interesting conclusion inferred from Fig.3(b) is that, with other parameters fixed, the period of oscillation approaches to a limit (15.60 in the present case) with the decrease of w. This can be explained in physics to be the oscillation of solitons occurring as long as the central position of the input beam is not superposed on that of waveguide. The potential well induced by a refractive index distribution remains almost invariable as w approaches zero. Therefore, the oscillatory period approaches a constant. The difference between the numerical and analytical results shown in Fig.3(c) for $\eta < 1.5$ is due to the fact that the condition $\eta T \gg 1$ is unsatisfied. If the value of T is chosen to be big enough for a fixed η, expression (12) can still accurately predict the motion of the soliton. The period of oscillation decreases when the parameter η or p increases. The oscillatory period approaches to infinity with $p \to 0$, which means that the soliton does not oscillate but moves along a rectilinear trajectory in the z-axis direction.

5. Conclusions

We have investigated the propagation dynamics of (1 + 1)D spatial solitons in a Kerr medium with a transversely symmetrical modulation of refractive index. By using an effective-particle approach, we derive the acceleration and oscillatory period of the soliton in such a scheme under the condition of $\eta T \gg 1$. The acceleration is a sinusoidal function of position of beam centre. The soliton oscillates periodically within the scope determined by the position of initial input soliton. Moreover, the oscillatory period in the z-direction is discussed in detail. The period is influenced by the parameters of soliton and modulation of refractive index profile. We also simulate the propagation of solitons numerically and excellent agreement is obtained between the theoretical and numerical results. Our research proves that the effective-particle approach can accurately describe the dynamics of soliton under appropriate conditions.

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