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Generation and concentration of atomic entangled state via adiabatic evolution

Ye Sai-Yun(叶赛云)†

Department of Electronic Science and Applied Physics, Fuzhou University, Fuzhou 350002, *China*

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Based on the idea of adiabatic evolution, we propose two probabilistic but simple schemes for generating maximally entangled states for two distant atoms and concentrating unknown atomic entangled states. Taking advantage of adiabatic passage, the atoms have no probability of being excited and thus the atomic spontaneous emission is suppressed. Furthermore, in the two schemes accurate adjustment of the interaction time is not required.

Keywords: concentration, entangled state, adiabatic evolution
PACC: 4250

Quantum entanglement, one of the intriguing features of quantum mechanics, plays a fundamental role in the demonstration of quantum nonlocality.[1] It is also recognized nowadays as a very useful resource in quantum information and computation, such as secret key distribution,[2] teleportation,[3] and dense coding.[4] Due to its diverse and promising applications, much efforts have been devoted to the generation of entangled states both theoretically and experimentally. For instance, Zheng and Guo have proposed a robust scheme for generating two-atom entangled states, where the cavity is only virtually excited and thus the decoherence time of cavity is greatly prolonged.[5] The scheme has been experimentally demonstrated in microwave cavity QED by a group led by Haroche.[6] In recent years, photon-interference is also utilized for the generation of entangled state for distant atoms.[7] Very recently, Zheng has proposed a novel scheme for engineering multi-atom entanglement via adiabatic passage, in which the atomic spontaneous emission and cavity decay are suppressed due to the absence of the atom and cavity being excited.[8] In the scheme, by taking the advantage of adiabatic passage, accurate adjustment of the atom–cavity interaction time is not required.

Furthermore, in practice, the maximally entangled state is often used as the quantum resource for perfect quantum information processing. For example, to obtain the optimal fidelity, maximally entangled states are the preferred resources in quantum teleportation. Generally, the carriers of entangled states are shared by distant parties. However, the maximally entangled states are very frangible due to the existence of decoherence during the storage and transmission of qubits. Thus, accompanying with the intensive interest in the generation of maximally entangled states, much attention has been directed to purification processes. Aiming to obtain a high degree of entanglement, now there are two methods to get maximally entangled states from nonmaximally entangled states, which are termed as entangled concentration[9] for the pure nonmaximally entangled state case, and entanglement purification or distillation[10] for the mixed entangled state case. Here, we only focus on the case of entanglement concentration. Bose *et al*[11] and Yang *et al*[12] have proposed schemes for concentration of photonic and atomic entangled states by entanglement swapping, respectively. Recently, Cao *et al* have proposed a probabilistic scheme for entanglement concentration of unknown atomic entangled states via cavity decay,[13] in which the atoms and photons are used as stationary and flying qubits, respectively.

In this paper, we first propose a simple scheme for generating two-atom maximally entangled state via adiabatic passage. Then employing the idea of adiabatic evolution, we propose another distinct scheme for concentrating unknown atomic entangled states. To the best of our knowledge, this is the first proposal for entanglement concentration by using adiabatic passage, in which the evolution is very different from the normal dynamical evolution. Taking the advantage of adiabatic evolution, the two schemes might

†Corresponding author. E-mail: yesaiyun@fzu.edu.cn

be robust against certain fluctuations of experimental parameters.[14,15]

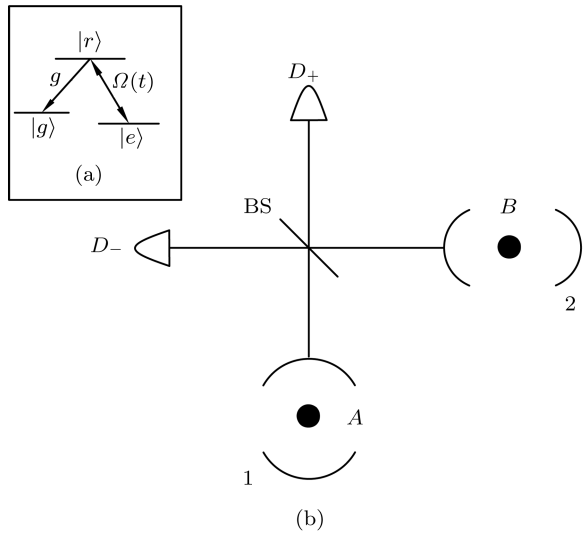


Fig.1. (a) The level configuration of each atom. The transition $|g\rangle \rightarrow |r\rangle$ is resonant with a quantized cavity field with coupling strength g, while the transition $|e\rangle \rightarrow |r\rangle$ is driven by a resonant classical laser field with a time-dependent coupling coefficient Ω. (b) The experimental setup for the generation of maximally entangled state of distant atoms A and B. The atoms A and B are separately trapped in leaking cavities 1 and 2. The photons leaking from the cavities interfere in a 50/50 beam splitter and then are detected by the detectors $D+$ and $D-$.

