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Preparation of W state in resonant bimodal cavity quantum electrodynamics*

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A scheme is proposed for generating entangled W states with four cavity modes. In this scheme, we send a V-type three-level atom through two identical two-mode cavities in succession. After the atom exits from the second cavity, the four cavity modes are prepared in the W state. On the other hand we can obtain three-atom W states by sending three V-type three-level atoms through a two-mode cavity in turn. The present scheme does not require conditional measurement, and it is easily generalized to preparing $2n$-mode W states and n-atom W states.

Keywords: resonant interaction, V-type three-level atom, two-mode cavity, W state
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Entanglement is at the heart of quantum information theory and has many applications in quantum information processing, including quantum cryptography,[1] dense coding,[2] and teleportation.[3] In recent years a large number of new entangled states have been proposed, particulary GHZ states,[4] and the W states.[5,6] Dür *et al*[6] have shown that there are two inequivalent kinds of tripartite entanglement states, i.e. the GHZ class and the W class, under stochastic local operation and classical communication. The W state has some interesting properties. For example, it retains bipartite entanglement when any one of the three qubits is traced out, and thus it is much more robust than the GHZ states. W states can be used to test quantum nonlocality without Bell inequality.[7] More recently it has been shown that multiqubit W states lead to stronger nonclassicality than Greenberger-Horne-Zeilinger states (The N -qubit states of the W class, for $N > 10$, lead to more robust violations to local realism than the GHZ states.).[8]

W states can be generated in the ion trap system,[9] and cavity quantum electrodynamics (QED).[10−12] In Ref.[11], Guo and Zhang have proposed a scheme for preparing three-mode W state of cavity fields via cavity (QED). In this scheme, an appropriately prepared two-level atom is sent through three resonant single cavities in turn. After the atom exits from the last cavity, the three cavity fields are entangled in the W state. This scheme can also be used for preparing three atoms in the entangled W state. In this paper we present an alternative scheme for generating entangled four-mode W states. Our scheme is based on the resonant interaction of a two-mode cavity with a V-type three-level atom.[13−16] On the other hand we can obtain three-atom W states by sending three V-type three-level atoms through a two-mode cavity in turn. As far as preparing W states of cavity modes is concerned, compared with Ref.[11] the advantage of our scheme is that we require few cavities. This scheme can be easily generalized to generating $2n$-mode entangled W states and n-atom entangled W states.

We first consider a three-level V-type atom interacting with a two-mode cavity field. The three levels of the atom are denoted by $|1\rangle$, $|2\rangle$ and $|3\rangle$, separately, with $|1\rangle$ and $|2\rangle$ being two upper levels and $|3\rangle$ being the lower level. The transitions $|1\rangle \longleftrightarrow |3\rangle$ and $|2\rangle \longleftrightarrow |3\rangle$ are coupled to the cavity modes A and B, respectively. The transition between levels $|1\rangle$ and $|2\rangle$ is dipole forbidden. The Hamiltonian for the system is (assuming $\hbar$=1)[14−16]

$$H_0 = \omega_1 |1\rangle\langle 1| + \omega_2 |2\rangle\langle 2| + \omega_3 |3\rangle\langle 3| + \omega_A a^\dagger a + \omega_B b^\dagger b, \quad (1)$$

$$H_i = g_1(a^\dagger |3\rangle\langle 1| + a |1\rangle\langle 3|) + g_2(b^\dagger |3\rangle\langle 2| + b |2\rangle\langle 3|). \quad (2)$$

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Here $a^\dagger(b^\dagger)$ and $a(b)$ are, respectively, the creation and annihilation operators for the cavity mode $A(B)$ with frequency $\omega_A(\omega_B)$. ω_i is the energy of the level $|i\rangle$ $(i = 1, 2, 3)$, and g_1 and g_2 are the corresponding coupling constants.

We work on condition that

$$\omega_A = \omega_1 - \omega_3, \quad \omega_B = \omega_2 - \omega_3, \tag{3}$$

so the free Hamiltonian H_0 commutes with the interaction Hamiltonian H_i. Then in the interaction picture, the Hamiltonian can be written as

$$\begin{aligned} H_{\mathrm{I}} = g(&a^\dagger |3\rangle \langle 1| + a |1\rangle \langle 3| + b^\dagger |3\rangle \langle 2| \\ &+ b |2\rangle \langle 3|), \end{aligned} \tag{4}$$

where we assume $g_1 = g_2 = g$. The time evolution of this system is determined by Schrödinger's equation

$$\mathrm{i}[\mathrm{d} |\psi(t)\rangle / \mathrm{d}t] = H_{\mathrm{I}} |\psi(t)\rangle . \tag{5}$$

With the help of a Ramsey field we prepare the upper two levels of the three-level atom in linear superposition state before it has entered into the first cavity field. The two identical cavities are initially in the vacuum state. We may express the initial state of the system as

$$\begin{aligned} \left|\varphi^{(0)}\right\rangle = &[\cos\theta |1\rangle \\ &+ \sin\theta \mathrm{e}^{\mathrm{i}\phi} |2\rangle] |0_A, 0_B\rangle_1 |0_A, 0_B\rangle_2 , \end{aligned} \tag{6}$$

where ϕ is the relative phase between two atomic states.

After the atom is sent through the first bimodal cavity, the atom-cavity field system evolves into the state

$$\begin{aligned} |\varphi\rangle = &\{\cos\theta[\cos(gt_1) |1\rangle |0_A\rangle_1 |0_B\rangle_1 - \mathrm{i}\sin(gt_1) |3\rangle |1_A\rangle_1 |0_B\rangle_1] \\ &+ \sin\theta \mathrm{e}^{\mathrm{i}\phi}[\cos(gt_1) |2\rangle |0_A\rangle_1 |0_B\rangle_1 - \mathrm{i}\sin(gt_1) |3\rangle |0_A\rangle_1 |1_B\rangle_1]\} |0_A\rangle_2 |0_B\rangle_2 , \end{aligned} \tag{7}$$

where t_1 is the interaction time.

After the atom passes through the second cavity the state of the whole system is

$$\begin{aligned} |\Psi\rangle = &\cos\theta\{\cos(gt_1)[\cos(gt_2) |1\rangle |0_A\rangle_1 |0_B\rangle_1 |0_A\rangle_2 |0_B\rangle_2 \\ &-\mathrm{i}\sin(gt_2) |3\rangle |0_A\rangle_1 |0_B\rangle_1 |1_A\rangle_2 |0_B\rangle_2] - \mathrm{i}\sin(gt_1) |3\rangle |1_A\rangle_1 |0_B\rangle_1 |0_A\rangle_2 |0_B\rangle_2\} \\ &+ \sin\theta \mathrm{e}^{\mathrm{i}\phi}\{\cos gt_1[\cos(gt_2) |2\rangle |0_A\rangle_1 |0_B\rangle_1 |0_A\rangle_2 |0_B\rangle_2 \\ &-\mathrm{i}\sin(gt_2) |3\rangle |0_A\rangle_1 |0_B\rangle_1 |0_A\rangle_2 |1_B\rangle_2] - \mathrm{i}\sin(gt_1) |3\rangle |0_A\rangle_1 |1_B\rangle_1 |0_A\rangle_2 |0_B\rangle_2\}, \end{aligned} \tag{8}$$

where t_2 is interaction time between the atom and the second cavity. Choose $\theta = \dfrac{\pi}{4}$, $\phi = 0$, $gt_1 = \dfrac{\pi}{4}$ and $gt_2 = \dfrac{\pi}{2}$, then expression (8) will become

$$\begin{aligned} |\Psi\rangle = &-\frac{1}{2}\mathrm{i} |3\rangle [|1_A\rangle_1 |0_B\rangle_1 |0_A\rangle_2 |0_B\rangle_2 + |0_A\rangle_1 |1_B\rangle_1 |0_A\rangle_2 |0_B\rangle_2 \\ &+ |0_A\rangle_1 |0_B\rangle_1 |1_A\rangle_2 |0_B\rangle_2 + |0_A\rangle_1 |0_B\rangle_1 |0_A\rangle_2 |1_B\rangle_2]. \end{aligned} \tag{9}$$

Thus after the atom exits from the second cavity, the four cavity modes are entangled in the W state. From expression (9) we can know that the probability of successfully obtaining W state is 1.