Consider a Λ-type three-level atom trapped in a single-mode optical cavity. As shown in insert (a) of Fig.1, the atomic energy levels are denoted by $\mid r\rangle$ for the excited state, $\mid e\rangle$ and $\mid g\rangle$ for the two ground states. The $\mid e\rangle \rightarrow\mid r\rangle$ and $\mid g\rangle \rightarrow\mid r\rangle$ transitions are resonantly interacting with a classical laser field and a quantized cavity field, respectively. In the interaction picture, the effective Hamiltonian of the atom–cavity system is

$$H_{\text{eff}} = ga \mid r\rangle\langle g \mid + \Omega(t) \mid r\rangle\langle e \mid + \text{H.c.} \tag{1}$$

Here, a^+ and a are the creation and annihilation operators for the cavity mode, g is the coupling constant for the $\mid g\rangle \rightarrow\mid r\rangle$ transition under the condition that the atom is trapped at a specific position in the cavity, while $\Omega(t)$ is the coupling coefficient for the $\mid e\rangle \rightarrow\mid r\rangle$ transition and is time-dependent. The Hamiltonian has an adiabatic eigenstate with zero eigenvalue, which is given by

$$\mid \Phi(t)\rangle = \alpha \mid e\rangle \mid 0\rangle - \beta \mid g\rangle \mid 1\rangle, \tag{2}$$

where

$$\alpha = \frac{g}{\sqrt{g^2 + \Omega^2(t)}}, \tag{3}$$

and

$$\beta = \frac{\Omega(t)}{\sqrt{g^2 + \Omega^2(t)}}. \tag{4}$$

We note that the dark state is decoupled from the excited state $\mid r\rangle$ and thus the atomic spontaneous emission is suppressed.

In the first part of this paper, we consider two identical atoms A and B separately trapped in two distant single-mode cavities 1 and 2 as shown in Fig.1. In each cavity, the atom and fields have the interaction described in Eq.(1). Then the photons leaking from the respective cavities interfere in a beam splitter, followed by the detection by two single-photon detectors D_+ and D_-. Suppose that initially the atoms are in the state $\mid e\rangle$ and the cavity fields are in vacuum state. In each cavity, we first choose $g \gg \Omega(0)$ to prepare atom–cavity system in the dark state $\mid D(0)\rangle = \mid e\rangle \mid 0\rangle$. Then we slowly increase the Rabi frequency $\Omega(t)$ in each cavity. Under the adiabatic condition, the time evolution of such system is given by Eq.(2). Here, we further assume that the atom–cavity coupling constant g is much larger than the cavity decay rate κ and thus the cavity decay can be ignored during the atom–cavity interaction stage. At the time T, we assume $\Omega(T) = g$. This leads the total atom–cavity system to the state

$$\begin{aligned} \mid \Psi(T)\rangle = \; & \frac{1}{2}(\mid e\rangle_A \mid 0\rangle_1 - \mid g\rangle_A \mid 1\rangle_1)(\mid e\rangle_B \mid 0\rangle_2 \\ & - \mid g\rangle_B \mid 1\rangle_2). \end{aligned} \tag{5}$$

Now we wait for the detectors to click. Due to the cavity decay, during the waiting-for-detection stage, the atom–cavity system is governed by the non-Hermitian Hamiltonian $\sum_{j=1,2} -\mathrm{i}\kappa a^+ a$.[16] Thus the state of atom–cavity system evolves into

$$\mid \Psi(t_j \leq t_{\text{D}})\rangle = \frac{1}{1 + \mathrm{e}^{-2\kappa t_j}} \left(\mid e\rangle_A \mid 0\rangle_1 - \mathrm{e}^{-\kappa t_j} \mid g\rangle_A \mid 1\rangle_1 \right) \left(\mid e\rangle_B \mid 0\rangle_2 - \mathrm{e}^{-\kappa t_j} \mid g\rangle_B \mid 1\rangle_2 \right), \tag{6}$$

where t_j is the waiting-for-detection time and $t_{\rm D}$ is the detection time for that one of the detectors clicks. If there is a click at one of the photon-detectors, the corresponding coherent evolution is interrupted by a quantum jump, which can be formulated as $(a_A \pm a_B)/\sqrt{2}$ for the detector $D_\pm$. Once $D_\pm$ clicks, the total system will collapse onto

$$\begin{aligned} |\Psi(t_{\rm D})^{\pm}\rangle = & \frac{1}{\sqrt{2\left(1+{\rm e}^{-2\kappa t_{\rm D}}\right)}}\Big[(|g\rangle_A|e\rangle_B \pm |e\rangle_A|g\rangle_B)\otimes|0\rangle_1|0\rangle_2 \\ & -{\rm e}^{-\kappa t_{\rm D}}|g\rangle_A|g\rangle_B\Big(|0\rangle_1|1\rangle_2 \pm |1\rangle_1|0\rangle_2\Big)\Big]. \end{aligned} \tag{7}$$