We can generalize the idea to generating multi-mode W states. Sending a V-type three-level atom through n identical two-modes cavities in turn, and choosing appropriately interaction time such that $gt_1 = \dfrac{\pi}{4}$, and $gt_i = \dfrac{\pi}{2}(i = 2, \ldots, n)$ yield $2n$-mode W state

$$\begin{aligned} \left|\psi^{(n)}\right\rangle = &\frac{-1}{N}\mathrm{i} |3\rangle (|1_A\rangle_1 |0_B\rangle_1 |0_A, 0_B\rangle_{2\ldots n} + |0_A\rangle_1 |1_B\rangle_1 |0_A, 0_B\rangle_{2\ldots n} \\ &+ \cdots |0_A, 0_B\rangle_{1\ldots n-1} |1_A\rangle_n |0_B\rangle_n + |0_A, 0_B\rangle_{1\ldots n-1} |0_A\rangle_n |1_B\rangle_n , \end{aligned} \tag{10}$$

where N is a normalization factor, $|0_A, 0_B\rangle_{1\ldots n-1}$ denotes that the first $(n-1)$ identical two-mode cavities are in the vacuum states, and $|0_A, 0_B\rangle_{2\ldots n}$ denotes that all identical two-mode cavities are in the vacuum states except the first one.

We now describe the process of preparing three V-type three-level atoms in entangled W states. We assume that the first atom is initially prepared in the state $\frac{1}{\sqrt{2}}(|1\rangle_1+|2\rangle_1)$. We then send the atom through a two-mode cavity initially in the vacuum states $|0_A\rangle|0_B\rangle$. When the atom exits from the cavity the state of the system is given by

$$\begin{aligned}|\psi_1\rangle = & \frac{1}{\sqrt{2}}[(\cos(g\tau_1)|1\rangle_1|0_A\rangle - \mathrm{i}\sin(gt_1)|3\rangle_1|1_A\rangle)|0_B\rangle \\ & + |0_A\rangle(\cos(gt_1)|2\rangle_1|0_B\rangle - \mathrm{i}\sin(gt_1)|3\rangle_1|1_B\rangle],\end{aligned} \tag{11}$$

where τ_1 is the interaction time. We choose the atomic speed carefully such that $g\tau_1 = \arcsin(\sqrt{2/3})$ is fulfilled. Then we have

$$\begin{aligned}|\psi_1\rangle &= \frac{1}{\sqrt{2}}\left[\left(\sqrt{\frac{1}{3}}|1\rangle_1|0_A\rangle - \mathrm{i}\sqrt{\frac{2}{3}}|3\rangle_1|1_A\rangle\right)|0_B\rangle + |0_A\rangle\left(\sqrt{\frac{1}{3}}|2\rangle_1|0_B\rangle - \mathrm{i}\sqrt{\frac{2}{3}}|3\rangle_1|1_B\rangle\right]\right., \\ &= \frac{1}{\sqrt{2}}\left[\sqrt{\frac{1}{3}}(|1\rangle_1+|2\rangle_1)|0_A\rangle|0_B\rangle - \mathrm{i}\sqrt{\frac{2}{3}}|3\rangle_1|1_A\rangle|0_B\rangle - \mathrm{i}\sqrt{\frac{2}{3}}|3\rangle_1|0_A\rangle|1_B\rangle\right].\end{aligned} \tag{12}$$

Then a second atom, initially prepared in the state $|3\rangle_2$, is sent through the cavity. After the atom emerges from the cavity the state of the system is

$$\begin{aligned}|\psi_2\rangle = & \frac{1}{\sqrt{2}}\left\{\sqrt{\frac{1}{3}}(|1\rangle_1+|2\rangle_1)|3\rangle_2|0_A\rangle|0_B\rangle - \mathrm{i}\sqrt{\frac{2}{3}}|3\rangle_1[\cos(g\tau_2)|3\rangle_2|1_A\rangle|0_B\rangle\right. \\ & -\mathrm{i}\sin(g\tau_2)|1\rangle_2|0_A\rangle|0_B\rangle] - \mathrm{i}\sqrt{\frac{2}{3}}|3\rangle_1[\cos(g\tau_2)|3\rangle_1|0_A\rangle|1_B\rangle \\ & \left.-\mathrm{i}\sin(g\tau_2)|2\rangle_2|0_A\rangle|0_B\rangle]\right\},\end{aligned} \tag{13}$$

with the choice $g\tau_2 = \frac{\pi}{4}$, we have

$$\begin{aligned}|\psi_2\rangle &= \frac{1}{\sqrt{2}}\left\{\sqrt{\frac{1}{3}}(|1\rangle_1+|2\rangle_1)|3\rangle_2|0_A\rangle|0_B\rangle - \mathrm{i}\sqrt{\frac{2}{3}}|3\rangle_1\left[\sqrt{\frac{1}{2}}|3\rangle_2|1_A\rangle|0_B\rangle\right.\right. \\ &\quad \left.-\mathrm{i}\sqrt{\frac{1}{2}}|1\rangle_2|0_A\rangle|0_B\rangle\right] - \mathrm{i}\sqrt{\frac{2}{3}}|3\rangle_1\left[\sqrt{\frac{1}{2}}|3\rangle_2|0_A\rangle|1_B\rangle - \mathrm{i}\sqrt{\frac{1}{2}}|2\rangle_2|0_A\rangle|0_B\rangle\right\} \\ &= \frac{1}{\sqrt{2}}\left\{\sqrt{\frac{1}{3}}(|1\rangle_1+|2\rangle_1)|3\rangle_2|0_A\rangle|0_B\rangle - \sqrt{\frac{2}{3}}|3\rangle_1\left[\sqrt{\frac{1}{2}}(|1\rangle_2+|2\rangle_2)|0_A\rangle|0_B\rangle\right]\right. \\ &\quad \left.-\mathrm{i}\sqrt{\frac{2}{3}}|3\rangle_1\left[\sqrt{\frac{1}{2}}|3\rangle_2|1_A\rangle|0_B\rangle + \sqrt{\frac{1}{2}}|3\rangle_2|0_A\rangle|1_B\rangle\right]\right\}.\end{aligned} \tag{14}$$

We then sent a third atom, initially in the state $|3\rangle_3$, through the cavity. After an interaction time t_3 the whole system is in the state

$$\begin{aligned}|\psi_3\rangle = & \frac{1}{\sqrt{2}}\left\{\sqrt{\frac{1}{3}}(|1\rangle_1+|2\rangle_1)|3\rangle_2|3\rangle_3|0_A\rangle|0_B\rangle - \sqrt{\frac{2}{3}}|3\rangle_1\left[\sqrt{\frac{1}{2}}(|1\rangle_2+|2\rangle_2)|3\rangle_3|0_A\rangle|0_B\rangle\right]\right. \\ & -\mathrm{i}\sqrt{\frac{2}{3}}|3\rangle_1\left[\sqrt{\frac{1}{2}}|3\rangle_2(\cos(g\tau_3)|3\rangle_3|1_A\rangle|0_B\rangle - \mathrm{i}\sin(g\tau_3)|1\rangle_3|0_A\rangle|0_B\rangle)\right. \\ & \left.\left.+\sqrt{\frac{1}{2}}|3\rangle_2(\cos(g\tau_3)|3\rangle_3|0_A\rangle|1_B\rangle - \mathrm{i}\sin(g\tau_3)|2\rangle_3|0_A\rangle|0_B\rangle\right]\right\}.\end{aligned} \tag{15}$$

Choosing $g\tau_3 = \dfrac{\pi}{2}$, we obtain

$$|\psi_3\rangle = \frac{1}{\sqrt{3}}[|\varphi\rangle_1 |3\rangle_2 |3\rangle_3 - |3\rangle_1 |\varphi\rangle_2 |3\rangle_3 - |3\rangle_1 |3\rangle_2 |\varphi\rangle_3] |0_A\rangle |0_B\rangle , \quad (16)$$

where

$$|\varphi\rangle_j = \sqrt{\frac{1}{2}}(|1\rangle_j + |2\rangle_j) \quad (j = 1, 2, 3). \quad (17)$$

In this way the three atoms are prepared in the W state by using the cavity again in the vacuum state.