Here, we have discarded the common factor -1. Now we wait for another time $t_{\rm w}$. Assume that there is no photon decaying from cavities during the waiting stage, the state of the total system evolves into

$$\begin{aligned} |\Psi'(t_{\rm D})^{\pm}\rangle = & \frac{1}{\sqrt{2\left(1+{\rm e}^{-2\kappa(t_{\rm D}+t_{\rm w})}\right)}}\Big[\Big(|g\rangle_A|e\rangle_B \pm |e\rangle_A|g\rangle_B\Big)\otimes|0\rangle_1|0\rangle_2 \\ & -{\rm e}^{-\kappa t_{\rm D}}{\rm e}^{-\kappa t_{\rm w}}|g\rangle_A|g\rangle_B\Big(|0\rangle_1|1\rangle_2 \pm |1\rangle_1|0\rangle_2\Big)\Big] \end{aligned} \tag{8}$$

If $t_{\rm w}$ is sufficiently long and thus satisfies ${\rm e}^{-\kappa(t_{\rm D}+t_{\rm w})} \ll 1$, the state of atoms A and B approximates to

$$|\phi\rangle = \frac{1}{\sqrt{2}}(|g\rangle_A|e\rangle_B \pm |e\rangle_A|g\rangle_B), \tag{9}$$

which is a maximally entangled state we want. For ideal single-photon detection, the success probability is $P_{\rm suc} = 1/2$. However, due to the imperfection of single-photon detectors, there are two unwanted cases: (1) no photon being detected while actually one photon leaks from cavity; (2) two photons leaking from cavities but only one of them being detected. Therefore, taking the two cases into consideration and assuming the efficiency of single-photon detector being η, the success probability is $P'_{\rm suc}(\eta) = \eta P_{\rm suc} = \eta/2$ and fidelity is $F = 1/(2-\eta)$.

In the second part of this paper, we focus on the concentration of unknown entangled atomic states via adiabatic passage. Assume that two pairs of nonmaximally entangled atomic states

$$|\psi\rangle_{A'C'} = a|e\rangle_{A'}|g\rangle_{C'} + b|g\rangle_{A'}|e\rangle_{C'} \tag{10}$$

and

$$|\psi\rangle_{B'D'} = a|e\rangle_{B'}|g\rangle_{D'} + b|g\rangle_{B'}|e\rangle_{D'} \tag{11}$$

are shared by three distant parties, where a and b are unknown normalization coefficients with $|a^2| + |b^2| = 1$. As shown in Fig.2, atoms A' and B', cavities $1'$ and $2'$ and detectors $D'_\pm$ are possessed by Alice, while atoms C' and D' are possessed by Bob and Charlie, respectively. The four atoms have the same energy level structure as shown in insert (a) of Fig.1. Therefore,

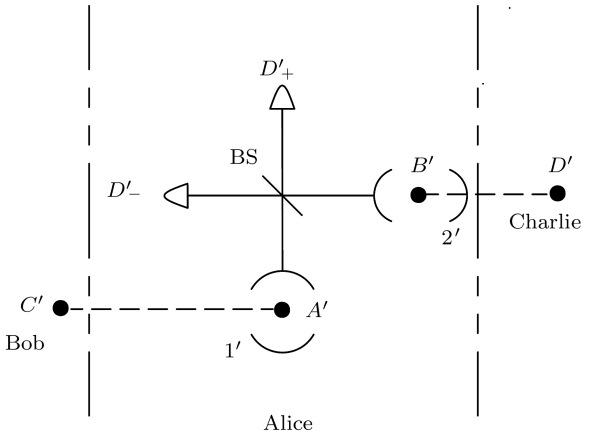


Fig.2. The experimental setup for concentration unknown entangled atomic states. Atoms A' and B' are possessed by Alice, while atoms C' and D' are separately possessed by Bob and Charlie.

the effective Hamiltonian in the interaction picture is in the same formulation as Eq.(1). Now we switch on the effective Hamilton in cavities 1 and 2 simultaneously. First, we also assume $\Omega(0) \ll g$. Then we slowly increase $\Omega(t)$ and thus the system undergoes the adiabatic evolution given by Eq.(2). At the time T', we assume that $\Omega(T') \gg g$. Thus, the two atom–cavity systems evolve into the states

$$|\,\psi(T')\rangle_{A'C'} = (-a\,|\,g\rangle_{C'}\,|\,1\rangle_{1'} + b\,|\,e\rangle_{C'}\,|\,0\rangle_{1'})\,|\,g\rangle_{A'}, \tag{12}$$

and

$$|\,\psi(T')\rangle_{B'D'} = (-a\,|\,g\rangle_{D'}\,|\,1\rangle_{2'} + b\,|\,e\rangle_{D'}\,|\,0\rangle_{2'})\,|\,g\rangle_{B'}, \tag{13}$$

During the waiting stage, the total atom–cavity system is in the state

$$\begin{aligned}|\,\psi\left(t'_j \le t'_{\rm D}\right)\rangle_{\rm total} = \; & \frac{1}{|a|^2\,{\rm e}^{-2\kappa t'_j} + |b|^2}\left(-a{\rm e}^{-\kappa t'_j}\,|\,g\rangle_{C'}\,|\,1\rangle_{1'} + b\,|\,e\rangle_{C'}\,|\,0\rangle_{1'}\right) \\ & \times |\,g\rangle_{A'} \otimes \left(-a{\rm e}^{-\kappa t'_j}\,|\,g\rangle_{D'}\,|\,1\rangle_{2'} + b\,|\,e\rangle_{D'}\,|\,0\rangle_{2'}\right)|\,g\rangle_{B'},\end{aligned} \tag{14}$$

where t'_j is the waiting-for-detection time and $t'_{\rm D}$ is the time for only one photon being detected. If $D'_{\pm}$ clicks at the time $t'_{\rm D}$, the atom–cavity system will collapse onto the state

$$\begin{aligned}& |\,\psi(t'_{\rm D})^{\pm}\rangle_{\rm total} \\ = \; & \frac{a^2{\rm e}^{-\kappa t'_{\rm D}}\,|\,g\rangle_{C'}\,|\,g\rangle_{D'}\,(|\,0\rangle_{1'}\,|\,1\rangle_{2'} \pm |\,1\rangle_{1'}\,|\,0\rangle_{2'}) - ab\,(|\,g\rangle_{C'}\,|\,e\rangle_{D'} \pm |\,e\rangle_{C'}\,|\,g\rangle_{D'})\,|\,0\rangle_{1'}\,|\,0\rangle_{2'}}{\sqrt{2\left(|a|^4\,{\rm e}^{-2\kappa t'_{\rm D}} + |ab|^2\right)}}.\end{aligned} \tag{15}$$

Next we wait for another time $t'_{\rm w}$. If there is no photon decaying from two cavities during the waiting stage, the total system will evolve into the state

$$\begin{aligned}& |\,\psi'(t'_{\rm D} + t'_{\rm w})^{\pm}\rangle_{\rm total} \\ = \; & \frac{a^2{\rm e}^{-\kappa\left(t'_{\rm D}+t'_{\rm w}\right)}\,|\,g\rangle_{C'}\,|\,g\rangle_{D'}\,(|\,0\rangle_{1'}\,|\,1\rangle_{2'} \pm |\,1\rangle_{1'}\,|\,0\rangle_{2'}) - ab\,(|\,g\rangle_{C'}\,|\,e\rangle_{D'} \pm |\,e\rangle_{C'}\,|\,g\rangle_{D'})\,|\,0\rangle_{1'}\,|\,0\rangle_{2'}}{\sqrt{2\left(|a|^4\,{\rm e}^{-2\kappa\left(t'_{\rm D}+t'_{\rm w}\right)} + |ab|^2\right)}}.\end{aligned} \tag{16}$$

In the case that $t'_{\rm w}$ is sufficiently long and thus ${\rm e}^{-\kappa\left(t'_{\rm D}+t'_{\rm w}\right)} \ll 1$, the atoms C' and D' approximate the maximally entangled state

$$\begin{aligned}& |\,\varphi(t'_{\rm D} + t'_w)\rangle \\ = \; & \frac{1}{\sqrt{2}}\,(|\,g\rangle_{C'}\,|\,e\rangle_{D'} \pm |\,e\rangle_{C'}\,|\,g\rangle_{D'})\,,\end{aligned} \tag{17}$$

leaving cavities $1'$ and $2'$ in vacuum states. The discussion of fidelity and success probability for the concentration scheme is not analysed here, as it is similar to the case of the first scheme for generation of maximally entangled state. We note that accurate control of atom-field interaction time is not needed in the two present schemes as long as the adiabatic condition is satisfied.

In summary, by taking the adiabatic passage, we have proposed two probabilistic but simple schemes: (1) one for generating two-atom maximally entangled states for atoms trapped in two separated cavities; (2) the other for concentrating unknown entangled atomic states. In both schemes, the atoms have no probability of being excited and thus the atomic spontaneous emission is suppressed. Furthermore, the accurate adjustment of atom and classical field interaction time is not required due to the adiabatic evolution.

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