We note that this method can be generalized to preparing entangled W states for n atoms. Assume that the first atom initially is in state $\dfrac{1}{\sqrt{2}}(|1\rangle_1 + |2\rangle_1)$, the remaining $n-1$ atoms initially are in the state $|g\rangle_j$ $(j = 2, \ldots, n)$, which are sequentially sent through the initially empty two-mode cavity. Choose the interaction time appropriately, then the n atoms will be prepared in the W state.

We now address the experimental feasibility of the scheme. A cavity with a quality factor $Q = 4.2 \times 10^{10}$ is experimentally achievable,[17] In the present case, the cavity field frequency is $\nu = 51\,\mathrm{GHz}$. Thus the lifetime of photon is $T_\mathrm{C} = \dfrac{Q}{2\pi\nu} \simeq 130 \times 10^{-3}\,\mathrm{s}$. The three atomic levels $|1\rangle$, $|2\rangle$ and $|3\rangle$ are Rydberg levels, and the radiative lifetimes are on the order of $T_\mathrm{at} = 30\,\mathrm{ms}$. Set $g = 2\pi \times 25\,\mathrm{kHz}$, then we will have $t_1 = \dfrac{\pi}{4g} = 5\,\mu\mathrm{s}$, $t_2 = \dfrac{\pi}{2g} = 10\,\mu\mathrm{s}$, and the total travelling time for the atom to pass through two identical cavities is about $t = 10 \times 15 = 150\,\mu\mathrm{s}$. All these parameters satisfy the requirements for our scheme.

In conclusion, we have proposed a scheme for generating W states with four cavity modes. It is based on resonant interaction of two two-mode cavities with a V-type three-level atom. In our scheme the cavity modes are initially in vacuum states. Since the atom resonantly interacts separately with two cavity modes, the required interaction time is very short. The scheme can be generalized to preparing $2n$-mode W states and three-atom W states.

References

[1] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661

[2] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881

[3] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895

[4] Greenberger D M, Horne M A and Zeilinger A 1989 *Bell's Theorem, Quantum Theory, and Conceptions of the Universe* (Dordrecht: Kluwer)

[5] Zeilinger A, Horne M A and Greenberger D M 1992 *Proceedings of Squeezed States and Quantum Uncertainty* (NASA Conf. Publ. 3135, 73)

[6] Dür W, Vidal G and Cirac J I 2000 *Phys. Rev.* A **62** 062314

[7] Zheng S B 2002 *Phys. Rev.* A **66** 014103

[8] Sen (De) A, Sen U, Wiesniak M, Kaszlikowski D and Zukowski M 2003 *Phys. Rev.* A **68** 062306

[9] Zheng S B 2004 *Phys. Rev.* A **70** 045804

[10] Guo G P, Li C F, Li J and Guo G C 2002 *Phys. Rev.* A **65** 042102

[11] Guo G C and Zhang Y S 2002 *Phys. Rev.* A **65** 054302

[12] Zheng S B 2005 *J. Opt.* B **7** 10

[13] Yang Z B 2006 *J. Phys.* B **39** 603

[14] Zheng S B 2003 *Chin. Phys.* **12** 977

[15] Wu Y 1996 *Phys. Rev.* A **54** 1586

[16] Wu Y and Yang X X 1997 *Phys. Rev.* A **56** 2443

[17] Kuhr S, Gleyzes S, Guerlin C, Bernu J, Hoff U B, Deléglise S, Osnaghi S, Brune M, Raimond J M and Haroche S quant-ph/0612138